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## Research on identification method of multiple cracks in a shaft by vibration characteristics

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# Research on identification method of multiple cracks in a shaft by vibration characteristics

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**Abstract.** As the main transmission device in mechanical equipment, the shaft plays an important role in the function realization and stable operation of the machine, but it is particularly prone to failure due to the long-term alternating or random axial load, bending moment and torsion, so it is of great significance to find the crack on the shaft in its early state, to reduce economic losses and prevent catastrophic accidents. The vibration characteristics of the circular cross-section shaft with multiple cracks are studied, and a damage detection method is proposed. Based on the basic beam theory and the local compliance theory, the stress intensity factors(SIFs) for the cracked shaft subjected to axial force, shear force and bending moment are derived, and the evaluation of corresponding compliance coefficient is given. The differential equation of motion is discussed. Results indicate that the derived stress intensity factor is in good agreement with the experimental results. As the shaft has cracks, the mode shape will show discontinuous at the crack location. This method can identify the crack position, depth and the number of cracks.

## 1. Introduction

Shaft is one of the common components which is widely used in a dynamic machine. It plays an important role in the realization of function and stable operation of dynamic machine. Therefore, the reliability of shaft is very significant to the machine. Due to the environment of vibration and repeated force, cracks are often initiated in a shaft, making the machine work unstable and reducing the strength of the shaft. Especially when the shaft works in a resonant environment, the crack will be propagated rapidly which may lead some accidents. To guarantee the safety reliability of the shaft, it is of great significance to find the crack position and crack size in the shaft in its early state. Numerous studies have been done about the identification method of crack. Zhong et al. proposed a novel non-projection vision-based system, the purpose of this system is to realize simultaneous measurement of the radial and axial displacements with high accuracy and good reliability by using a tailored artificial constant density sinusoidal fringe pattern. Results indicated that this method was an effective and accurate technique for real-time vibration monitoring of rotating shaft[1]. Zhang et al. studied damage detection method of a beam based on the coherence function of random vibration. Results indicated that this method can be used to detect single-crack and multi-crack of the cantilever beam accurately and reliably[2]. Gounaris et al. simulated the cracks using local flexibility theory, and established a finite element model of the cracked beam, and analyzed the dynamic response of a cantilever beam under harmonic excitation. Results indicated that the modal frequencies and modal shape are greatly affected

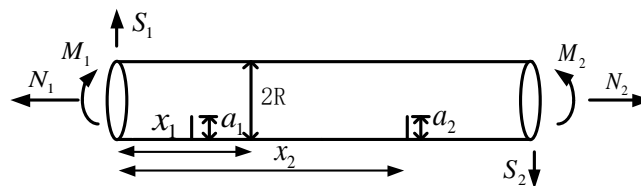


by the presence of medium cracks[3]. An sandwich panel with some cracks was used as the object by Zhang et al. to study three identification methods of cracks. Results shown that the crack location can be accurately identified according to the difference of vibration shape[4]. A simple and effective identification algorithm of crack damage before a break occurs was employed by Cruz-Vega et al. to identify several levels of crack of a rotor bar under different load conditions[5]. A vector due to measured values of vibration response of the rotor was used to establish the system matrix by Ratan et al., results indicated that this method can even detect and locate cracks smaller than 4% of the diameter of the shaft[6]. Hwang detected the location and the size of crack damage by minimizing the differences between trials and analysis of FRFs[7]. Yang et al. studied the effects of open crack on the vibration characteristics of elastic beams[8]. Steady-state response study displayed that the combined frequency and longitudinal response of the rotational and torsional excitations in the lateral response are reversed. The frequency of excitation can be used to detect the oblique cracks in shaft of the rotor system[9]. Bovsunovsky et al. studied a simple procedure to evaluate the effectiveness of vibration diagnostics of damage. The procedure is based on the determination of the change of shaft's compliance caused by a crack via the use of linear fracture mechanics[10].

