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Dynamics Problem of Mode III Crack Subjected to Superimpose Uniform Loads

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Abstract. The problem of the mode III flaw under superimpose uniform loads was studied by the means of the theory of complex variable functions. By the ways, the problems studied can be fast transformed into R-H issues and K-S mixed boundary value issues. The expression of stresses, displacement and stress intensity factor were gained on the theory of the theory of analytical functions.

1. Introduction

The studied problem is Correlation with III mode flaw extending issue. A restricted flaw occurs suddenly and extends at a invariant velocity to both ends. We can found the solution at several cases in [1, 2]. According to this dynamic problem's complicacy of half-plane extension, By applying complex function, we formulate anti-plane shear and reduce it to a R-H issue. The conditions of the flaw beared the superimpose uniform loads, could be solved.

2. Elementary Equations of Half-plane Shear Issue

For the Orthonormal anisotropy body, the Cartesian co-ordinates founded are consistent with the Elastic symmetry axis. The half-plane equation of moving is given such as:

$$C_{55}\partial^2 w / \partial x^2 + C_{44}\partial^2 w / \partial x^2 = \rho \partial^2 w / \partial t^2 \quad (1)$$

Here ρ is the density, w is the displacement component of z and C_{44} , C_{55} are material constants. Basing on the jobs in [1, 2] and using Atkinson alternation in [3], we build that:

$$\zeta = x - \eta t + \lambda y \quad (2)$$

Here η is a complex variable and λ is a function of η .

The expression of w can be written by

$$w = \text{Re} \left[\int_{-\infty}^{\infty} \phi(\zeta) d\eta \right] \quad (3)$$

Here ϕ is an aleatoric function that is determined by the constraint condition. Then bringing Eq. (3) in (1), here is



$$C_{55} + C_{44}\lambda^2 - \rho\eta^2 = 0 \quad (4)$$

Supposing that Eq. (4) exists complex roots, so

$$\lambda(\eta) = i\sqrt{(C_{55} - \rho\eta^2)/C_{44}} \quad (5)$$

Bringing Eq. (3) in the stress-displacement relations of an orthotropic body, the conclusions are

$$\tau_{yz} = \text{Re}\left\{\int_{-\infty}^{\infty} C_{44}\lambda[\partial\phi(\varsigma)/\partial\varsigma]d\eta\right\}, \quad \tau_{xz} = \text{Re}\left\{\int_{-\infty}^{\infty} C_{55}[\partial\phi(\varsigma)/\partial\varsigma]d\eta\right\} \quad (6)$$

2.1. The First Case

While the displacements are homogeneous function, let

$$\phi'(\xi) = f(\eta)/\xi \quad (7)$$

Now bringing Eq. (7) to Eqs. (6) and (3), and use Cauchy Standard at $y = 0$, it is

$$\begin{aligned} \tau_{yz} &= \text{Re}[-2\pi i(C_{44}/t)\lambda(\tau)f(\tau)], \quad \tau_{xz} = \text{Re}[-2\pi i(C_{44}/t)f(\tau)] \\ \partial w/\partial t &= \text{Re}[2\pi i(x/t^2)f(\tau)] \end{aligned} \quad (8)$$

Applying alike measure [3] for $\tau = x/t$, we can build

$$F(\tau) = -2\pi i C_{44}\lambda(\tau)f(\tau) \quad (9)$$

We can rewrite Eq. (8) as

$$\begin{aligned} \tau_{xz} &= [C_{55}/(C_{44}t)]\text{Re}[F(\tau)/\lambda(\tau)], \quad \tau_{yz} = (1/t)\text{Re}[F(\tau)] \\ \partial w/\partial \tau &= \text{Re}[F(\tau)/\lambda(\tau)]/C_{44} \end{aligned} \quad (10)$$

