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Extended Order High Gain Observer Based Stabilization of 2 DOF Pan Tilt Platform for Aerial Imaging System

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Abstract. Aerial imaging systems are mounted on two axis serial robotic manipulators with high powered pan and tilt actuations, known as a Gimbal. The angular displacement of the gimbal is measured through very high precision sensors, typically shaft encoders with high PPR specifications. The purpose of the gimbal is to keep the orientation of the payload towards the target irrespective of the displacement of the UAV (unmanned aerial vehicle) and any other vibrations/ disturbances that might be introduced into the system. Because of 2-DOF (degrees of freedom) dynamical model of the gimbal, frictional force, noise, cable restraint, disturbances from the external environment along with the motions of the vehicle's body (as a result of manoeuvring or vibration), the spotting and tracking accuracy of the gimbal platform system may significantly degrade. The nature of these redundant disturbances is mostly of nonlinear nature and their modelling is a difficult task as they keep on changing. In order to cancel out these redundant disturbances and uncertainties, extended order high gain observer (EHGO) based feedback linearization control is used and extensive simulations are performed to show the comparison of tracking control without EHGO and after augmenting EHGO.

1. Introduction
Inertial stabilization platforms (ISPs) have various engineering applications in modern world like these platforms can be used as housing for military weapons, astronomical equipment and cameras. Designs of these systems are configured according to the requirements, applications and desired performance [1]. The primary objective of these models is to mitigate the effects of aerial manoeuvres, platform motions, engine vibrations and nonlinear disturbances. All of these disturbances results in decreasing the accurate pointing capability of the ISPs [2].

The stability of imaging systems mounted on gimbal, on board UAVs is fundamental for accurate surveillance/ target, more so, for long range sensors/ target designators termed as payload. In this regard a robust nonlinear control scheme based on detailed mathematical model for the two axis gimbal of the UAV that houses the payload is the solution since the gimbal is subjected to numerous un-modelled disturbance which include and are not limited to parameter uncertainties, wind gusts, engine vibrations etc.

The paper includes mathematical modelling of the 2-axis gimbal, nonlinear tracking controller design for it and simulations of the system with its capability to reject unwanted disturbances.

Paper is divided into following subsections, mathematical modelling is defined in section 2 followed by feedback linearization in section three. Section 4 explains extended order high gain observer design and simulation results are shown in section 5. Conclusion is given in section 6.

2. Mathematical Modelling

As is customary in the field of aerospace and robotics, quantification of moving elements is accomplished by attaching frames to links. In case of gimbal the base frame which is attached with the UAV is marked as frame 0. The motion of first rotary joint, termed as pan, is represented by frame 1. The first joint is also named as outer gimbal in some texts. The second joint has frame 2 attached with it. It is pertinent to note here that the first link rotates about the z_1 axis by an angle θ_1 (pan angle) and the second link rotates about the y_2 axis by and angle θ_2 (tilt angle). The affixing of frames provides the foundation for dynamic modelling of the gimbal. [3, 4].
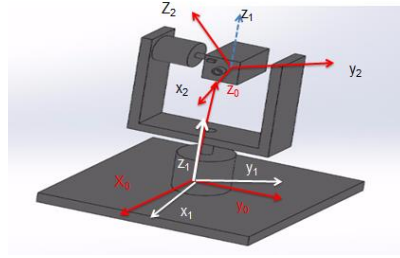

Figure 1. Schematic model of a 2 DOF Gimbal

Mathematical modelling of a system is a preliminary step for control design. As the 2-axes gimbal is a two link serial robotic manipulator, its modelling is done in the classical style of robotic manipulators. The method is well-known "Iterative Newton Euler Dynamics". [3].

The dynamic model of 2-axes gimbal is evolved using the kinematic and dynamic expressions listed above. The kinematic expressions are evolved from base frame to link 2. The base frame is considered static for a slowly maneuvering aircraft [3].

