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# Geometrically Nonlinear Constitutive Equations of the Plastic Flow Theory in Terms of Asymmetric Stress and Strain Measures

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# Geometrically Nonlinear Constitutive Equations of the Plastic Flow Theory in Terms of Asymmetric Stress and Strain Measures

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**Abstract.** A formulation of the geometrically nonlinear plastic flow theory (PFT) based on asymmetric measures of stress and strain states is proposed. A main emphasis is placed on the physically reasonable decomposition of the deformation gradient into three components: elastic distortions, which determine stresses, an orthogonal tensor characterizing the quasi-rigid motion of a material and the plastic strain gradient. The quasi-rigid motion of the material is defined by introducing for a representative volume element a generalized lattice, which represents its symmetry elements. The hypoelastic anisotropic law is introduced in terms of the movable coordinate system associated with the material. The rate of plastic deformations is determined by the associated law of plastic flow. As a result, the closed system of constitutive equations of the geometrically nonlinear PFT of is obtained.

## 1. Introduction

Currently, there is a high need for physically correct description of severe deformation processes (SPD). The improvement of SPD technologies leads to the need to solve optimization problems that require the implementation of a variety of calculations with different process parameters. To the solve of corresponding boundary value problems, it is necessary to use computational effective mathematical models. At the same time, these material models must be physically relevant. Special attention should be paid to the correct description of the geometric nonlinearity arising in the study of SPD processes using the methods of deformable solid body mechanics. Usually, various modifications of classical plastic flow theories based on symmetric of stress and strain measures are used to describe of SPD. The use of a symmetric measure of the strain state (or its velocity) excludes any information about the quasi-rigid material motion and the effects associated with the material rotation. In the present work, a modification of the classical plastic flow theory based on asymmetric stress and strain measures. Correct geometric nonlinearity description is physically ground separation movement on quasirigid (rotational) and deformation. Separating the quasi rigid motion is necessary to introduce the correct constraint between the stress and the strain in terms of a rigid moving coordinate system (MCS). The complete motion of the continuum is represented as a combination of a quasi-rigid motion of the material (together with the MCS) and the deformation one (with respect to the MCS). The stress measure is the Cauchy stress tensor, defined in terms of the MSC. The transposed velocity gradient is the total strain measure. The quasi-rigid material rotation should not cause changes in the response of the material (for the observer in the MCS), so quasi-rigid motion must be excluded from the complete motion; only the strain movement effects on the material response. Determining the quasi-solid motion of the continuum is a difficult due to it is impossible to select an undeformable (throughout the whole process) aggregate of any material elements, preferably connected with axes or symmetry planes.

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## 2. Continuum kinematic

### *2.1. Multiplicative decomposition of the deformation gradient*

The classical multiplicative decomposition of the deformation gradient into elastic $\mathbf{F}^{\mathrm{e}}$ and plastic $\mathbf{F}^{\mathrm{p}}$ components [1] is represented by the relation:

$$\mathbf{F} = \mathbf{F}^{\mathrm{e}} \cdot \mathbf{F}^{\mathrm{p}}, \tag{1}$$

where the inelastic component $\mathbf{F}^{\mathrm{p}}$ connects the basis vectors of the Lagrangian coordinate system in the reference and the plastically deformed configurations.

In [2, 3], for crystalline materials with an internal structure, a multiplicative decomposition different from the classical one (1) was proposed. The motion of deformable crystalline body, which is described by the deformation gradient $\mathbf{F}$, is represented by the sequence of plastic deformations (it is assumed that plastic deformations do not rotate the MCS associated with the lattice of the crystallite), a rotation of the MCS (together with the material) and an elastic distortion of the material with respect to the MCS:

$$\mathbf{F} = \bar{\mathbf{F}}^{\mathrm{e}} \cdot \mathbf{R} \cdot \mathbf{F}^{\mathrm{p}}, \tag{2}$$

where the inelastic component $\mathbf{F}^{\mathrm{p}}$ connects the reference and plastically deformed (intermediate unloaded) configurations, $\mathbf{R}$ is the orthogonal tensor, which converts the reference basis of the rigid moving coordinate system into the current (rotating tensor of the MCS from the reference to actual configuration), $\bar{\mathbf{F}}^{\mathrm{e}}$ – the deformation gradient, which transforms the plastically deformed configuration, which has experienced the quasi-rigid motion, into the actual configuration. It should be noted that determining the relationship between the MCS motion and the continuum (orthogonal transformation $\mathbf{R}$ at any time) and the initial orientation of the MCS for the continuum that does not have an internal structure is difficult, moreover, it has many possible solutions.

