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Analytical and graphical modelling of the cutting scheme when generating with a pinion cutter

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Abstract. The teeth generation in the case of the cylindrical gears with involute profile is frequently realized with tools of pinion cutter type, on specialized slotting machine tools. In general, during gear teeth machining, several teeth of the tool are concomitantly in contact with the worked part, each one detaching a separate chip. The process is characterized by important unevenness of these chips cumulated area, depending on the relative position between the pinion cutter and the worked part. This transforms into cutting force and cutting torque unevenness and negatively influences both machine tool operating and tool life. The paper addresses the problem of smoothing the detached chips cumulated area at the consecutive strokes of the tool, along the rolling motion between it and the worked part, in order to obtain a main cutting torque as uniform as possible. In this purpose, the analytical modelling of the tool/ worked part contact is realized at first, in the cases of generating both exterior and interior gears with involute straight teeth. Then, because the analytical solution is very laborious, a modelling algorithm is developed in one of the most performant graphical environments, which is CATIA. By implementing the graphical solution, the variation of detached chips cumulated area can be determined and the required variation law of the rolling feed, leading to process smoothing, can also be found. A numerical approached case study, concerning the teeth machining for an exterior gear is also presented.

1. Introduction

The cutting schemes modelling when generating with tools having multiple active cutting edges – the case of the tools for machining gears teeth – and the conditions in which this can be modified in order to improve the toothing process have always been a subject of constant interest for researchers working in cutting domain [1-3].

Diverse solutions laying on the general theory of the enwrapped surfaces have already been presented [4]. They consist in dedicated algorithms, developed in the case of generating by rolling method, with tools as the rack tool, the pinion cutter or the mill hob.

Constructive modifications of the cutting tools were imagined, without disclaiming the fundamental principles of generating the surfaces by enwrapping. In particular, regarding the gears with involute profile of the teeth, improvements of the specific cutting scheme aiming to reduce the mill hob wear [1] and to smooth the toothing process [3, 5] were proposed.

The problem was approached on the base of the fundamental principles surfaces enwrapping, either by analytical developments, or by graphical modelling in AutoCAD. Specific, dedicated soft products were realized in these purposes [5].

The graphical design evolution and the availability of remarkable versatile environments, such as CATIA, are enabling a new approach in the graphical analysis of the toothing cutting schemes – in the case addressed by this paper, the cutting scheme when generating with a pinion cutter.

In this paper, the analysis objective was changed, meaning that the modification of teeth generating process is aimed by varying the feed in the rolling motion, without altering the specific mechanism of rolling between the centrodes attached to both tool and worked part. The followed target is to obtain a more uniform ongoing of the toothing process, by ensuring a very limited variation of the cumulated area of the concomitantly detached chips. The ultimate goals were the machining process improvement from energetic point of view (meaning to reach a uniform cutting torque) and the diminishing of cutting tool wear, both in conditions of controlled geometric roughness of teeth flanks surfaces. The gears teeth machining by hobbing [6] with a pinion cutter is further addressed.

In what concerns paper structure, the next section is dedicated to present the analytical model of the cutting scheme. The third section describes the specific kinematics of the generating process. The fourth section presents the modelling of detached chips area, while the last section is for conclusion.

2. The analytical model of the cutting scheme

An analytical model for the generating of straight teeth having involute profile with a pinion cutter is below presented. Due to problem specific, a plain model is the most suitable choice. The model addresses both cases of exterior and interior teeth. Related to this, the figure 1 shows the rolling centrodes: C (tool), C_1 (gear case) and C_2 (gear rim case). They are all circles, having the radii R_{rt}, R_{rp1} and R_{rp2}, respectively.
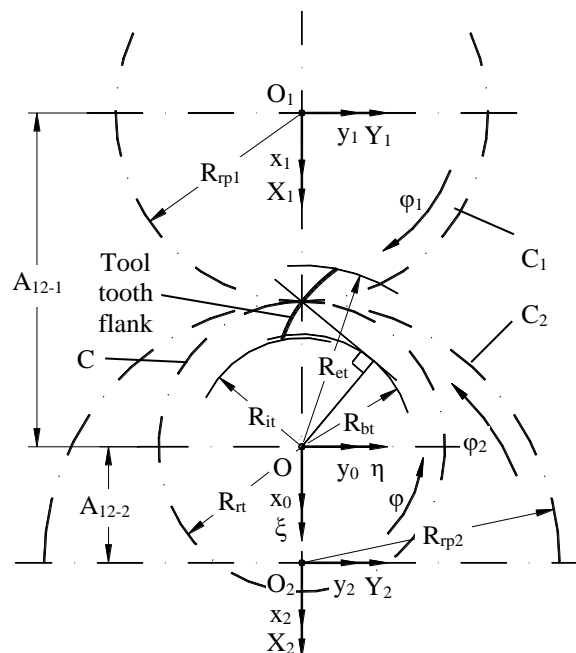
Figure 1. The rolling centrodes and the reference systems.