2.1 Base Frame

Linear velocity, angular velocity, linear acceleration, angular acceleration turns out to be

$$
\begin{aligned}
{}^0\upsilon_0 &= [0,0,0]^T \;,\; {}^0\omega_0 = [0,0,0]^T \\
{}^0\dot{\upsilon}_0 &= [0,0,0]^T \,,\, {}^0\dot{\omega}_0 = [0,0,0]^T
\end{aligned}
\tag{1}
$$

2.2 Inner Gimbal

The rotation matrix of frame 1 with respect to base frame is given by

$$
{}^0_1R = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\tag{2}
$$

Linear velocity, angular velocity, linear acceleration and angular acceleration of frame 1 are given below

$$
\begin{aligned}
{}^1\upsilon_1 &= [0\ 0\ 0]^T, \; {}^1\dot{\upsilon}_1 = [0\ 0\ 0]^T \\
{}^1\omega_1 &= [0\ 0\ \dot{\theta}_1]^T, \; {}^1\dot{\omega}_1 = [0\ 0\ \ddot{\theta}_1]^T
\end{aligned}
\tag{3}
$$

2.3 Outer Gimbal

The outer gimbal is serially connected with the inner gimbal. The outer gimbal rotates about its *y* axis. The rotation matrix of the outer gimbal with respect to the inner gimbal is given as:

$$
{}^1_2R = \begin{bmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{bmatrix}
\tag{4}
$$

The linear velocity, angular velocity, linear acceleration and the angular acceleration of the outer gimbal are expressed as:

$$^2\upsilon_2 = [0\ 0\ 0]^T,\ ^2\dot{\upsilon}_2 = [0\ 0\ 0]^T$$
$$^2\omega_2 = [-s_2\dot{\theta}_1\ \ \dot{\theta}_2\ \ c_2\dot{\theta}_2]^T$$
$$^2\dot{\omega}_2 = [-s_2\ddot{\theta}_1 - c_2\dot{\theta}_1\dot{\theta}_2\ \ \ddot{\theta}_2\ \ c_2\ddot{\theta}_1 - s_2\dot{\theta}_1\dot{\theta}_2]^T$$

$$(5)$$

According to the Newton's laws the resultant force acts on the centre of gravity (CG) of a rigid body. This warrants locating the CG of each link. The CG of the inner and outer gimbals expressed with respect to their frames of rotation is given as follows.

$$^1p_{C_1} = \begin{bmatrix} 0 \\ 0 \\ z_{c1} \end{bmatrix},\ ^2p_{C_2} = \begin{bmatrix} 0 \\ 0 \\ z_{c2} \end{bmatrix}$$

Similarly the Euler equations for rigid bodies require inertia matrices. The inertia matrices for the two gimbals are presented as follows:

$$^{c1}I_1 = \begin{bmatrix} I_{1x} & 0 & 0 \\ 0 & I_{1y} & 0 \\ 0 & 0 & I_{1z} \end{bmatrix},\ ^{c2}I_2 = \begin{bmatrix} I_{2x} & 0 & 0 \\ 0 & I_{2y} & 0 \\ 0 & 0 & I_{2z} \end{bmatrix}$$

The evaluation of kinematic equations for the 2-axes gimbal leads to the expressions for resultant forces and moments on each link. The resultant forces on inner and outer gimbals are expressed as:

$$^1F_1 = {}^2F_2 = [0\ 0\ 0]^T$$

$$(6)$$

The expressions for resultant moment on gimbal is given by

$$^1N_1 = [0\ 0\ J_{1z}\ddot{\theta}_1]^T$$

$$(7)$$

Similarly the expression for net moment on the outer gimbal is as follows

$$^2N_2 = \begin{bmatrix} \left(-s_2\ddot{\theta}_1 - c_2\dot{\theta}_1\dot{\theta}_2\right)J_{2x} + c_2\dot{\theta}_1\dot{\theta}_2 J_{2z} - c_2\dot{\theta}_1\dot{\theta}_2 J_{2y} \\ \ddot{\theta}_2 J_{2y} - c_2 s_2 \dot{\theta}_1^2 J_{2x} + c_2 s_2 \dot{\theta}_1^2 J_{2z} \\ \left(c_2\ddot{\theta}_1 - s_2\dot{\theta}_1\dot{\theta}_2\right)J_{2z} + s_2\dot{\theta}_1\dot{\theta}_2 J_{2x} - s_2\dot{\theta}_1\dot{\theta}_2 J_{2y} \end{bmatrix}$$