Although many works about crack detection method for shaft have been done by the researchers in the past few decades. However, most of the works only consider single crack in the model, the influence of the neutral axis on the stress intensity factor are seldom considered. In this article, the model of SIFs is derived based on the elastic beam theory, and the local compliance model is also established. An identification method of multiple cracks in shaft is discussed in the end.

## 2. Calculation of SIFs

The cracked shaft model is shown in Figure 1. The radius of the shaft is assumed to be 'R' and the crack depth are 'a1' and 'a2'. The shaft is subjected to the shear forces 'S1' and 'S2' bending moments 'M1' and 'M2', and axial tensile forces 'N1' and 'N2'.



**Figure 1.** Cracked shaft model

When the crack grows, strain energy concentration occurs in the vicinity of the crack tip, the energy release due to crack propagation will cause structural compliance changes. It is assumed that the structural crack region is in the elastic phase. Based on the linear elastic fracture mechanics theory, the strain energy release rate is defined as the strain energy dissipated during increase the crack surface area.

$$G = \frac{\partial U}{\partial A} \quad (1)$$

The strain energy release rate due to crack growth can also be expressed as

$$G = \frac{1}{E'} (K_I^2 + K_{II}^2 + (1 + \nu) K_{III}^2) \quad (2)$$

where E' is elastic modulus, E' = E/(1-ν<sup>2</sup>) for plane strain, E'=E for plane stress. According to the beam theory, a shaft subjected to a bending moment M, the strain energy due to Δb is

$$\Delta V_e = \frac{M^2}{2} \frac{1}{EI} \Delta b \quad (3)$$

where  $\Delta U$  denotes the strain energy required for crack expansion. When  $\Delta b \rightarrow 0$

$$\frac{\partial U}{\partial b} = -\frac{M^2}{2E} \left( \frac{1}{I^c} - \frac{1}{I} \right) \quad (4)$$

Assuming that the bending stiffness of the fracture section is ' $EI^c$ ' and the bending stiffness of the uncracked section is ' $EI$ '. Therefore the strain energy due to crack growth is expressed as

$$\Delta U = \frac{M^2}{2} \left( \frac{1}{EI} - \frac{1}{EI^c} \right) \Delta b \quad (5)$$

The strain energy take the following form when it is subjected to a axial tension or shear

$$\frac{\partial U}{\partial b} = -\lambda \frac{S^2}{2u} \left( \frac{1}{A^c} - \frac{1}{A} \right) \quad (6)$$

where  $A$  is the uncracked cross-section area of the shaft, ' $A^c$ ' is the area of the cracked cross section, and  $u$  is the shear modulus. According to the method proposed by Kienzler and Herrmann[11]. The energy release rate  $\partial U / \partial a$  due to crack growth is related to that one due to crack widening  $\partial U / \partial b$  can be expressed as

$$\frac{\partial U}{\partial a} = 2\beta \frac{\partial U}{\partial b} \quad (7)$$

Therefore

$$\frac{\partial U}{\partial a} = -4R \frac{K^2}{E'} \sqrt{\eta - \eta^2} \quad (8)$$

where  $\eta = a / 2R$  is crack ratio, Substituting (4),(5),(6) into (9), we obtain

$$K_{I_N} = \frac{N}{2} \sqrt{\frac{\beta'}{R\sqrt{\eta - \eta^2}}} \left( \frac{1}{A^c} - \frac{1}{A} \right) \quad (9)$$

$$K_{I_M} = \frac{M}{2} \sqrt{\frac{\beta'}{R\sqrt{\eta - \eta^2}}} \left( \frac{1}{I^c} - \frac{1}{I} \right) \quad (10)$$

$$K_{II_S} = \frac{S}{2} \sqrt{\frac{2\lambda\beta'(1+\nu)}{R\sqrt{\eta - \eta^2}}} \left( \frac{1}{A^c} - \frac{1}{A} \right) \quad (11)$$