Several reference systems had to be defined in order to obtain the analytical model of the cutting scheme, namely (see also figure 1):
- x_0y_0, which is a global system having the origin O into the centre of tool centrode;
- x_1y_1 – global system having the origin O_1 into the centre of machined gear centrode;
- x_2y_2 – global system having the origin O_2 into the centre of machined gear rim centrode;
- $\xi\eta$ – local, mobile system, initially overlapped to x_0y_0 and attached to the tool, hence having rotation motion of φ angular parameter;

- X_1Y_1 – local, mobile system, initially overlapped to x_1y_1 and attached to the machined gear, hence having rotation motion of φ_1 angular parameter;
- X_2Y_2 – local, mobile system, initially overlapped to x_1y_1 and attached to the machined gear, hence having rotation motion of φ_1 angular parameter.

The relative position between the global systems of the rolling centrodes is defined by the distance between their centres, which, depending on the addressed case, is calculated as equation (1) or (2):

$$A_{12-1} = R_{rp1} + R_{rt} \text{ (gear case)} \tag{1}$$

or
$$A_{12-2} = R_{rp2} - R_{rt} \text{ (gear rim case).} \tag{2}$$

In what concerns the pinion gear model, R_{bt}, R_{it} and R_{et} mean the radii of base, foot and head circles, respectively.

According to the introduced notations, the rolling condition has the equation (3) or (4):

$$R_{rt} \cdot \varphi = R_{rp1} \cdot \varphi_1 \text{ (gear case),} \tag{3}$$

or
$$R_{rt} \cdot \varphi = R_{rp2} \cdot \varphi_2 \text{ (gear rim case).} \tag{4}$$

3. The kinematics of the generating process
The two generating cases (gear and gear rim) are successively addressed. In both cases, the machined straight teeth are considered as having involute profile with 0 displacements.

3.1. Gear case
The equations (5) and (6) of the absolute motions are:

$$x_0 = \omega_3{}^T(\varphi) \cdot \xi \text{ (pinion cutter)} \tag{5}$$

and
$$x_1 = \omega_3{}^T(-\varphi_1) \cdot X_1 \text{ (gear).} \tag{6}$$

In the equations from above, ω_3 is the well known matrix of coordinates transform, associated to the rotation around z-axis (here, around system origin).

The relative position between x_0y_0 and x_1y_1 systems is described by the equation (7):

$$x_0 = x_1 + \begin{pmatrix} -(A_{12-1e} - k \cdot \varphi_1) \\ 0 \end{pmatrix} \tag{7}$$

with
$$A_{12-1e} = m(z_p + z_t)/2 + 2.25 \cdot m. \tag{8}$$

In equations (7) and (8), k is a constant defining the speed of tool radial feed, m – the gear module, z_p and z_t – the gear and tool teeth numbers, respectively. If the cutting scheme modelling is intended without considering the tool radial feed, then $k = 0$.

The equation of tool motion relative to machined gear results from equations $(5 - 7)$ as:

$$X_1 = \omega_3(-\varphi_1)\left[\omega_3{}^T(\varphi) \cdot \xi + \begin{pmatrix} A_{12-1e} - k \cdot \varphi_1 \\ 0 \end{pmatrix} \right] \tag{9}$$

the relation between φ and φ_1 being the one stated in (3).