It is known that polycrystalline materials, in particular, metals, consist of separate crystallites with a certain structure. For a single crystallite, inelastic deformations realized by the motion of dislocations do not lead to the rotation of the crystal lattice [2, 3]. In the case of a polycrystalline material, the actual plastic deformations of each individual crystallite, realized by yields along crystal slip systems (SS), also do not lead to rotate of lattice. For the polycrystalline, it is possible to introduce a "generalized crystal lattice" (GL), which characterizes the average (by representative volume) symmetry properties of the material. Thus, the plastic deformation of a polycrystalline, described by the plastic component $\mathbf{F}^{\mathrm{p}}$ determined by the plastic deformations of crystallites of the representative volume. It is assumed, that $\mathbf{F}^{\mathrm{p}}$ (by analogy with the crystallites) not to turn the "generalized crystal lattice" of the representative volume element (RVE). The MCS is associated with the introduced "generalized lattice" of the representative volume element. The MCS is responsible for the quasi-rigid motion of the representative volume element. As a result, relation (2) represents the movement of the polycrystalline by the sequence of plastic deformation, described by the plastic component $\mathbf{F}^{\mathrm{p}}$, carried out without distortion and rotation of the "generalized lattice" (in other words, it is assumed that during plastic deformation the material "flows" through the "generalized lattice", leaving the latter invariant), and the affine transformation of $\mathbf{F}^{\mathrm{e}}$, which includes the rotation of $\mathbf{R}$ and the distortion of this lattice, described by $\bar{\mathbf{F}}^{\mathrm{e}}$.

We note that the "generalized lattice" is experiencing both rotation and distortion, which are collectively described by the elastic component of the position gradient $\mathbf{F}^{\mathrm{e}}$ (1), while the MCS is experiencing only rotation (the translation motion of the MCS do not lead to a change of basis, and therefore are not considered further). The decomposition $\mathbf{F}^{\mathrm{e}} = \bar{\mathbf{F}}^{\mathrm{e}} \cdot \mathbf{R}$ differs significantly from the classical polar decomposition, since $\bar{\mathbf{F}}^{\mathrm{e}}$ is not symmetric in the general case and is calculated after the determination of the quasi-rigid motion of the MCS. By analogy with the deformation of single crystals [2], when determining the quasi-hard rotation, it is proposed to exclude purely plastic deformation.

### *2.2. Additive decomposition of the strain rate measure in the current configuration*

The introduced multiplicative decomposition (2) is used to obtain the additive decomposition of the transposed gradient of total velocity in the current configuration:

$$\mathbf{L} = \hat{\nabla}\mathbf{V}^{\mathrm{T}} = \text{[illegible]} + \bar{\mathbf{F}}^{\mathrm{e}} \cdot \mathbf{\Omega} \cdot \bar{\mathbf{F}}^{\mathrm{e}-1} + \bar{\mathbf{F}}^{\mathrm{e}} \cdot \mathbf{R} \cdot \mathbf{L}^{\mathrm{p}} \cdot \mathbf{R}^{\mathrm{T}} \cdot \bar{\mathbf{F}}^{\mathrm{e}-1}, \tag{3}$$

where $\mathbf{V}$ – is velocity of continuum points, $\hat{\nabla}$ – the operator nabla in the current configuration, $\bar{\mathbf{L}}^{\mathrm{e}} = \text{[illegible]}$, $\mathbf{L}^{\mathrm{p}} = \text{[illegible]}$. From decomposition (2) it follows that $\bar{\mathbf{F}}^{\mathrm{e}}$ reflects only elastic distortions of the GL. In the case of large gradients of displacements and severe deformations of metals, the elastic distortions are small compared to the rotations $\mathbf{R}$ and the plastic deformations $\mathbf{F}^{p}$. In this regard, $\bar{\mathbf{F}}^{e}$ is close to unity, and the unloaded configuration consists with current configuration. However, the rate cannot be neglected; it is comparable to the values of the other terms in the decomposition (3). The additive decomposition is introduced:

$$\mathbf{Z} = \mathbf{Z}^{e} + \mathbf{Z}^{in}, \tag{4}$$

where, based on [2], $\mathbf{Z} = \mathbf{L} - \mathbf{\Omega}$, $\mathbf{Z}^{in} = \mathbf{R} \cdot \mathbf{L}^{p} \cdot \mathbf{R}^{T}$, $\mathbf{Z}^{e} =$ – are measures of velocities of complete, inelastic and elastic distortions, defined in terms of the actual (unloaded) configuration, $\mathbf{\Omega}$ is the spin tensor of MCS tensor, $\mathbf{L}^{p}$ is the plastic component of the displacement velocity gradient, defined in terms of the reference configuration.