$$(8)$$

2.4 Dynamical modeling

The next step in dynamic modeling [3] of the gimbal is inward iteration for calculation of moments applied by link 1 (inner gimbal) on link 2 (outer gimbal) and by base on link. The moment applied by inner gimbal on outer gimbal is given as:

$$^2n_2 = \begin{bmatrix} -\left(\ddot{\theta}_1 s_2 + \dot{\theta}_1\dot{\theta}_2 c_2\right)I_{2x} - \dot{\theta}_1\dot{\theta}_2 c_2 I_{2y} + \dot{\theta}_1\dot{\theta}_2 c_2 I_{2z} \\ -\dot{\theta}_1^2 c_2 s_2 I_{2x} + \ddot{\theta}_2 I_{2y} + \dot{\theta}_1^2 c_2 s_2 I_{2z} \\ \dot{\theta}_1\dot{\theta}_2 s_2 I_{2x} - \dot{\theta}_1\dot{\theta}_2 s_2 I_{2y} + \left(\ddot{\theta}_1 c_2 - \dot{\theta}_1\dot{\theta}_2 s_2\right)I_{2z} \end{bmatrix}$$

$$(9)$$

The second row represents the rotary joint about which rotation can take place. This rotation is caused by the actuator torque τ_2. Equating the second row with the actuator torque and rearranging leads to the following expression which is the desired dynamic equation for the second joint.

$$\ddot{\theta}_2 = \frac{\tau_2 + \dot{\theta}_1^2 c_2 s_2 I_{2x} - \dot{\theta}_1^2 c_2 s_2 I_{2z}}{I_{2y}} \tag{10}$$

Proceeding inwards further, the moment applied by the base frame on the inner gimbal is calculated and after evaluating, the following dynamic equation for the first joint is obtained [3].

$$\ddot{\theta}_1 = \frac{\tau_1 - 2\dot{\theta}_1\dot{\theta}_2 c_2 s_2 I_{2x} + 2\dot{\theta}_1\dot{\theta}_2 c_2 s_2 I_{2z}}{I_{1z} + s_2^2 I_{2x} + c_2^2 I_{2z}} \tag{11}$$

3. Feedback Linearization

Feedback linearization is a control technique used for controlling nonlinear systems by choosing an appropriate control input and transforming the nonlinear system into a linearized system [6, 7]. In order to linearize the system model, first it should be written in state space form.

3.1 State space model

The two joint angles and their respective velocities are chosen as state variable as follows:

$$x_1 = \theta_1, x_3 = \theta_2, u_1 = \tau_1, u_2 = \tau_2$$

The state space representation of the 2-axis gimbal model is given by

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{(-2J_{2x}\cos x_3 \sin x_3.x_2x_4 - 2J_{2z}\cos x_3 \sin x_3.x_2x_4) + u_1}{J_{1z} + J_{2z}(\cos x_3)^2 + J_{2x}(\sin x_3)^2} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{(J_{2z}\sin x_3 \cos x_3.(x_2)^2 - J_{2x}\sin x_3 \cos x_3.(x_2)^2) + u_2}{J_{2y}}
\end{aligned}
\tag{12}
$$

3.2 Control Design

The next step towards designing EHGO is control design [8, 9].

The control objective for the 2-axes gimbal is the tracking of desired pan and tilt angles for the gimbal. The desired angles are computed by the geo-pointing algorithm on the basis of target location, UAV location and UAV attitude provided by the on board gyros. The desired reference angles for pan and tilt are respectively termed as r_1, r_2. The control objective is to asymptotically track these angles which are mathematically defined as

$$\lim_{t\to\infty}(\theta_1 - r_1) \to 0, \quad \lim_{t\to\infty}(\theta_2 - r_2) \to 0 \tag{13}$$

In order to achieve tracking the state space representation transformed into error coordinates is

$$
\begin{aligned}
e_1 &= x_1 - r_1 \\
e_2 &= x_2 - \dot{r}_1 \\
e_3 &= x_3 - r_2 \\
e_4 &= x_4 - \dot{r}_2
\end{aligned}
\tag{14}
$$