If in equation (9) ξ means the profile of one among tool teeth (composed by its flanks joined by the head arc), then, after developing, the relation will give the position of respective tool tooth relative to the machined gear. Furthermore, if appropriate, discrete values are given to φ (hence to φ_1 also), then the tool tooth positions corresponding to the successive tool strokes will result. These values are in connection with the circular feed speed (be it constant or variable), which, at its turn, depends on the number of double strokes made by the pinion gear per minute, n_{cd}.

Usually, two cutting regime parameters need to be set for machining gears onto slotting machines: the number of double strokes made by the ram slide per minute, n_{cd} [dbl. strokes/min] and the circular feed, s_c [mm/dbl. stroke]. Starting from here, the value of the angle φ_1 with which the worked gear rotates around its axis in a minute can be calculated with equation (10):

$$\varphi_1 = s_c \cdot n_{cd} / R_{rp1}. \tag{10}$$

As consequence, the increment $\Delta\varphi_1$ for successively increasing the angle φ_1 is, equation (11):

$$\Delta\varphi_1 = \varphi_1 / n_{cd} = s_c / R_{rp1}. \tag{11}$$

Note: The recommended radial feed s_r of the pinion gear depends on machine tool type, for example at 5A12 slotting machine [7], equation (12):

$$s_r = \left(85° \cdot 2.25 \cdot m \cdot s_c\right) / \left(90° \cdot m \cdot z_p\right) \tag{12}$$

where 90° is the rotation angle of the cam for radial feed motion. The rotation angle of worked gear corresponding to the completion of entire radial motion of the tool, φ_{1p} is calculated with, equation (13):

$$\varphi_{1p} = i \cdot 90° \tag{13}$$

(at 5A12 slotting machine, $i = 4/3$). Finally, the value of k constant from (7) results as equation (14):

$$k = 2.25 \cdot m / \left(\pi \cdot 120/180\right) \text{ [mm/dbl. stroke]} \tag{14}$$

3.2. Gear rim case
In this case, the equation of gear rim absolute motion is, equation (15):

$$x_2 = \omega_3{}^T(\varphi_2) \cdot X_2. \tag{15}$$

The relative position between x_0y_0 and x_2y_2 systems is described by the equation (16):

$$x_0 = x_2 + \begin{pmatrix} A_{12-2e} + k \cdot \varphi_2 \\ 0 \end{pmatrix} \tag{16}$$

with

$$A_{12-2e} = m\left(z_p - z_t\right)/2 - 2.25 \cdot m \tag{17}$$

The equation of tool motion relative to machined rim gear (equation (18) results from equations (5), (15) and (16):

$$X_2 = \omega_3(\varphi_2)\left[\omega_3{}^T(\varphi) \cdot \xi - \begin{pmatrix} A_{12-2e} + k \cdot \varphi_2 \\ 0 \end{pmatrix}\right] \tag{18}$$

the relation between φ and φ_2 being the one stated in (4).

If in relation (18) ξ means the profile of one among tool teeth (composed by its flanks joined by the head arc), then, after developing, the relation will give the position of respective tool tooth relative to the machined rim gear.

The increment $\Delta\varphi_2$ for successively increasing the angle φ_2 is, in this case equation (19):

$$\Delta\varphi_2 = \varphi_2 / n_{cd} = s_c / R_{rp2}.\tag{19}$$

The discussion from above referring to the calculus of k constant value remains valid.

4. The modelling of detached chips area

The figure 2 illustrates the principle of analytically calculating the cumulated area of the simultaneously detached chips, depending on the current number of the cutting tool dbl. stroke, in the case of gear teeth generating. The case of gear rim teeth generating can be similarly addressed.
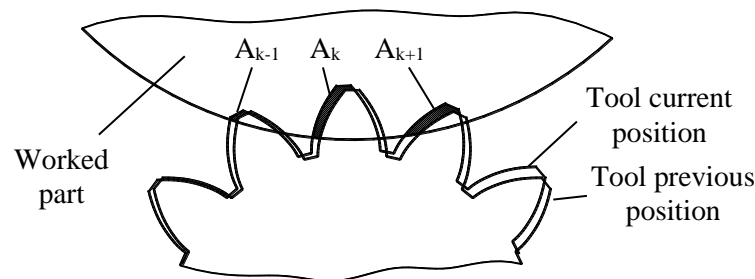
Figure 2. The chips detached by tool successive teeth.

