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Comparison of current stiffness modulus values in national and international literature

V-I Iftode[1], I Olteanu-Dontov[1], P Mihai[2], M Budescu[1], I Sococol[2], I-D Dinu[1]
[1] "Gheorghe Asachi" Technical University of Jassy, Faculty of Civil Engineering and Building Services, Department of Structural Mechanics, Str. Prof. Dr. Doc.Dimitrie Mangeron, nr. 1, 700050, Romania,
[2] "Gheorghe Asachi" Technical University of Jassy, Faculty of Civil Engineering and Building Services, Department of Concrete Structures, Building Materials, Technology and Management, Str. Prof. Dr. Doc.Dimitrie Mangeron, nr. 1, 700050, Romania

E-mail: iftodevlad@yahoo.com

Abstract. The paper objective is to realize a comparative synthesis of the current coefficients used to reduce the stiffness modulus. For this, national and international codes were considered as well as reference papers on the subject. In the end, the paper presents a case study in which behaviour differences were observed for a reinforced concrete frame structure between national and international codes.

1. Introduction
Most of reinforced concrete structure are statically indeterminate structures, so the stiffness of the structural elements influences not only the displacements, but also the internal forces distribution in the elements.
The structural walls, columns and beams respond in working state II, cracked state. For this reason, in the structural computation is mandatory to consider the stiffness corresponding to working state II, the cracked one.
Admissible values are obtained for the displacement computation of a structure from the point of view of the real behaviour, if approximate values of the stiffness are used. These approximate values are computed by multiplying the stiffness modulus of the initial cross section with subunitary factors.
The subunitary factors recommended in national and international codes, reduce in fact the elasticity modulus or the geometric characteristics of the section.

2. Research methodology

2.1. National codes
In Romania, according to the design code for reinforced concrete wall structures, CR 2-1.1/2013, the stiffness values for reinforced concrete structural walls, coupled or not, are given by several computation hypothesis, presented below:

If
$$\upsilon_d = \frac{N_{Ed}}{A_c \cdot f_{cd}} = 0,4: \tag{1}$$

$$I_{eq} = 0,80 \cdot I_c \tag{2}$$

$$A_{eq} = 0,90 \cdot A_c \tag{3}$$

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$$A_{eq,s} = 0{,}80 \cdot A_{c,s} \tag{4}$$

If
$$\upsilon_d = \frac{N_{Ed}}{A_c \cdot f_{cd}} = 0{,}0: \tag{5}$$

$$I_{eq} = 0{,}40 \cdot I_c \tag{6}$$

$$A_{eq} = 0{,}60 \cdot A_c \tag{7}$$

$$A_{eq,s} = 0{,}50 \cdot A_{c,s} \tag{8}$$

If
$$\upsilon_d = \frac{N_{Ed}}{A_c \cdot f_{cd}} = -0{,}2: \tag{9}$$

$$I_{eq} = 0{,}10 \cdot I_c \tag{10}$$

$$A_{eq} = 0{,}40 \cdot A_c \tag{11}$$

$$A_{eq,s} = 0{,}20 \cdot A_{c,s} \tag{12}$$

For intermediate values of υ_d ratio, the calculus values for the inertia moment I_{eq}, the area of the cross section A_{eq} and the area of the shearing section $A_{eq,s}$ are established through linear interpolation. I_c, A_c and $A_{c,s}$ values correspond to the section without cracks.

N_{Ed} and f_{cd} represents the design value for the axial force (positive for compression) in the section and the concrete compression strength value respectively.

The characteristics for the coupling beams are computed based on the reinforcement type:

- orthogonal bars reinforcing (longitudinal bars and stirrups):

$$I_{eq} = 0{,}20 \cdot I_c \tag{13}$$

$$A_{eq} = 0{,}20 \cdot A_c \tag{14}$$

- diagonally reinforced:

$$I_{eq} = 0{,}40 \cdot I_c \tag{15}$$

$$A_{eq} = 0{,}40 \cdot A_c \tag{16}$$

According to the Seismic design code, part I, P100-1/2013, the subunitary reduction factor of the stiffness modulus depends on the selected structural type (frame or structural walls). The recommended values for reinforced frame structures are given in Table 2.1. and take into consideration the connection types between non-structural components and the structural system.