The error dynamics are given as

$$\dot{e}_1 = e_2$$
$$\dot{e}_2 = f_1(x) + g_1(x)u_1 - \ddot{r}_1$$
$$\dot{e}_3 = e_3$$
$$\dot{e}_4 = f_2(x) + g_2(x)u_2 - \ddot{r}_2$$

(15)

where for convenience of notation, the nonlinear terms are defined as

$$f_1(x) = \frac{2(\sin x_3 \cos x_3 . x_2 . x_4).(J_{2z} - J_{2x})}{J_{1z} + J_{2z} \cos x_3^2 + J_{2x} \sin x_3^2}$$

$$f_2(x) = \frac{(\sin x_3 \cos x_3 . x_2^2).(J_{2z} - J_{2x})}{J_{2y}}$$

$$g_1(x) = \frac{1}{J_{1z} + J_{2z} \cos x_3^2 + J_{2x} \sin x_3^2}$$

(16)

$$g_2(x) = \frac{1}{J_{2y}}$$

Tracking is achieved by the following state feedback control law which is based on feedback linearization technique [5,6]

$$u_1 = g_1^{-1}(x)\left(-f_1(x) + \ddot{r}_1 + v_1\right)$$
$$u_2 = g_2^{-1}(x)\left(-f_2(x) + \ddot{r}_2 + v_2\right)$$

(17)

The above control input linearizes the error dynamics of the system, which are given by

$$\dot{e}_1 = e_2$$
$$\dot{e}_2 = v_1$$
$$\dot{e}_3 = e_4$$
$$\dot{e}_4 = v_2$$

(18)

Equation (3.7) is of a controllable linear system, the origin of which if stabilized, guarantees asymptotic tracking. To this effect following input is applied [7]

$$v_1 = 33.35e_1 + 11.58e_2 - 2.92e_3 - 0.51e_4$$
$$v_2 = -2.47e_1 - 0.47e_2 + 38.39e_3 + 11.41e_4$$

(19)

where the gains are selected such that the eigenvalues of the closed loop system are at -5, -5.5, -6, -6.5.

4. Extended Order High Gain Observer

In any practical control system, the availability of all system states is not possible, to account for the unmeasured system states, observers are used, for nonlinear systems like the 2-axes gimbal, a variant of the high gain observer, known as extended order high gain observer (EHGO) has been employed. The strength of the high gain observer is that it can estimate modelling uncertainties and disturbances in addition to system states [8]. The EHGO for the 2-axes gimbal is presented as follows [8, 9]

$$\dot{e}_1 = e_2 + (\frac{\alpha_1}{\varepsilon}).(e_1 - e_1)$$

$$\dot{e}_2 = g_1.u_1 - \ddot{r}_1 + (\frac{\alpha_2}{\varepsilon^2}).(e_1 - e_1) + \sigma_1$$

$$\dot{\sigma}_1 = (\frac{\alpha_3}{\varepsilon^3}).(e_1 - e_1)$$

$$\dot{e}_3 = e_4 + (\frac{\alpha_1}{\varepsilon}).(e_3 - e_3)$$

$$\dot{e}_4 = g_2.u_2 - \ddot{r}_2 + (\frac{\alpha_2}{\varepsilon^2}).(e_3 - e_3) + \sigma_2$$

$$\dot{\sigma}_2 = (\frac{\alpha_3}{\varepsilon^3}).(e_3 - e_3)$$

(20)

The observer design parameters are given as ϵ=0.1, α_1=6, α_2=11, α_3=6. With the estimates of all perturbations made available by the EHGO, the control law is modified as:

$$u_1 = g_1^{-1}(x)(-f_1(x) + \ddot{r}_1 + v_1 - \sigma_1)$$

$$u_2 = g_2^{-1}(x)(-f_2(x) + \ddot{r}_2 + v_2 - \sigma_2)$$

(21)

5. Simulation Results

The tracking control law is simulated for tracking constant and sinusoidal signals. Two different scenarios for perturbation are considered. In one case EHGO is not employed and it becomes evident that tracking error persists, in the other case when estimates of perturbations obtained from the EHGO are incorporated into the control law, perfect tracking is achieved. The tracking results for various scenarios are presented as follows