where  $\beta' = \beta E' / E$  and  $\beta$  is the slope of the stress diffusion lines. We first consider the shaft subjected to a tension. The position of the neutral axis changes because of the presence of cracks. The offset distance is

$$e = 16R^3 (\eta - \eta^2)^{\frac{3}{2}} / 3A^c \quad (12)$$

Then, a combined tension and bending moment occurs

$$K_{I_N} = \frac{N}{2} \sqrt{\frac{\beta'}{R\sqrt{\eta - \eta^2}}} \left( \frac{1}{A^c} - \frac{1}{A} \right) + \frac{eN}{2} \sqrt{\frac{\beta'}{R\sqrt{\eta - \eta^2}}} \left( \frac{1}{I^c} - \frac{1}{I} \right) \quad (13)$$

therefore

$$K_{I_N} = \frac{N}{R^{3/2}} F_N(\eta) \quad (14)$$

where

$$F_N(\eta) = \frac{1}{2} \sqrt{\frac{1}{\Delta} \left( \frac{1}{\Pi_1} - \frac{1}{\pi} \right) + \frac{8\Delta^3}{3\Pi_1} \sqrt{\frac{1}{\Delta} \left( \frac{1}{4} \Pi_2 + (1-2\eta)^3 \Delta - \frac{256\Delta^6}{9\Pi_1} \right)^{-1} - \frac{4}{\pi}}} \quad (15)$$

with

$$\begin{aligned} \Pi_2 &= \pi - \cos^{-1}(1-2\eta) - 2(1-2\eta)\sqrt{\eta-\eta^2} \\ \Pi_2 &= \pi - \cos^{-1}(1-2\eta) - 2(1-2\eta)\sqrt{\eta-\eta^2} \\ \Delta &= \sqrt{\eta-\eta^2} \end{aligned} \quad (16)$$

Similarly, when the shaft is subjected to a bending moments, we obtain

$$K_{I_M} = \frac{M}{2} \sqrt{\frac{\beta'}{R\sqrt{\eta-\eta^2}} \left( \frac{1}{I^c} - \frac{1}{I} \right)} = \frac{M}{R^{5/2}} F_M(\eta) \quad (17)$$

where

$$F_M(\eta) = \frac{1}{2} \sqrt{\frac{1}{\Delta} \left( \frac{1}{4} \Pi_2 + (1-2\eta)^3 \Delta - \frac{256\Delta^6}{9\Pi_1} \right)^{-1} - \frac{4}{\pi}} \quad (18)$$

When the shaft is subjected to a shear, its stress intensity factor is

$$K_{II_s} = \frac{S}{2} \sqrt{\frac{2\lambda\beta'(1+\nu)}{R\sqrt{\eta-\eta^2}} \left( \frac{1}{A^c} - \frac{1}{A} \right)} = \frac{S}{R^{3/2}} F_s(\eta) \quad (19)$$

where

$$F_s(\eta) = 0.8832 \sqrt{\frac{1}{\Delta} \left( \frac{1}{\Pi_1} - \frac{1}{\pi} \right)} \quad (20)$$

### 3. Local compliance of the cracked shaft

According to the Castigliano's theorem, if strain energy is expressed as a function related to the force, and then the strain energy can be divided into the displacement  $u_i$  in the direction of the force with respect to the forces as follows

$$u_i = \frac{\partial U}{\partial Q_i} \quad (21)$$

where  $Q_i$  is the force direction of the displacement  $u_i$ . The compliance coefficients of the shaft subjected to the axial tensile, bending moment and the shear force is given as

$$c_{ij} = \frac{\partial u_i}{\partial Q_j} = \frac{\partial^2}{\partial Q_i \partial Q_j} \int_A \frac{1}{E'} [(K_{I_N} + K_{I_M})^2 + K_{II_s}^2] dA \quad (22)$$