- E_c - modulus of elasticity for the concrete
- I_g - inertia moment of the initial concrete section (without cracks).

Table 1. Design values for the stiffness modulus according P100-1/2013.

Structural type	Connection type between the non-structural component and the reinforced concrete structure	
	Non-structural elements influence the overall stiffness of the structure	Non-structural elements do not interact with the structure
Reinforced concrete structures		
Frames structures	$E_c I_g$	$0{,}50 \cdot E_c I_g$
Structural walls	$E_c I_g$	

2.2. International codes

The New Zealand standard, NZS 3101-1/2006, provides a computation equation (17), which takes into account the modulus of elasticity and the moment of inertia of the concrete section without cracks in relation to the normalized axial force. For the accuracy of the calculation, the characteristics of the reinforcement are also considered, equation 18.

$$EI = \frac{\left(E_c I_g / 2{,}5 \right)}{1 + \beta_d} \tag{17}$$

$$EI = \frac{\left(E_c I_g / 5 + E_e I_{se} \right)}{1 + \beta_d} \tag{18}$$

In which I_g is the inertia moment of the initial concrete section (without cracks), I_{se} - moment of inertia for the reinforcement, E_c - elasticity modulus for concrete, E_s - elasticity modulus for steel and β_d - ratio of design axial dead load to total axial design axial load of a column or pier:

$$\beta_d = \frac{N_{Ed}}{A_c \cdot f_{cd}} \tag{19}$$

- f_{cd} - design compression strength for concrete;
- A_c - initial concrete area section;
- N_{Ed} - design value for axial force.

Recommended values of effective section properties that may be used for the elastic analysis of structures subjected to seismic forces corresponding with the ultimate limit state are as shown in columns 2 & 3 of Table C6.6 from standard NZS 3101-2/2006.

Notes from table 2:

- (§) With these values the E value should be the elastic modulus for concrete with strength of 40 MPa regardless of the actual concrete strength.
- (‡) The values in brackets apply to columns which have a high level of protection against plastic hinge formation in the ultimate limit state.
- (¶) For additional flexibility, within joint zones and for conventionally reinforced coupling beams refer to the text.

Table 2. Effective section properties, I_e

Type of member	Ultimate limit state		Serviceability limit state		
	$f_y = 300MPa$	$f_y = 500MPa$	$\mu = 1{,}25$	$\mu = 3{,}00$	$\mu = 6{,}00$
Beams					
Rectangular¶	$0{,}40I_g$ (use with E_{40})§	$0{,}32I_g$ (use with E_{40})§	I_g	$0{,}70I_g$	$0{,}40I_g$ (use with E_{40})§
T or L Beams¶	$0{,}40I_g$ (use with E_{40})§	$0{,}32I_g$ (use with E_{40})§	I_g	$0{,}70I_g$	$0{,}40I_g$ (use with E_{40})§
Columns					
$N^*/A_g f_c' > 0{,}50$	$0{,}80I_g\left(1{,}00I_g\right)^‡$	$0{,}80I_g\left(1{,}00I_g\right)^‡$	I_g	$1{,}00I_g$	As for the ultimate limit state value in brackets
$N^*/A_g f_c' = 0{,}20$	$0{,}55I_g\left(0{,}66I_g\right)^‡$	$0{,}50I_g\left(0{,}66I_g\right)^‡$	I_g	$0{,}80I_g$	
$N^*/A_g f_c' = 0{,}00$	$0{,}40I_g\left(0{,}45I_g\right)^‡$	$0{,}30I_g\left(0{,}35I_g\right)^‡$	I_g	$0{,}70I_g$	
Walls¶					
$N^*/A_g f_c' > 0{,}50$	$0{,}48I_g$	$0{,}40I_g$	I_g	$0{,}70I_g$	As for the ultimate limit state value
$N^*/A_g f_c' = 0{,}20$	$0{,}40I_g$	$0{,}33I_g$	I_g	$0{,}60I_g$	
$N^*/A_g f_c' = 0{,}00$	$0{,}32I_g$	$0{,}25I_g$	I_g	$0{,}70I_g$	
Diagonally reinforced coupling beams	$0{,}60I_g$ – for flexure Shear area, A_{shear}		I_g $1{,}50A_{shear}$ for ULS	$0{,}75I_g$ $1{,}25A_{shear}$ for ULS	As for ultimate limit