Following [2], the defining relation is introduced:

$$\mathbf{\Sigma}^{CR} = \mathrm{d}\mathbf{\Sigma} / \mathrm{d}t + \mathbf{\Sigma} \cdot \mathbf{\Omega} - \mathbf{\Omega} \cdot \mathbf{\Sigma} = \bar{\mathbf{\Pi}} : \mathbf{Z}^{e}. \tag{5}$$

where $\mathbf{\Sigma}$ is the Cauchy stress tensor, $\bar{\mathbf{\Pi}} = \bar{\Pi}_{ijkl}\mathbf{k}^{i}\mathbf{k}^{j}\mathbf{k}^{k}\mathbf{k}^{l}$ is a fourth-rank tensor of elastic properties, symmetric with respect to pairs of indices, whose components are constant in the MCS basis $\{\mathbf{k}^{i}\}$.

## 3. The plastic flow theory based on asymmetric measures

In this section, the constitutive equations of the plastic flow theory based on asymmetric strain and stress rate measures are introduced. The previously obtained additive decomposition of the strain rate measure into the elastic and inelastic components (4) is used. The deviators of stress and strain measures and their velocities are used to derive the equations of the plasticity theory. The deviators of the Cauchy stress tensor $\mathbf{\Sigma}$ and the velocity gradient of relative displacements $\mathbf{Z}$ are introduced:

$$\begin{aligned} &\mathbf{Z}' = \mathbf{Z} - 1/3\mathrm{I}_1(\mathbf{Z})\mathbf{E} = \mathbf{L} - \mathbf{\Omega} - 1/3\mathrm{I}_1(\mathbf{L} - \mathbf{\Omega})\mathbf{E} = \mathbf{L} - \mathbf{\Omega} - 1/3\mathrm{I}_1(\mathbf{L})\mathbf{E} = \mathbf{L}' - \mathbf{\Omega}, \\ &\mathbf{\Sigma}' = \mathbf{\Sigma} - 1/3\mathrm{I}_1(\mathbf{\Sigma})\mathbf{E}, \end{aligned} \tag{6}$$

where $\mathbf{E}$ is the isotropic second rank tensor.

As the intensities of measures, the classical expressions are used, extended for the asymmetric case:

$$\begin{aligned} &\Sigma_{u} = \sqrt{3/2\,\mathbf{\Sigma}' : \mathbf{\Sigma}'^{T}}, \\ &\mathrm{Z}_{u} = \sqrt{2/3\,\mathbf{Z}' : \mathbf{Z}'^{T}}, \\ &\lambda = \int \sqrt{2/3\,\mathbf{Z}^{in} : \mathbf{Z}^{inT}}\,\mathrm{d}\tau. \end{aligned} \tag{7}$$

It should be noted that the scalar measures of stresses and the accumulated strain λ need further refinement in the case of severe deformations in order to correctly introduce the single curve hypothesis. In numerous articles [4-6], there are experimental data indicating that it is impossible to introduce a single dependence $\Sigma_u(\lambda)$ for all loading types without significant errors (up to 20%).

As a basis for constructing the asymmetric plasticity theory, the associated law of flow is used [7]:

$$\mathbf{Z}^{in} = \frac{\mathbf{\Sigma}'}{\Sigma_{u}}, \tag{8}$$

where $\Sigma_u = \mathrm{R}(\lambda)$ during an active loading and R(λ) determines the single dependence of the intensity of the flow stresses on the accumulated plastic strain [7].