Substituting equations (9), (10) and (11) into equation (23), the compliance coefficients can be computed as

$$c_{11} = \frac{2(1-\nu^2)}{ER^3} \int_0^a \int_{-b}^b F_N^2(\mu/D) dx d\mu \quad (23)$$

$$c_{22} = \frac{2(1-\nu^2)}{ER^3} \int_0^a \int_{-b}^b F_S^2(\mu/D) dx d\mu \quad (24)$$

$$c_{33} = \frac{2(1-\nu^2)}{ER^3} \int_0^a \int_{-b}^b F_M^2(\mu/D) dx d\mu \quad (25)$$

$$c_{13} = \frac{2(1-\nu^2)}{ER^4} \int_0^a \int_{-b}^b F_{MN}(\mu/D) dx d\mu \quad (26)$$

where

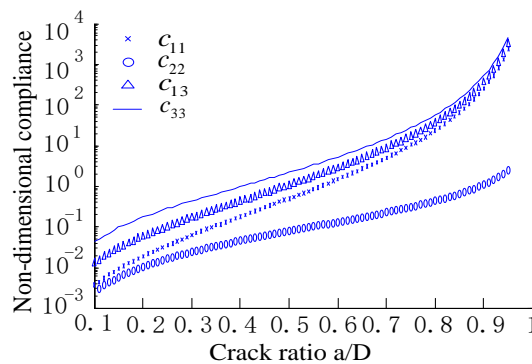
$$b = \sqrt{R^2 - y^2} = \sqrt{2Rz - \mu^2} \quad (27)$$

Therefore, the compliance matrix of nodal point is determined as

$$\mathbf{c} = \begin{bmatrix} c_{11} & 0 & c_{13} \\ 0 & c_{22} & 0 \\ c_{13} & 0 & c_{33} \end{bmatrix} \quad (28)$$

therefore

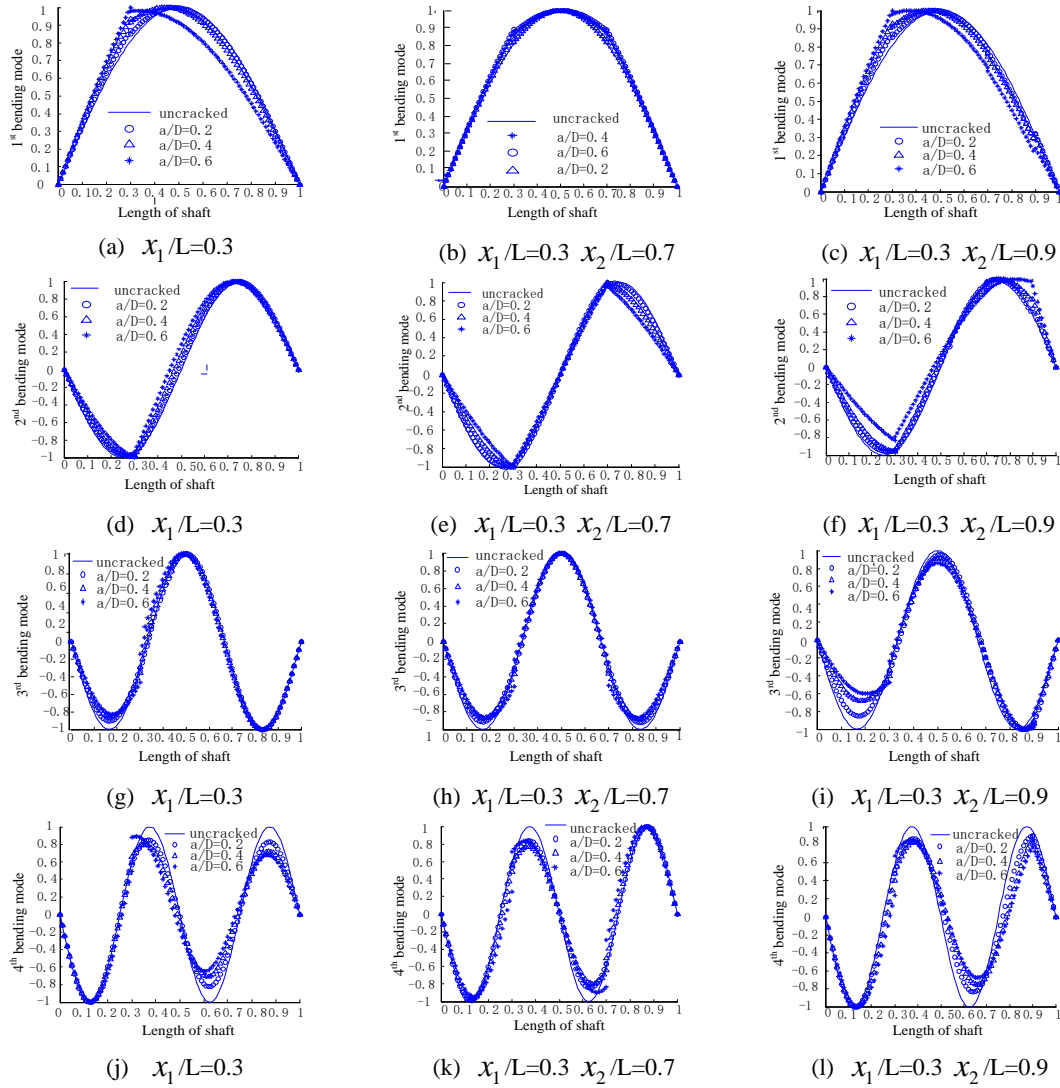
$$\mathbf{K}_f = \mathbf{c}^{-1} \quad (29)$$