In the Japanese code, JSCE-No. 15-2010, a detailed procedure for the reduction of the stiffness modulus assessment is presented.

If the effective bending stiffness is considered as a function of the bending moment, the evaluation relation is:

$$E_e I_e = \left(\frac{M_{crd}}{M_d}\right)^4 \cdot \frac{E_e I_g}{1 - \dfrac{\Delta M_{csg}}{M_d - P\left(d_p - c_g\right)}} + \left[1 - \left(\frac{M_{crd}}{M_d}\right)^4\right] \cdot \frac{E_e I_{cr}}{1 - \dfrac{\Delta M_{csg,r}}{M_d - P\left(d_p - c_g\right)}} \tag{20}$$

If the effective bending stiffness is considered constant on longitudinal direction, the evaluation relation becomes:

$$E_e I_e = \left(\frac{M_{crd}}{M_{d,\max}}\right)^3 \cdot \frac{E_e I_g}{1 - \dfrac{\Delta M_{csg}}{M_{d,\max} - P\left(d_p - c_g\right)}} + \left[1 - \left(\frac{M_{crd}}{M_{d,\max}}\right)^3\right] \cdot \frac{E_e I_{cr}}{1 - \dfrac{\Delta M_{csg,r}}{M_{d,\max} - P\left(d_p - c_g\right)}} \tag{21}$$

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where:
E_e - effective elasticity modulus for concrete and is computed by 22 relation.

$$E_e = \frac{E_{ct}}{1+\varphi} = \frac{E_{ct}}{1+\left(\frac{E_{ct}}{E_c}\right)\cdot\varphi_{28}} \quad (22)$$

The following notations were made: E_{ct} - elasticity modulus for the concrete from permanent loading moment, φ - creep coefficient, φ_{28} - creep coefficient at 28 days, I_g - moment of inertia of gross section, I_{cr} - moment of inertia for the section with cracks, I_e - effective moment of inertia, M_{crd} - critical moment for bending when a crack appears in the transversal section, M_d - design bending moment, P - axial force or the pretension force and ΔM_{csg} - apparent bending moment produces by the contraction in the section without cracks taking into consideration the reinforcement, computed by 23 relation:

$$\Delta M_{csg} = \left(\frac{I'_{sg}}{c_g - d'} - \frac{I_{sg}}{d-c_g} - \frac{I_{pg}}{d_p - c_g}\right)\cdot\varepsilon'_{cs} \quad (23)$$

$\Delta M_{csg,r}$ - apparent bending moment produces by the contraction in the section with cracks taking into consideration the reinforcement, computed by 24 relation:

$$\Delta M_{csg,r} = \left(\frac{I'_{scr}}{c_{sr} - d'} - \frac{I_{scr}}{d-c_{cr}} - \frac{I_{pcr}}{d_p - c_{cr}}\right)\cdot\varepsilon'_{cs} \quad (24)$$

The creed coefficient depends on the concrete grade and the reinforcement percentage from the elements. Its values are given in table 3, when $E_{ct} = E_c$.

