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Abstract. The fusion barrier distribution (D_{fus}) for the systems $^{16}O+^{144}Sm$ and $^{16}O+^{208}Pb$ systems had been analyzed in this work to test the benefit of using second derivative of Wong's formula for fusion cross section to extract D_{fus} from experimental data. The calculated D_{fus} using Wong's formula compared with the result of three-point difference formula, which is the usual method for extract D_{fus} from experimental cross section. The results showed that this method generated the fusion barrier distribution for both systems in an acceptable approach with respect to the relative position of distribution peaks when it is compared with the theoretical calculations and three-point difference formula.

1. Introduction

Fusion process between two nuclei is achieved by formed a compound nucleus and it is governed by overcoming a barrier formed as a resultant of two forces between collision partners, these forces are; repulsive Coulomb and attractive nuclear forces, and the barrier is called fusion barrier [1-3]. The simple theoretical model for manipulate fusion process is the potential model which treats each of the collision partners as a structureless particle, where the potential of the system is given in terms of relative distance between these partners. This model gives good prediction of fusion cross section compared with experimental data for light systems, while for heavy systems, the model fails at sub-barrier region, where it had been observed a large enhancement in fusion cross section over calculated one by potential model[4].

However, as attempt to interpret this enhancement in fusion cross section, researches indicated that there is a set of barrier in place of single barrier, these barriers arise from intrinsic states of nuclei such as vibrations and rotations. This spectrum of barriers is called fusion barrier distribution, and its shape is strongly depending on the properties of collision partners [5-7].

The usual theoretical model used to calculate fusion cross section for heavy system is coupled channel model, which has a potential dependents not only on relative distance but as well as on the internal degrees of freedom for collision partners (vibration or rotation deformations or particles transfer). The results of this model depend on the type and strength of deformations involved in calculations[8,9]. So to approach the shape of the extracted fusion barrier distribution, a certain deformation in the target or in the projectile or in both must to be assuming in the calculations.

Therefore, fusion barrier distribution for heavy ion systems have been widely investigated due to its significant role to probe the systems structures and to compare the theoretical calculations [9-12].

To get accurate calculation for the fusion barrier distribution, it is required an accurate and high quality fusion cross section data, furthermore, a good numerical method to calculate the second

derivative. So authors either try to perform experiments with high-intensity stable beams [13], or to improve the method used to extract fusion barrier distribution [14,15].

The aim of this work is to test the second derivative of Wong's formula and compare it with the three-point difference formula and theoretical calculations using CCFULL fortran90 code [16]. Where we have used Chi-square fitting of experimental fusion cross section with Wong's formula.

2. Theoretical part

Fusion barrier distribution $D_f(E)$ can be extracted from experimental fusion cross section σ according to the definition that is supposed by Rowley et al., [7] by taking the second derivative of $(E\sigma)$ with respect to center of mass energy E.

$$D_f(E) = \frac{d^2(E\sigma)}{dE^2} \tag{1}$$

Almost all calculations of barrier distribution depending on calculate the second derivative using three-point difference formula given by [17]:

$$\frac{d^2(E\sigma)}{dE^2} = 2\left(\frac{(E\sigma)_3 - (E\sigma)_2}{E_3 - E_2} - \frac{(E\sigma)_2 - (E\sigma)_1}{E_2 - E_1}\right)\left(\frac{1}{E_3 - E_1}\right) \tag{2}$$

which is calculated at energy $(E_1 + 2E_2 + E_3)/4$, and for equally spaced energy data, this formula is approximated to the following form:

$$\frac{d^2(E\sigma)}{dE^2} = \left(\frac{(E\sigma)_3 - 2(E\sigma)_2 + (E\sigma)_1}{\Delta E^2}\right) \tag{3}$$

This method has two shortcoming, the first is it has statistical error given by

$$\delta_c \approx \left(\frac{E}{\Delta E^2}\right)[(\delta\sigma)_1^2 + 4(\delta\sigma)_2^2 + (\delta\sigma)_3^2]^{1/2} \tag{4}$$

growing with increasing in uncertainty of cross section $(\delta\sigma)$, which becomes more significant at higher energy. The second shortcoming is that this method needs more points for experimental cross section, in other word, this method is inefficient with the nuclear systems that have few points.