### *3.1. The derivation of the constitutive equation in the modified plastic flow theory*

To derive the constitutive equation connecting the stress and strain rates, we will follow the scheme outlined in [8]. The aim of the subsequent transformations is to determine the tensor of elastoplastic properties, which connects the measures of these rates. Using the yield criterion $\Sigma_u = \mathrm{R}(\lambda)$, we determine the change rate of λ during an active plastic deformation $\left(\mathbf{\Sigma}' : \mathbf{Z}^{inT} \geq 0\right)$:

$$\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{R}(\lambda)=\mathrm{R}_\lambda(\lambda)\dot{\lambda} \ [illegible] \ \frac{\left(3/2\,\boldsymbol{\Sigma}':\boldsymbol{\Sigma}'^{\mathrm{T}}\right)^{\bullet}}{2\sqrt{3/2\,\boldsymbol{\Sigma}':\boldsymbol{\Sigma}'^{\mathrm{T}}}}=\frac{1}{2\Sigma_u}3/2\left(\dot{\Sigma} \ [illegible]\right),$$

$$=\frac{1}{2\Sigma_u}3/2\left(\dot{\Sigma} \ [illegible]\right) \ [illegible] \ \frac{\ }{2\Sigma_u}\left([illegible]\right) \ [illegible] \ \frac{[illegible]}{2\ \Sigma_u}, \qquad (9)$$

$$\dot{\lambda} \ [illegible] \ \frac{[illegible]}{\mathrm{R}_\lambda(\lambda)}=\frac{[illegible]}{2}\frac{\dot{\Sigma} \ [illegible]}{\Sigma_u\mathrm{R}_\lambda(\lambda)},$$

where $\dot{\Sigma}$ is the total material derivative of the Cauchy stress tensor deviator. The elastic relation (5) determines the rate of the total stresses, and not their deviator part, relationship is needed:

$$\left(\boldsymbol{\Sigma}'\right)^{\bullet}=\left(\boldsymbol{\Sigma}-1/3\left(\boldsymbol{\Sigma}:\mathbf{E}\right)\mathbf{E}\right)^{\bullet}=\dot{\Sigma} \ [illegible] \ \overset{def}{=} \ [illegible]$$
$$=\dot{\Sigma}\cdot\left([illegible]\right) \ [illegible] \ \left(\mathbf{C}_{\mathrm{III}} \ [illegible] \ \mathbf{C}_{\mathrm{I}}\right), \qquad (10)$$

where $\mathbf{E}=\mathbf{e}_i\mathbf{e}^i$ is the isotropic tensor of the second rank, $\mathbf{C}_{\mathrm{I}}=\mathbf{EE}$, $\mathbf{C}_{\mathrm{III}}=\mathbf{e}_i\mathbf{e}^j\mathbf{e}_j\mathbf{e}^i$ are isotropic tensors of the fourth rank. From relations (8)-(10), the relationship between the rate of plastic strain and the stress rate is obtained:

$$\mathbf{Z}^{\mathrm{in}}=\frac{\boldsymbol{\Sigma}'}{\Sigma_u}\dot{\lambda} \ [illegible] \ \frac{\boldsymbol{\Sigma}'}{\Sigma_u}\frac{[illegible]}{2}\frac{\dot{\Sigma} \ [illegible]}{\Sigma_u\mathrm{R}_\lambda(\lambda)}=\frac{3}{2}\frac{\boldsymbol{\Sigma}'}{\Sigma_u}\frac{\boldsymbol{\Sigma}'^{\mathrm{T}}:\dot{\Sigma}}{\Sigma_u\mathrm{R}_\lambda(\lambda)}=\mathbf{P}:\dot{\Sigma},$$
$$\mathbf{P}\overset{об}{=}\frac{3}{2}\frac{\boldsymbol{\Sigma}'\boldsymbol{\Sigma}'^{\mathrm{T}}}{\Sigma_u^2}\frac{1}{\mathrm{R}_\lambda(\lambda)}=\frac{3}{2}\frac{\boldsymbol{\Sigma}'\boldsymbol{\Sigma}'^{\mathrm{T}}}{\Sigma_u^2}\frac{1}{\mathrm{R}_\lambda(\lambda)}, \qquad (11)$$

where $\mathbf{P}$ is the fourth rank tensor, which completely determines the inelastic properties of the material when the flow stress is reached with active loading; this tensor includes the tensor product of the stress tensor $\boldsymbol{\Sigma}'/\Sigma_u$ on itself and the value of the instantaneous hardening modulus $\mathrm{R}_\lambda(\lambda)$.