Table 3. Values for the creed coefficient

Creed coefficient for concrete					
Exposure conditions	Concrete age for the pretension or loading applications				
	4-7 days	14 days	28 days	3 months	1 year
Exterior	2,7	1,7	1,5	1,3	1,1
Interior	2,4	1,7	1,5	1,3	1,1
Creed coefficient for light concrete					
	4-7 days	14 days	28 days	3 months	1 year
Exterior	2,0	1,3	1,1	1,0	0,8
Interior	1,8	1,3	1,1	1,0	0,8
Creed coefficient for concrete with 1% reinforcement					
	4-7 days	14 days	28 days	3 months	1 year
Exterior	2,1	1,4	1,2	1,1	0,9
Interior	1,9	1,4	1,2	1,1	0,9

Where: I'_{sg} - moment of inertia of the compressed reinforcement of the section without cracks, I_{sg} - moment of inertia of the tensioned reinforcement of the section without cracks, I_{pg} - moment of inertia of the prestressing reinforcement of the section without cracks, I'_{scr} - moment of inertia of the compressed reinforcement of the section with cracks, I_{scr} - moment of inertia of the tensioned reinforcement of the section with cracks, I_{pcr} - moment of inertia of the prestressing reinforcement of the section with cracks, c_g - distance between the compressed face to the centroid of the section without cracks, c_{cr} - distance between the compressed face to the centroid of the section with cracks, d' - distance from the compressed fibre to the compressed reinforcement, d - distance from the compressed fibre to the tensioned reinforcement, d_p - distance from the compressed fibre to the prestressing reinforcement and ε'_{cs} - contraction strain.

2.3. In reference works

T. Paulay and M. Priestley, in their book "Seismic design of reinforced concrete structures and masonry buildings" from 1992, give an interval of values which depends on the constituent elements of the structure, and on the recommended values, table 4.

Table 4. Design values for the stiffness modulus according to Paulay and Priestley/1992

Element	Interval	Recommended value
Beams	$0{,}30 \div 0{,}50 \cdot I_g$	$0{,}40 \cdot I_g$
Beams T, L	$0{,}25 \div 0{,}45 \cdot I_g$	$0{,}35 \cdot I_g$
Columns, $P \geq 0{,}50 \cdot A_g f_c$	$0{,}70 \div 0{,}90 \cdot I_g$	$0{,}80 \cdot I_g$
Columns, $P = 0{,}20 \cdot A_g f_c$	$0{,}50 \div 0{,}70 \cdot I_g$	$0{,}60 \cdot I_g$
Columns, $P = -0{,}05 \cdot A_g f_c$	$0{,}30 \div 0{,}50 \cdot I_g$	$0{,}40 \cdot I_g$

I_e depends on the structural element type, so

- for structural walls:

$$I_e = \left(\frac{100}{f_y} + \frac{P_u}{A_g f} \right) \cdot I_g \tag{25}$$

- for coupling beams with diagonal reinforcements:

$$I_e = \frac{0{,}40 \cdot I_g}{\left[1 + 3 \cdot \left(\frac{h}{l_n} \right)^2 \right]} \tag{26}$$

- for coupling beams with orthogonal bars reinforcements:

$$I_e = \frac{0{,}20 \cdot I_g}{\left[1 + 3 \cdot \left(\frac{h}{l_n} \right)^2 \right]} \tag{27}$$

Where f_c is the compressions strength for the concrete, f_y - elasticity limit of the steel in case of reinforced concrete, P_u - axial force obtained from the seismic combination, h - section height and l_n - coupling beam length.

3. Quantitative results, qualitative result and discussions

For the case study a reinforced concrete frame structure was considered with one opening of 6m and height of 3m. Columns of 50x50 cm^2 were considered, reinforced with 12 longitudinal bars of Ø16 diameter and stirrups of Ø8 at a distance of 15cm. 30x60 cm^2 cross section beams were considered reinforced top and bottom with 2 bars of Ø16 and 1 bar of Ø10, and stirrups of Ø8 at a distance of 15cm. A uniformly distributed load of 50kN/m is applied on the beams as live load and 50kN/m for permanent load.