5.1 Tracking without EHGO
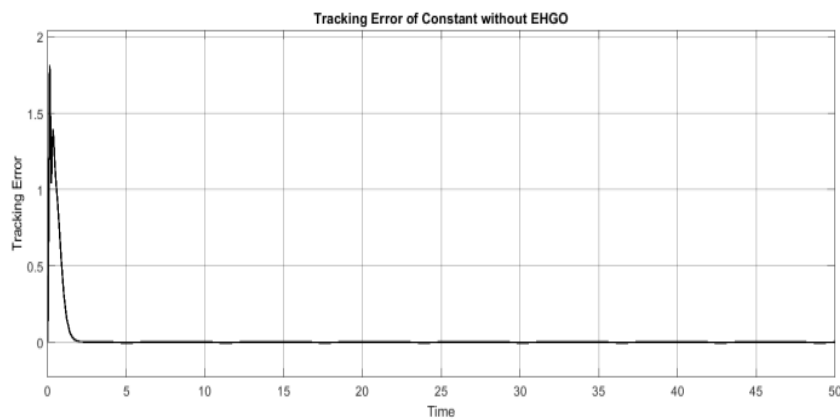
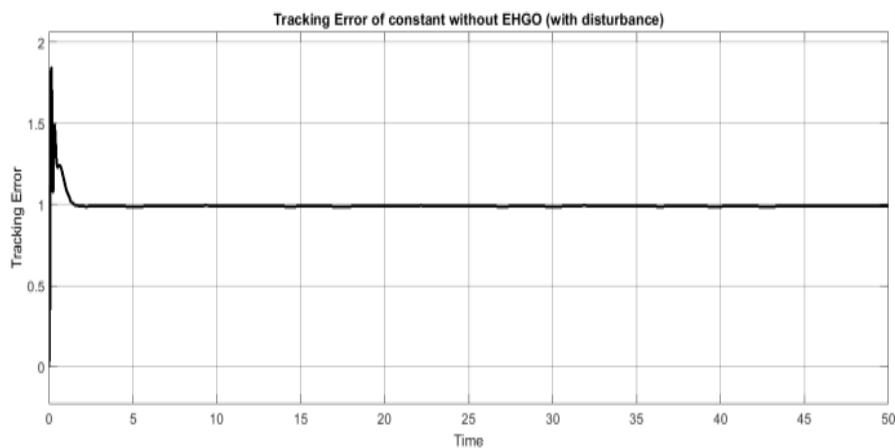**Figure 2.** Tracking error of a constant without EHGO

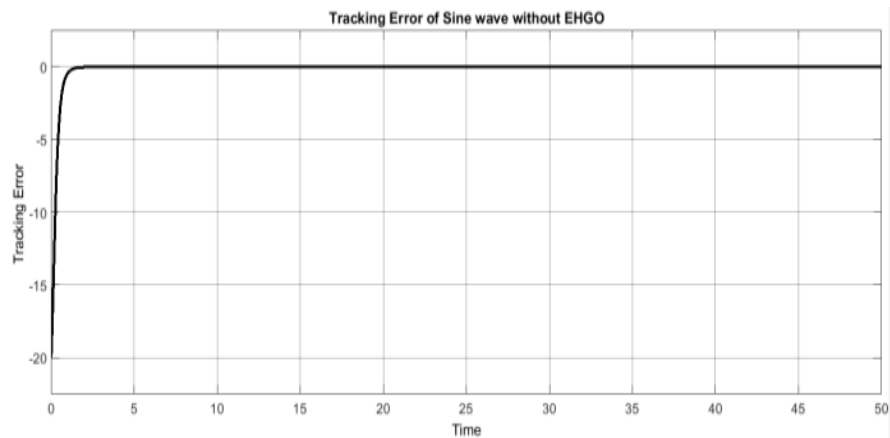**Figure 3.** Tracking error of a constant without EHGO with disturbance