If two successive positions of the tool model relative to the worked part model are considered, then the shape and the area of the chip detached by each tool tooth (in figure 2 – A_{k-1}, A_k, A_{k+1}) can be found. The cumulated area of simultaneously detached chips, A_Σ results by summing the areas of the chips detached by the N tool teeth in contact with the worked part at the current moment, equation (20):

$$A_\Sigma = \sum_{i=1}^{N} A_{k_i}.\tag{20}$$

The analytical approach is rigorous. However, the development of numerical applications dedicated to evaluate the shapes and the areas of tool teeth cuts in the topological domain representing the worked part is complex and highly laborious. For this reason, we have imagined a graphical solution for modelling the detached chips area, developed in CATIA environment and introduced below.

Some specific steps have to be made in order to simulate in CATIA the generating of exterior teeth having involute profile. For an easier understanding, we further address the actual case of machining a gear with $z_p = 30$teeth of module $m = 10$mm, with a slotting tool having $z_t = 20$teeth.

• The tool tooth profile is determined. For that, the coordinates of a certain number of points defining the profile of one tool tooth flank (e.g. left flank) are calculated with involute equations, at first, for example by using a dedicated MatLab application. Then, the resulted points are joined into a spline curve, which is filled to the foot circle of tool model. The second flank (right flank) of the tool tooth results by mirroring the first one against the tooth symmetry axis.

• The tool profile results by multiplying the tooth profile around tool axis, in accordance to tool number of teeth, and by joining the resulted items with the help of "*Wireframe and Surface Design*" CATIA module.

• The tool model is generated by extruding the tool profile with "*Pad*" command. The other tool model dimensions have been set according to the standards.

- The worked part model is generated as cylindrical plate, having the radius $R_{ep1} = m(z_p + 2)/2$, hence R_{ep1} = 160mm.

- The tool / worked part relative position from the beginning of the toothing process is created. In this purpose, the tool model is rotated until the overlapping between the axis of a tooth and the line joining tool and part centres is reached. Finally, the two models are brought into contact (figure 3).

- The motions executed by both tool and worked part during the toothing process are simulated in discrete manner. Thus, the angular increment of part rotation was set as $\Delta\varphi_1 = 1°$. Corresponding to it, due to rolling condition (3), the angular increment of tool rotation is $\Delta\varphi = 1.5°$. At the same time, the distance between tool and part axis is diminished with 0.25mm for each successive rotation (corresponding to tool radial feed), until it reaches the final value A_{12-1} = 250mm.

- For each successive relative position between tool and worked part, their models are intersected and, with "*Remove*" command, the common surface (meaning the cumulated area of detached chips) is cut out from part model. Its area is calculated as difference between part areas before and after the common surface cutting out (figure 4).
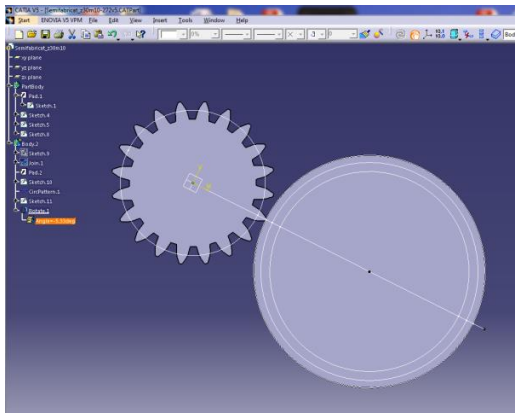
**Figure 3.** Tool & part relative positioning.
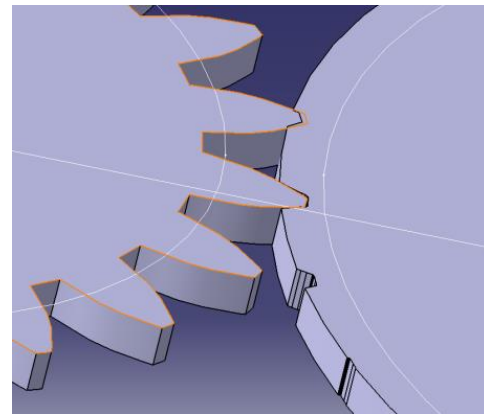
Figure 4. Finding of removed chips area.