A modal and a static nonlinear analysis were performed in SAP2000. The software is based on the finite element method. The stiffness modulus of the frame elements was reduced according to the synthesis from chapter 2.

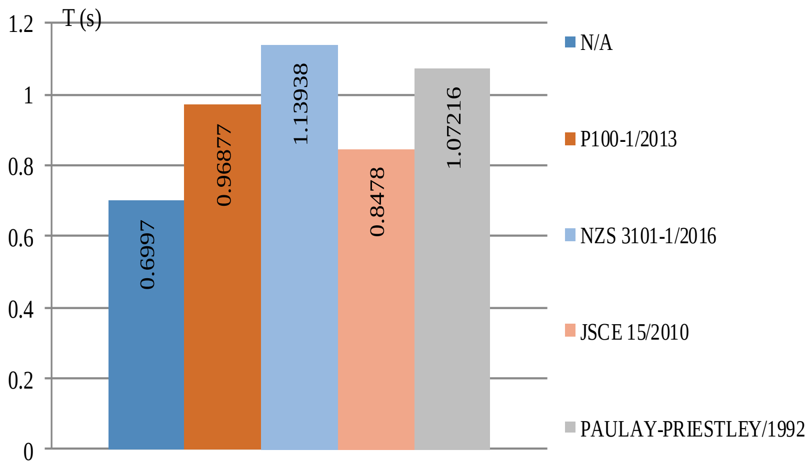

Figure 1. Results synthesis – comparison of the fundamental period of vibration

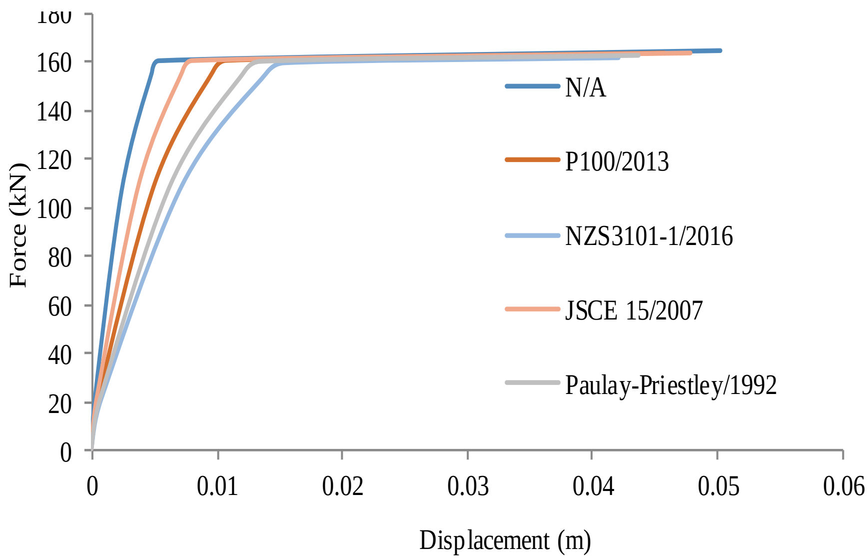

Figure 2. Results synthesis – comparison of the capacity (pushover) curve

4. Conclusion

The reinforced concrete frame structures are flexible one for which as long as their period of vibration is greater, they experience the maximum dynamic displacement. According to the Romanian code, the period of vibration is greater with 27.8% with respect to the initial structure. The highest increase of the period of vibration is observed according to the New Zeeland code, of 38.6% with respect to the neutral state. Based on the results obtained from the nonlinear static analysis, for the maximum recorded lateral force a significant difference is observed by applying the New Zeeland code. The lateral force is smaller with 1.5% and the displacement with 19.2% smaller with respect to the neutral state.

Further studies should be carried out to establish the optimum reduction factor for the stiffness modulus.

5. References

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