The rate of elastic deformations is expressed from the elastic relation (5):

$$\mathbf{Z}^{\mathrm{e}}=\bar{\boldsymbol{\Pi}}^{-1}:\boldsymbol{\Sigma}^{\mathrm{CR}}. \qquad (12)$$

Considering the additivity of strain rates (4) and the expressions for the component strain rates (11), (12):

$$\mathbf{Z}=\bar{\boldsymbol{\Pi}}^{-1}:\boldsymbol{\Sigma}^{\mathrm{CR}}+\mathbf{P}:\dot{\Sigma}, \qquad (13)$$

whence, using the expression for the corotational derivative (5), the total rate of change of stresses from the point of view of a fixed observer in the laboratory coordinate system (LCS) is expressed:

$$\mathbf{Z}=\bar{\boldsymbol{\Pi}}^{-1}:\left(\dot{\Sigma} \ [illegible]\right) \ [illegible],$$
$$\mathbf{Z}=\bar{\boldsymbol{\Pi}}^{-1}:\left(\dot{\Sigma} \ [illegible] \left(\left(\mathbf{C}_{\mathrm{III}} \ [illegible] \ \mathbf{C}_{\mathrm{I}}\right) [illegible]\right)\right), \qquad (14)$$
$$\mathbf{Z}-\bar{\boldsymbol{\Pi}}^{-1}:\boldsymbol{\Sigma}^{\Omega}=\left(\bar{\boldsymbol{\Pi}}^{-1}+\mathbf{P}:\left(\mathbf{C}_{\mathrm{III}}-1/3\,\mathbf{C}_{\mathrm{I}}\right)\right):\dot{\Sigma},$$

where the designation of the term $\boldsymbol{\Sigma}^{\Omega}\overset{def}{=}\boldsymbol{\Sigma}\cdot\boldsymbol{\Omega}-\boldsymbol{\Omega}\cdot\boldsymbol{\Sigma}$ characterizing the geometric nonlinearity is introduced. From the relation (14), the final expression for the stress rate with active loading is obtained, i.e. when the imaging stress point is found on the yield surface and the condition $\boldsymbol{\Sigma}':\mathbf{Z}^{\mathrm{inT}}\geq 0$ [9] is fulfilled:

$$\dot{\Sigma} \ [illegible] \ \mathbf{P}:\left(\mathbf{C}_{\mathrm{III}}-1/3\,\mathbf{C}_{\mathrm{I}}\right)\right)^{-1}:\left(\mathbf{Z}-\bar{\boldsymbol{\Pi}}^{-1}:\boldsymbol{\Sigma}^{\Omega}\right),$$
$$\text{by conditional: } \ \Sigma_u=\mathrm{R}(\lambda),\ \boldsymbol{\Sigma}:\mathbf{Z}^{\mathrm{inT}}\geq 0. \qquad (15)$$

In the case of elastic deformation, when the stress vector is located inside the yield surface, the plastic deformations are identically zero, then the connection between the stress rate and the total strain rate is performed in the following form:

$$\boldsymbol{\Sigma}^{\mathrm{CR}}=\bar{\boldsymbol{\Pi}}:\mathbf{Z}^{\mathrm{e}}=\bar{\boldsymbol{\Pi}}:\mathbf{Z},$$
$$\text{by conditional: } \ \Sigma_u<\mathrm{R}(\lambda). \qquad (16)$$
$$\text{by conditional: } \ \Sigma_u=\mathrm{R}(\lambda),\ \boldsymbol{\Sigma}:\mathbf{Z}^{\mathrm{inT}}<0.$$

The final set of relations of the geometrically non-linear plastic flow theory is:

$$
\begin{aligned}
&\mathbf{Z} = \mathbf{L} - \mathbf{\Omega},\\
&\mathbf{\Sigma}^{\Omega} \overset{def}{=} \mathbf{\Sigma}\cdot\mathbf{\Omega} - \mathbf{\Omega}\cdot\mathbf{\Sigma},\\
&\Sigma_u = \sqrt{3/2\,\mathbf{\Sigma}' : \mathbf{\Sigma}'^{\mathrm{T}}},\\
&\mathbf{Z}^{\mathrm{in}} = \mathbf{P} : \mathbf{\Sigma}'^{\mathrm{CR}}, \quad \mathbf{P} = \frac{3}{2}\frac{\mathbf{\Sigma}'\mathbf{\Sigma}'^{\mathrm{T}}}{\Sigma_u^2}\frac{1}{\mathrm{R}_{\lambda}(\lambda)},\\
&(\mathbf{\Sigma}')^{\mathrm{CR}} = \dot{\mathbf{\Sigma}} \text{[illegible]} \mathrm{III} \text{[illegible]} /3\mathbf{C}_{\mathrm{I}}) + \mathbf{\Sigma}^{\Omega},\\
&\lambda = \int\sqrt{2/3\,\mathbf{Z}^{\mathrm{in}} : \mathbf{Z}^{\mathrm{in}}}\,\mathrm{d}\tau,\\
&\begin{cases}
\Sigma_u < \mathrm{R}(\lambda) \text{ or } \Sigma_u = \mathrm{R}(\lambda), \mathbf{\Sigma} : \mathbf{Z}^{\mathrm{inT}} < 0 \;:\; \dot{\mathbf{\Sigma}} \text{[illegible]} \mathbf{\Sigma}^{\Omega},\\
\Sigma_u = \mathrm{R}(\lambda), \mathbf{\Sigma} : \mathbf{Z}^{\mathrm{inT}} \geq 0 \;:\; \dot{\mathbf{\Sigma}} \text{[illegible]} \mathbf{P} : \left(\mathbf{C}_{\mathrm{III}} - 1/3\mathbf{C}_{\mathrm{I}}\right)\big)^{-1} : \left(\mathbf{Z} - \bar{\mathbf{\Pi}}^{-1} : \mathbf{\Sigma}^{\Omega}\right).
\end{cases}
\end{aligned}
\tag{17}
$$

*3.2. On quasi-solid motion*

It should be noted that the main problem in the construction of the proposed model is the determination of the value of the spin tensor $\mathbf{\Omega}$ of MCS of representative volume element of the material at any time. As a solution of this problem, it is proposed to rely on the introduction of a generalized lattice of representative volume element, in which the MCS is rigidly associated with one direction and the plane of the generalized lattice. However, this does not take into account the orientation of the axes of symmetry at the reference configuration (the basis GL is assumed to coincide with the LCS), as well as at an arbitrary moment of process. It is obvious that the symmetry properties of the representative volume of the polycrystalline are determined by the law of distribution of orientations (changing in the process of deformation) in the current configuration and by the symmetry properties and orientations of its individual elements, i.e., crystallites. At the current stage of development of a modification of the PFT model, it is proposed an alternative method for determining the MCS spin tensor, assuming the use of the two-level crystal plasticity model [3]. The developing approach [10] is used to determine symmetry properties of the representative volume element. This approach makes it possible to determine compliance with the selected classes of symmetry of the representative volume element of the polycrystalline at any time during the process of arbitrary treatment. However, if at the initial moment the polycrystalline does not possess anisotropy of elastic properties, for which it is not possible to select character directions, the MCS is chosen to coincide with the LCS. In the future, it is planned to develop a macromodel to describe the evolution of symmetry properties. For identifying and verifying the parameters of the macromodel the two-level crystal plasticity model will used. The advantage of this approach is binding to the symmetry elements of the material rather than the objects induced by the loading kinematics, e.g., the polar decomposition [11] or the skew-symmetric part of the velocity gradient [12, 13], where the PCS is associated with the eigenvectors of certain strain measures that are not related to the material. The widely used logarithmic spin [14] allows to avoid the dissipation of the elastic energy on closed cycles (in the elastic region), as well as stress oscillations. However, the derivation of the logarithmic spin is based on the assumption of elastic isotropy, and thus it cannot be associated with the symmetric elements of the material. An approach, which is the most similar to the proposed one, is described in [15, 16], where the concept of a reference generalized lattice with corresponding material vectors is introduced. The evolution of these vectors is determined by the elastic component of the velocity gradient, the definition of which in the framework of the symmetrized measures raises questions undiscussed in the cited articles.

## 4. Conclusion

In this work, the structure of the macro-phenomenological geometrically non-linear plastic flow theory based on asymmetric stress and strain measures is presented. By adopting the Mises-Hyber-Hencky yield criterion, the associated plasticity law and the anisotropic hypoelastic law, conclusive resolving equations are obtained for the change rate of stresses. The developed model of elastoplastic material will be further used in solving boundary-value problems corresponding to real technological processes.

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