2.2. The Second Case

Where stresses is homogeneous, let

$$\phi''(\varsigma) = f(\eta)/\varsigma \quad (11)$$

Bringing Eq. (11) into Eq. (3) and (6) and use Cauchy Standard at $y = 0$, it can be built that

$$\begin{aligned} \partial\tau_{yz}/\partial t &= \text{Re}[2\pi i(\tau/t)C_{44}\lambda(\tau)f(\tau)], \quad \partial\tau_{xz}/\partial t = \text{Re}[2\pi i(\tau/t)C_{55}f(\tau)] \\ \partial^2 w/\partial t^2 &= \text{Re}[-2\pi i(\tau^2/t)f(\tau)] \end{aligned} \quad (12)$$

$$\text{We let } F(\tau) = 2\pi i \tau C_{44}\lambda(\tau)f(\tau) \quad (13)$$

Applying Eq. (11), the results are

$$\begin{aligned} \partial\tau_{xz}/\partial t &= [C_{55}/(C_{44}t)]\text{Re}[F(\tau)/\lambda(\tau)], \quad \partial\tau_{yz}/\partial t = (1/t)\text{Re}[F(\tau)] \\ \partial^2 w/(\partial t \partial \tau) &= \text{Re}[F(\tau)/\lambda(\tau)]/C_{44} \end{aligned} \quad (14)$$

Here $F(\tau)$ is self-similar functions.

2.3. The Third Case

Assume the limitless elastic half-space $y \geq 0$ start from stationary state. The end parts possessing coordinates x_n are shifting by the fixed speed V_n thus $x_n = V_n \cdot t$.

The secant tractions are expressed by aleatoric linear associations of

$$\frac{d^k g_{k_1}(x)}{dx^k} \bullet \frac{d^s g_{s_1}(x)}{dx^s} \quad (15)$$

Where

$$g_i(\xi) = \begin{cases} 0 & \text{for } \xi < 0 \\ \xi^i & \text{for } \xi > 0 \end{cases} \quad (16)$$

Here k , s , k_1 and s_1 are arbitrary integer positive numbers. Introduce the linear differential operator:

$$L = \frac{\partial^{m+n}}{\partial x^m \partial t^n}, \quad L^- = \frac{\partial^{-m-n}}{\partial x^{-m} \partial t^{-n}} \quad (17)$$

At $y = 0$, there is the expression

$$w^0 = Lw, \quad \tau_{xz}^0 = L\tau_{xz}, \quad \tau_{yz}^0 = L\tau_{yz} \quad (18)$$

When $L\tau_{xz}$ and $L\tau_{yz}$ are homogeneous, there results

$$w^0 = \frac{\partial}{\partial t} Lw, \quad \tau_{xz}^0 = \frac{\partial}{\partial t} L\tau_{xz}, \quad \tau_{yz}^0 = \frac{\partial}{\partial t} L\tau_{yz} \quad (19)$$

All the cases mentioned above is homogeneous [4]. It is shown that [5]

$$\tau_{xz}^0 = [C_{55}/(C_{44}t)] \operatorname{Re}[F(\tau)/\lambda(\tau)], \quad \tau_{yz}^0 = (1/t) \operatorname{Re}[F(\tau)]$$

$$\partial w^0 / \partial \tau = \operatorname{Re}[F(\tau)/\lambda(\tau)]/C_{44} \quad (20)$$

Where

$$g(\tau) = F(\tau)/\lambda(\tau) \quad (21)$$

Eqs. (20) can be expressed as

$$\tau_{xz}^0 = [C_{55}/(C_{44}t)] \operatorname{Re} g(\tau), \quad \tau_{yz}^0 = (1/t) \operatorname{Re} g(\tau)\lambda(\tau)$$

$$\partial w^0 / \partial \tau = \operatorname{Re} f(\tau)/C_{44} \quad (22)$$

3. The Concrete Problem

Let the moving flaw be on uniform loads $-P$ and $-q$ like:

$$\tau_{yz} = -P, \quad y = 0, \quad 0 < |x| < \alpha t$$

$$\tau_{yz} = -q, \quad y = 0, \quad \alpha t < |x| < Vt$$

$$w = 0, \quad y = 0, \quad |x| > Vt \quad (23)$$

Here α is the moving velocity of the loading. The foregoing stress restrict conditions can be expressed as:

$$\tau_{yz} = -q + (q - P)[H(x + \alpha t) - H(x - \alpha t)], \quad y = 0, \quad |x| < Vt \quad (24)$$

Consider $L = 1$ in Eq. (17), Eq. (24) can be substituted into (19) and (22) to express:

$$\begin{aligned} \operatorname{Re}[\lambda(\tau)g(\tau)] &= (q - P)\alpha t [\delta(x + \alpha t) + \delta(x - \alpha t)] \\ &= (q - P)\alpha [\delta(\tau + \alpha) + \delta(\tau - \alpha)], \quad y = 0, \quad |\tau| < V \end{aligned} \quad (25)$$