In this work, we introduce an alternative method to calculate D_f using least squares method of experimental cross section with Wong's formula to obtain an optimal values of fusion barrier parameters, which are fusion barrier height V_b, curvature $\hbar\omega$, and barrier radius R_b. The Wong's formula of fusion cross section is given by [18],

$$\sigma(E) = \frac{\hbar\omega R_b^2}{2E}\, ln\left[1 + exp\left(\frac{2\pi}{\hbar\omega}(E - V_b)\right)\right] \tag{5}$$

then the second derivative of $E\sigma$ is,

$$\frac{d^2(E\sigma)}{dE^2} = \frac{2\pi^2 R_b^2}{\hbar\omega}\,\frac{exp(x)}{\left(1 + exp(x)\right)^2} \tag{6}$$

where $x = 2\pi(E - V_b)/\hbar\omega$.

This method has been tested for two systems $^{16}O+^{208}Pb$ and $^{16}O+^{144}Sm$, which we show below their results.

3. Results and discussion

In this section, two systems have been analyzed. The experimental fusion cross section data have been treated in two different methods to extract the fusion barrier distribution, the first method is by using the traditional three-point difference formula eq. (2) , and the second method is by using the least squares fitting for Wong's formula eq. (5) with the experimental fusion cross section, then using the optimal parameters to get fusion barrier distribution from eq.(6). The theoretical treatment have been performed using CCFULL code [16].

The potential parameters which had been used in the calculations are displayed in table 1, and the parameters of CCFULL code which were used in the calculations are listed in table 2.

Table 1. Potential parameters.

System	V_0 (MeV)	r_0 (fm)	a_0 (fm)	V_b (MeV)
$^{16}O+^{208}Pb$	166	1.1	0.8	74.50 [19]
$^{16}O+^{144}Sm$	103	1.1	0.8	61.11 [20]

Table 2. Parameters of CCFULL calculations.

Nucleus	Radius parameter	λ^{π}	E^{*}(MeV)	$\boldsymbol{\beta_\lambda}$
^{16}O	1.20	-	-	-
^{144}Sm	1.06	3_1^-	1.810 [21]	0.205 [20]
		2_1^+	1.660 [21]	0.110 [20]
^{208}Pb	1.06	3_1^-	2.615 [22]	0.161 [23]
		5_1^-	3.198 [22]	0.056 [24]

3.1. $^{16}O+^{144}Sm$ System

The calculation of fusion cross section for this system is shown in Fig. 1(a), where the calculated cross section reproduced the experimental data very well when the vibration deformation of 144Sm had been taken into account with the parameters mentioned in Tables 1 and 2, the experimental data have been taken from Ref. [12] the main result of this study is shown in Fig. 1(b), where the second derivative of (Eσ) using three-point difference formula (solid black circle) produced almost two main peaks and the theoretical calculation with coupled channels (solid line) approaches these peaks, while Wong's formula eq.(6) (solid red circle) produced the first peak obviously, and the second peak with less candidness.

If we consider not the high of peaks, but instead their relative positions, then the Wong's formula can be consider as a good method to extract experimental fusion cross section specially this method does not depend on the number of data points conversely to three-point difference formula.