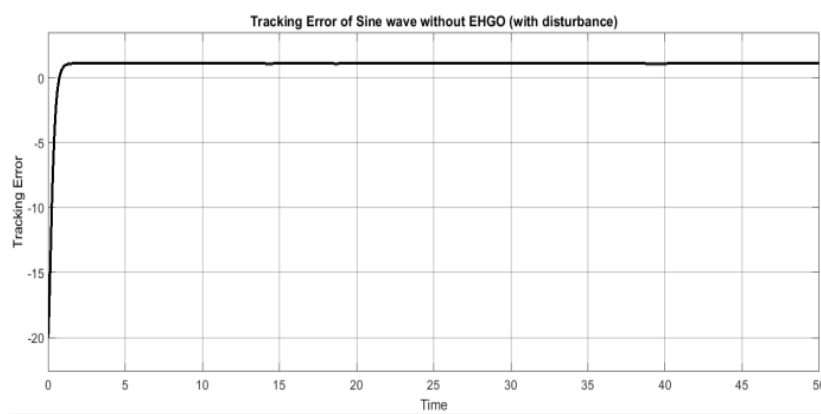**Figure 4.** Tracking error of a sine wave without EHGO

Figure 5. Tracking error of a sine wave without EHGO with disturbance
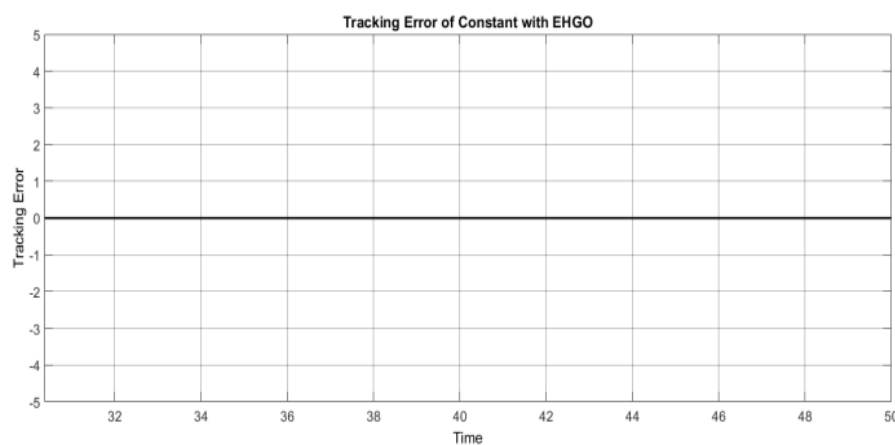
5.2 EHGO based tracking

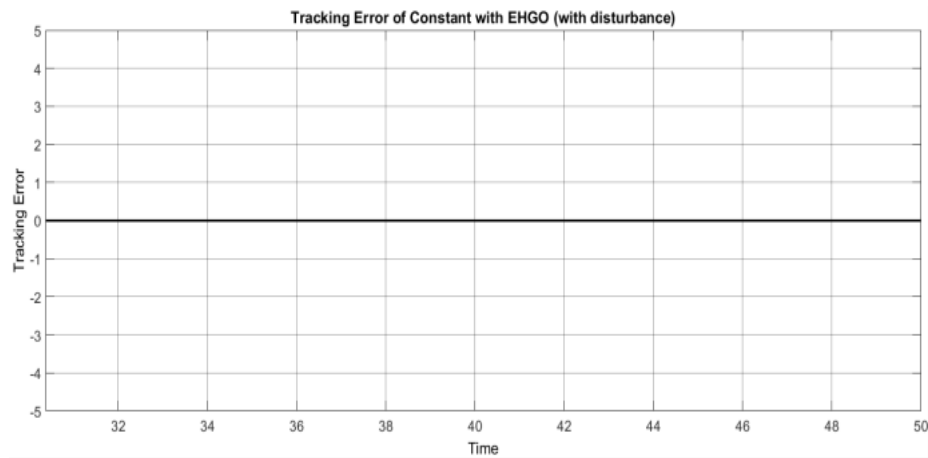**Figure 6.** Tracking error of constant with EHGO