**Figure 2.** Compliance coefficients

Figure 2 displays the effects of the cracks on compliance coefficients. Results show that the compliance coefficient increases with the increasing of the crack depth. And the local flexibility coefficient is the largest when the shaft is subjected to the bending moment, and that is minimum one when subjected to shear to the same crack form of the shaft. When an uncracked shaft element subjected to a bending moment  $M$ , a shear force  $S$  and a axial tensile  $N$  at the same time, the strain energy is

$$E = \frac{1}{2EI} (M^2 l - MS l^2 + \frac{S^2 l^3}{3}) + \frac{N^2 l}{2EA} + \frac{S^2 l}{\mu \Lambda} \quad (30)$$



**Figure 3.** First four bending modes for shafts

According to the equation (30), the stiffness matrix of the uncracked element can be expressed as

$$K = \begin{bmatrix} \frac{AE}{l} & 0 & 0 & -\frac{AE}{l} & 0 & 0 \\ 0 & \frac{12EI}{(1+\Gamma)l^3} & \frac{-6EI}{(1+\Gamma)l^2} & 0 & -\frac{12EI}{(1+\Gamma)l^3} & \frac{6EI}{(1+\Gamma)l^2} \\ 0 & -\frac{6EI}{(1+\Gamma)l^2} & \frac{(4+\Gamma)EI}{(1+\Gamma)l} & 0 & \frac{6EI}{(1+\Gamma)l^2} & \frac{(2-\Gamma)EI}{(1+\Gamma)l} \\ -\frac{AE}{l} & 0 & 0 & -\frac{AE}{l} & 0 & 0 \\ 0 & -\frac{12EI}{(1+\Gamma)l^3} & \frac{6EI}{(1+\Gamma)l^2} & 0 & \frac{12EI}{(1+\Gamma)l^3} & \frac{6EI}{(1+\Gamma)l^2} \\ 0 & \frac{-6EI}{(1+\Gamma)l^2} & \frac{(2-\Gamma)EI}{(1+\Gamma)l} & 0 & \frac{6EI}{(1+\Gamma)l^2} & \frac{(4+\Gamma)EI}{(1+\Gamma)l} \end{bmatrix} \quad (31)$$

Assuming that the cracked section is modeled as two planes joined together by a line-spring with zero length. Therefore, the dynamic equation of the cracked shaft can be expressed as

$$M \ddot{x} + (K + K_f)x = 0 \quad (32)$$

where  $\mathbf{M}$  is the mass matrix of the system,  $\mathbf{K}$  is the stiffness matrix. The problem of determining the natural vibration frequencies and the associated mode shapes of a system is transformed into solving the eigenproblem of the homogeneous linear system, such as

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{A} = 0 \quad (33)$$

#### 4. Results and discussion

The geometry and material properties of the cracked shaft is shown in Table 1.