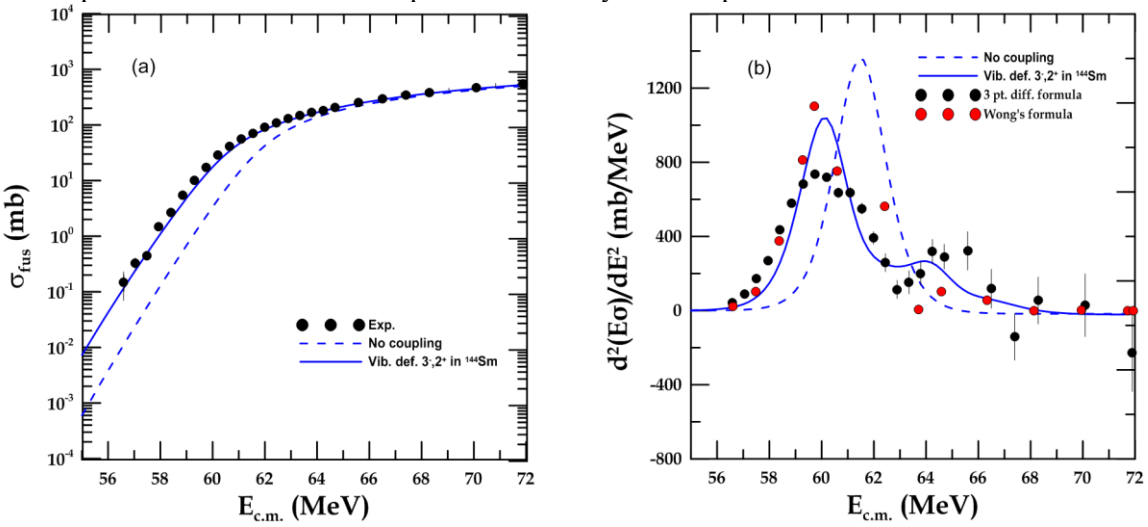

FIG. 1. The (a) fusion cross section and (b) barrier distributions for $^{16}O+^{144}Sm$ system, the single potential calculation (dashed line), the calculations using coupling to two single-phonon states (solid line), the results of three-point difference formula (solid black circle) and Wong's formula eq.(6) (solid red circle).

3.2. $^{16}O+^{208}Pb$ system

The second system which had been tested is $^{16}O+^{208}Pb$, Fig. 2(a) shows the theoretical calculations for the fusion cross section with parameters listed in Tables 1 and 2 as well as experimental data which have been taken from Ref. [19], where the solid line represents the coupling to 3^- and 5^- single phonon states in target nucleus ^{208}Pb, and dashed line is the calculation for single potential or (no coupling).

The calculations of fusion barrier distribution for this system is shown in Fig. 2(b), where it is obvious that this system showed also two peaks due to the deformation in target nucleus ^{208}Pb, for the theoretical calculation (solid line). Three-point difference method (solid black circle) showed clearly the main peak and more than one peaks beyond the first one. While Wong's method (solid red circle) also produced two peaks, one with significant high and other with relatively small high.

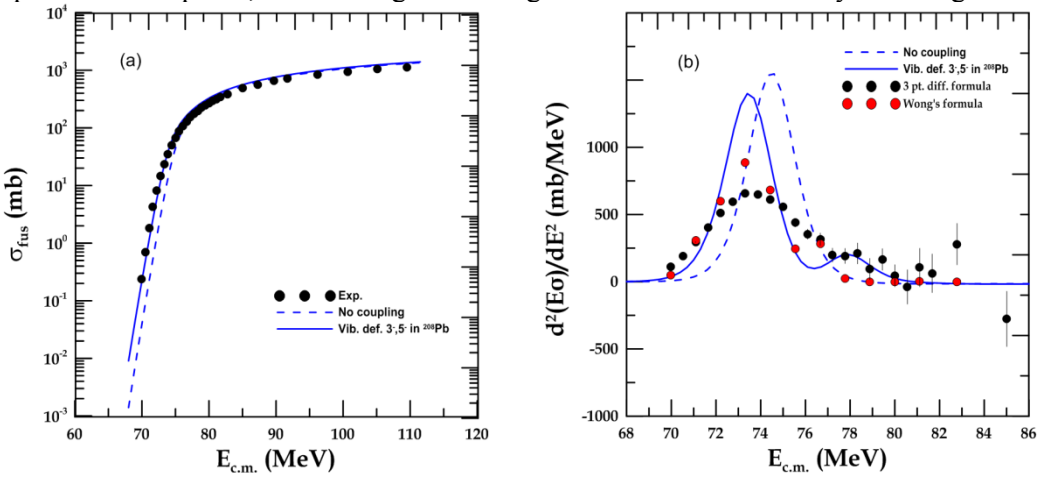

FIG. 2. The (a) fusion cross section and (b) fusion barrier distributions for $^{16}O+^{208}Pb$ system, the single potential calculation (dashed line), the calculations using coupling to two single-phonon states (solid line), the results of three-point difference formula (solid black circle) and Wong's formula eq.(6) (solid red circle).

4. Conclusions

In this work, fusion cross section and fusion barrier distribution for $^{16}O+^{144}Sm$ and $^{16}O+^{208}Pb$ systems have been analyzed theoretically, the theoretical calculation were performed using quantum mechanical coupled channel method with CCFULL code, the experimental data have been analyzed using the usual method (three-point difference formula) and new method which employed Wong's formula after it is fitted by least squares method with experimental data, then using eq. (6) to calculate the barrier distribution. It is being found that Wong's formula reproduced the barrier distribution in an acceptable manner compared to the three-point difference formula and theoretical calculations. The advantage of Wong's method is its independency on the number of data points, while the three-point difference formula is dependent significantly on the number of data points.

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