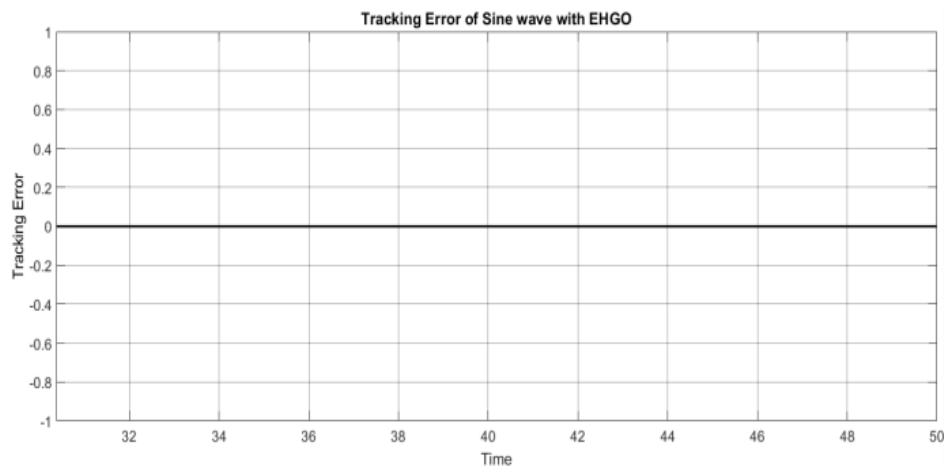**Figure 7.** Tracking error of constant with EHGO (with disturbance)

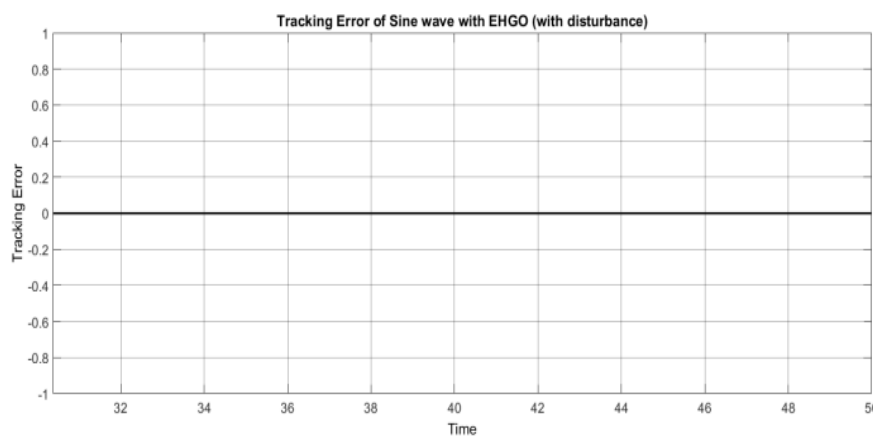**Figure 8.** Tracking error of a sine wave with EHGO

Figure 9. Tracking error of a sine wave with EHGO (with disturbance)

6. Conclusion

The simulations indicate the efficacy of the EHGO in estimating perturbations which are then used to achieve asymptotic tracking with feedback linearization tracking control law. The tracking of reference pan and tilt angles for the 2-axes gimbal is achieved.

7. References

[1]   J. Hilkert. Inertially stabilized platform technology concepts and principles, IEEE Control Systems, *28*(**1**), 26-46 ,(2008).

[2]   P. J. Kennedy, R. L. Kennedy. *Direct Versus Indirect Line of Sight (LoS) Stabilization*. IEEE Transactions on Control Systems Technology, **11(1):**3–15, (2003).

[3]   J.J Craig. *Introduction to Robotics: Mechanics & Control*, (1986)

[4]   B. Ekstrand. Equations of Motion for A Two Axes Gimbal System, IEEE Transactions on Aerospace and Electronic Systems, **37(3)**:1083–1091, (2001).

[5]   M.K. Masten, "Inertially Stabilized Platform for Optical Imaging Systems", IEEE Control Systems Magazine, February, (2008).

[6]   A. Isidori, *Nonlinear Control Systems*, Springer, (1985)

[7]   H. K Khalil, J. Grizzle, *Nonlinear systems*, vol.**3** (2002).

[8]   H. K Khalil. Extended High-Gain Observers as Disturbance Estimators, SICE Journal of Control, Measurement and System Integration, **Vol. 10**, **No. 3**, pp. 125–134, May (2017)

[9]   H. K Khalil, L. Praly, Generalized Extended State Observer Based High Precision Attitude Control of Quadrotor Vehicles Subject to Wind Disturbance, International Journal of Robust and Nonlinear Control, **24**, 993–1015, (2014).

[10] DI Shi, Zhong Wu AND Wusheng Chou, *High-gain observers in nonlinear feedback control*, IEEE Access, **Volume 6**, (2018).