The expression of Dirac functions is

$$\delta(\tau + \alpha) + \delta(\tau - \alpha) = \operatorname{Re}\left[\frac{2\tau i}{\pi(\tau^2 - \alpha^2)}\right] \quad (26)$$

A solution of Eq. (24) is [14]

$$g(\tau) = A\tau^n (V^2 - \tau^2)^{-3/2} \left\{ A + 2B/[\pi(\tau^2 - \alpha^2)] \right\} \quad (27)$$

Taking Eq. (27) into (20) and (25), n is decided by:

$$n = 1$$

Now take Eqs.(23), (24) and (25) into (5), the expression is

$$B = \frac{\alpha(q - P)(V^2 - \alpha^2)^{3/2}}{\sqrt{(C_{55} - \rho\alpha^2)/C_{44}}} \quad (28)$$

Eq. (28) may be Substituted to Eqs. (20) and (19) at $y = 0$, the stress τ_{yz} and the expression of $K_3(t)$ is:

$$\tau_{yz} = \operatorname{Re} \int_{C_d}^{x/t} - \frac{\sqrt{(C_{55} - \rho\tau^2)/C_{44}}}{(\tau^2 - V^2)^{3/2}} \left(A + \frac{2B}{\pi(\tau^2 - \alpha^2)} \right) d\tau \quad (29)$$

$$K_3(t) = \lim_{x \rightarrow Vt} \sqrt{2\pi(x - Vt)} \operatorname{Re} \int_{C_d}^{x/t} - \frac{\sqrt{(C_{55} - \rho\tau^2)/C_{44}}}{(\tau^2 - V^2)^{3/2}} \left(A + \frac{2B}{\pi(\tau^2 - \alpha^2)} \right) d\tau \quad (30)$$

Where $C_d = \sqrt{C_{55}/\rho}$ is the acoustic velocity (threshold value). by Eq. (29) into (23), we can get the constant A :

$$\begin{aligned} A &= (q - 2B\pi^{-1}J_2)/J_1, \quad J_1 = \operatorname{Re} \int_M^{C_d} - \frac{\sqrt{(C_{55} - \rho\tau^2)/C_{44}}}{(\tau^2 - V^2)^{3/2}} d\tau \\ J_2 &= \operatorname{Re} \int_M^{C_d} - \frac{\sqrt{(C_{55} - \rho\tau^2)/C_{44}}}{(\tau^2 - \alpha^2)(V^2 - \tau^2)^{3/2}} d\tau \end{aligned} \quad (31)$$

Where $\alpha < M < V$.

At the surface $y = 0$, We also can get w^0 by substituting Eq. (27) into (22) such as:

$$w^0 = \frac{1}{C_{44}} \int f(\tau) d\tau = \int \frac{\tau}{(\tau^2 - V^2)^{3/2}} \left[A + \frac{2B}{\pi(\tau^2 - \alpha^2)} \right] d\tau = \frac{A}{C_{44} \sqrt{V^2 - \tau^2}} + \frac{B}{\pi C_{44}} \cdot \left[\frac{2}{(V^2 - \alpha^2) \sqrt{V^2 - \tau^2}} + \frac{1}{\sqrt{(V^2 - \alpha^2)^3}} \ln \left| \frac{\sqrt{V^2 - \alpha^2} - \sqrt{V^2 - \tau^2}}{\sqrt{V^2 - \alpha^2} + \sqrt{V^2 - \tau^2}} \right| \right] + C \quad (32)$$

Because the flaw extends along the x-axis, we take constant $C = 0$. By substituting Eq. (32) into (19), at the surface $y = 0$, we can get the displacement w such as:

$$w = \frac{1}{C_{44}} \left[A + \frac{2B}{\pi(V^2 - \alpha^2)} \right] \frac{\sqrt{V^2 t^2 - x^2}}{V^2} - \frac{B}{\pi \sqrt{(V^2 - \alpha^2)^3}} \quad (33)$$

$$|x| < Vt$$

4. Conclusion

The expression of concrete problems of III mode dynamic flaw propagation was built up by the measure of complex variable function. The method used in the paper is based on the theory of the analytic functions; it is possible to get the expression of flaw studied. The means of solution is on account of theory of function-analytication theory. The flaw tip takes on singularity [6].

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