**Table 1.** Basic Parameters of shaft

Basic Parameters	value
Young's modulus E	210Gpa
Poisson's ratio $\nu$	0.33
Mass density $\rho$	7900kg/m <sup>3</sup>
Diameter d	0.1m
Length L	1m

Assuming that the boundary conditions of the shaft are simply supported on both end. Crack position is set as  $x_1/L=0.3$ . Shafts having two cracks, the crack positions are set as  $x_1/L=0.3$ ,  $x_2/L=0.7$  or  $x_2/L=0.9$ . And the crack depth is set as  $\eta=0.2$ , 0.4 and 0.6.

Figure 3 shows the first four bending modes of the shaft with single crack and double cracks respectively. When a crack is in the shaft, the vibration mode changes, a discontinuity occurs at the crack position. Comparing these curves in Figure 3, it can be found that the mode shape appears discontinuous at the 0.3. With the increasing of modal frequencies, discontinuous interval increases too. When the crack position is at 0.3 and 0.7, the vibration mode appears discontinuous at these two positions. When the position of the first crack remains unchanged, and the location of the second crack is changed to the position at 0.9. Discontinuous appear at the position of 0.3 and 0.9, it shows that this character not only can be used to identify the location of cracks, but also can be used to identify the number of cracks. It is noted that with the increase of the crack depth, discontinuity also become larger. This is helpful to find the crack and crack depth. However, it is also found that this phenomenon is not obvious when the cracks are located close to the nodal position.

#### 5. Conclusion

With the help of theoretical calculations, an identification method of cracks in a shaft is discussed. The main conclusions are as follows

- Based on the fracture mechanics and basic beam theory, the stress intensity factors of a shaft subjected to axial tension, bending moment and shear force are derived. The local compliance model of the transverse crack shaft is calculated by using the Castigliano's theorem and the expressions of local compliance of the cracked shaft are given.
- The vibration modal shapes of the cracked shaft are calculated by the differential equation of motion. The variation of the vibration mode of a shaft having different crack locations and crack numbers are discussed. The model established in this paper is very convenient for the damage identification of the shaft.

#### 6. References

- [1] Zhong J F, Zhong S F, Zhang Q K 2019 *Mech. Syst. Signal. Pr.* **115** 1
- [2] Zhang Y F, Wang S S, Gan S L 2016 *J. Vib. Shock.*, **35** 11



- [3] Gounaris G, Dimarogonas A 1988 *Comput. Struct.* **28** 3
- [4] Zhang Y J, Yan Y H, Li Y Q 2012 *J. Northeast. Univ.* **33** 3
- [5] Cruz-Vega I, Rangel-Magdaleno J, Ramirez-Cortes J and Peregrina-Barrito H 2017 *J. Mech. Sci. Technol.* **31** 6 :
- [6] Ratan S, Baruh H and Rodriguez J 1996 *J. Sound. Vib.* **194** 1
- [7] Hwang Y H and Kim 2004 *J. Sound, Vib.* **270** 1
- [8] Yang X F, Swamidas A S J, Seshadri R 2001 *J. Sound. Vib.* **244** 2
- [9] Lin Y L and Chu F 2010 *Mech. Syst. Signal. Pr.* **24** 2
- [10] Bovsunovsky A P 2018 *Electr. Pow. Syst. Res.* **154** 1
- [11] Kienzler R and Herrmann G 1986 *Acta. Mech.* **62**

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