Table 1. Numerical results (excerpt).

Crt. no.	φ_1 [°]	φ [°]	A_{12-1} [mm]	A_Σ [mm²]	Crt. no.	φ_1 [°]	φ [°]	A_{12-1} [mm]	A_Σ [mm²]
0	0	0	272.5	0	90	90	-135	250	30.523
1	1	-1.5	272.25	0.905	91	91	-136.5	250	36.033
2	2	-3	272	0.541	92	92	-138	250	37.538
3	3	-4.5	271.75	0.102	93	93	-139.5	250	36.540
4	4	-6	271.5	0.000	94	94	-141	250	34.586
5	5	-7.5	271.25	0.000		. . . .			
6	6	-9	271	0.000	111	111	-166.5	250	22.464
7	7	-10.5	270.75	0.085	112	112	-168	250	20.263
8	8	-12	270.5	2.764	113	113	-169.5	250	18.090
9	9	-13.5	270.25	3.960	114	114	-171	250	25.321
	. . . .				115	115	-172.5	250	36.033
85	85	-127.5	251.25	33.078	116	116	-174	250	37.539
86	86	-129	251	30.597	117	117	-175.5	250	36.539
87	87	-130.5	250.75	27.994	118	118	-177	250	34.586
88	88	-132	250.5	25.633	119	119	-178.5	250	32.177
89	89	-133.5	250.25	23.395	120	120	-180	250	29.580

The numerical results that were obtained are sampled in table 1. By representing the value of detached chips cumulated area against the current number of tool double stroke, the curve from figure 5 was obtained. As it can be noticed, the curve is composed by two regions with distinct aspect. The first region shows a general trend of area increase (due to the radial feed of the tool), over which an oscillating trend periodically overlaps. The second region (beginning with the 90-th tool double-stroke) presents only a periodic oscillation of the area, around a median value which does not change until all gear teeth are generated. However, the oscillations are showing an important variation of chips area, comprised between 18 and 37.5mm^2 – more than from simple to double. This entitles us to consider that a solution for smoothing the variation of detached chips area is necessary if intending to improve the toothing process performance.
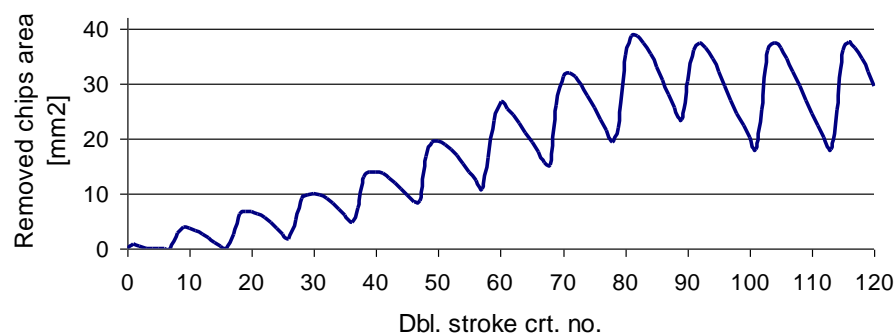
Figure 5. The variation of detached chips cumulated area.

5. Conclusion

The process of teeth machining with the pinion cutter tool is characterized by important unevenness of the detached chips cumulated area, depending on the relative position between the tool and the worked part. The aim of this paper was to create the support for implementing a new type of solution leading to the smoothening of detached chips cumulated area, namely the gears machining with variable rolling (circular) feed. In this purpose, an analytical model for the generating of straight teeth having involute profile with a pinion cutter has been developed. The model covers the both cases of gears and gear rims machining. The equations describing the generating process kinematics have been deducted, also in both the cases. An algorithm for the graphical modelling of the detached chips area, in CATIA environment, has been imagined, at first, and then applied in the actual case of machining a gear with $z_p = 30$teeth of module $m = 10$mm, with a slotting tool having $z_t = 20$teeth. Application results sustain the necessity of toothing process smoothening and, at the same time, enable the finding of the variation law for the circular feed that can lead to this smoothening.

6. References

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