IOP Conference Series: Materials Science and Engineering

PAPER • OPEN ACCESS

Characteristic Zero Resolution of Weyl Module in the Case of the Partition (8,7,3)

To cite this article: Haytham R. Hassan and Niran S. Jasim 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **571** 012039

Characteristic Zero Resolution of Weyl Module in the Case of the Partition (8,7,3)

Haytham R. Hassan [1], Niran S. Jasim [2]

[1] Department of Mathematic, College of Science, Al-Mustansiriyah University
[2] Department of Mathematic, College of Education for Pure Science/ Ibn Al-Haitham, University of Baghdad

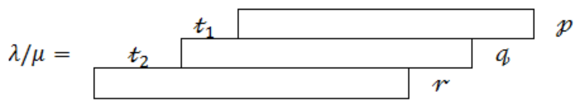
[1] haythamhassaan@uomustansiriyah.edu.iq, [2] sabahniran@gmail.com

Abstract. In this paper, we studied the resolution of Weyl module for characteristic zero in the case of partition (8,7,3) by using mapping Cone which enables us to get the results without depended on the resolution of Weyl module for characteristic free for the same partition.

1. Introduction

Let R be a commutative ring with 1 and $\mathcal{F}$ is a free R-module and $\mathcal{D}_i\mathcal{F}$ be the divided power algebra of degree i.

The resolution of partition $(p+t_1+t_2, q+t_2, r)$ which represented by below diagram and in our case $t_1 = t_2 = 0$.



Authors in [1 - 6] discussed the resolution of Weyl module for characteristic free for the partitions (4,4,4), (3,3,2), (6,6,3), (6,5,3), (7,6,3) and (8,7,3), respectively. Haytham R.H. and Niran S.J in [7] exhibit the terms and the exactness of the Weyl resolution in the case of partition (8,7). As well in [8] they illustrate the terms of characteristic-free resolution and Lascoux resolution of the partition (8,7,3).

Buchsbaum D.A. and Rota G.C. in [9] define the Capelli identities as:

Let $i, j, k, \ell \in \mathcal{P}^+$, then the divided powers of the place polarizations satisfy the following identities:

(1) If $k \neq j$, then

$$\partial_{ij}^{(r)}\partial_{jk}^{(s)} = \sum_{\alpha\geq 0}\partial_{jk}^{(s-\alpha)}\partial_{ij}^{(r-\alpha)}\partial_{ik}^{(\alpha)}$$

$$\partial_{jk}^{(s)}\partial_{ij}^{(r)} = \sum_{\alpha\geq 0}(-1)^{\alpha}\partial_{ij}^{(r-\alpha)}\partial_{jk}^{(s-\alpha)}\partial_{ik}^{(\alpha)}$$

(2) If $i \neq k$ and $j \neq \ell$ then $\partial_{ik}^{(s)}\partial_{i\ell}^{(r)} = \partial_{i\ell}^{(r)}\partial_{ik}^{(s)}$

In this work we survey the resolution of Weyl module for characteristic zero in the case of partition (8,7,3) by using mapping Cone without depending on the resolution of Weyl module for characteristic free for the same partition.

2. Characteristic-zero resolution of Weyl module with mapping Cone in the case of partition (8,7,3)

Before we study the resolution of Weyl module for characteristic-zero in isolation of characteristic-free, we need the definition of mapping Cone we review that as in [10]

Consider the following commute diagram

$$\begin{array}{lcccccccc}
C_0: & C_{n-1} & \xrightarrow{d_{n-1}} & C_n & \xrightarrow{d_n} & C_{n+1} & \xrightarrow{d_{n+1}} & C_{n+2} & \cdots \\
 & \downarrow{\scriptstyle f_{n-1}} & & \downarrow{\scriptstyle f_n} & & \downarrow{\scriptstyle f_{n+1}} & & \downarrow{\scriptstyle f_{n+2}} & \\
D_0: & D_{n-1} & \xrightarrow{d'_{n-1}} & D_n & \xrightarrow{d'_n} & D_{n+1} & \xrightarrow{d'_{n+1}} & D_{n+2} & \cdots
\end{array}$$

If the rows sequence are exact and

$\partial n-1: Cn \otimes Dn-1 \longrightarrow Cn+1 \otimes Dn$ defined by

$(\alpha,b) \mapsto (-dn(\alpha), d'_{n-1}(b)\, t + fn(\alpha))$ such hat $\partial n-1 \circ \partial n = 0; \forall\, n \in \mathbb{Z}^+$

Then the sequence

$$Cn-1 \xrightarrow{\partial_{n-1}} Cn \otimes Dn-1 \xrightarrow{\partial_n} Cn+1 \otimes Dn \xrightarrow{\partial_{n+1}} Cn+2 \otimes Dn+1 \xrightarrow{\partial_{n+2}} \ldots,$$

is exact.

Consider the complex of Lascoux in our partition (8,7,3) as the following diagram:

IOP Conf. Series: Materials Science and Engineering **571** (2019) 012039 doi:10.1088/1757-899X/571/1/012039

$$
\begin{tikzcd}
0 \arrow[r] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{7}\mathcal{F} \otimes \mathcal{D}_{1}\mathcal{F} \arrow[r, "h_1"] \arrow[d, "f_1"'] \arrow[dr, phantom, "\mathrm{A}"] & \mathcal{D}_{9}\mathcal{F} \otimes \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{1}\mathcal{F} \arrow[d, "g_1"] \\
0 \arrow[r] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{6}\mathcal{F} \otimes \mathcal{D}_{2}\mathcal{F} \arrow[r, "h_2"] \arrow[d, "f_2"'] \arrow[dr, phantom, "\mathrm{B}"] & \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{2}\mathcal{F} \arrow[d, "g_2"] \\
0 \arrow[r] & \mathcal{D}_{9}\mathcal{F} \otimes \mathcal{D}_{6}\mathcal{F} \otimes \mathcal{D}_{3}\mathcal{F} \arrow[r, "h_3"] & \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{7}\mathcal{F} \otimes \mathcal{D}_{3}\mathcal{F} \arrow[d, "d'"] \\
 & & \mathcal{K}_{(8,7,3)}(\mathcal{F}) \arrow[d] \\
 & & 0
\end{tikzcd}
$$

Diagram (2.1)

Where $h_1(v) = \partial_{21}(v)$; $v \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{7}\mathcal{F} \otimes \mathcal{D}_{1}\mathcal{F}$

$f_1(v) = \partial_{32}(v)$; $v \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{7}\mathcal{F} \otimes \mathcal{D}_{1}\mathcal{F}$

$h_2(v) = \partial_{21}^{(2)}(v)$; $v \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{6}\mathcal{F} \otimes \mathcal{D}_{2}\mathcal{F}$

$h_3(v) = \partial_{21}(v)$; $v \in \mathcal{D}_{9}\mathcal{F} \otimes \mathcal{D}_{6}\mathcal{F} \otimes \mathcal{D}_{3}\mathcal{F}$ and

$g_2(v) = \partial_{32}(v)$; $v \in \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{2}\mathcal{F}$

So we need to define g_1 which make the diagram A commute, i.e

$(\partial_{21}^{(2)}\,\partial_{32})(v) = (g_1 \circ \partial_{21})(v)$

From Capelli identities, we know that

$\partial_{21}^{(2)}\,\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31}$ and $\partial_{21}\,\partial_{31} = \partial_{31}\,\partial_{21}$

Then

$$
\begin{aligned}
\partial_{21}^{(2)}\,\partial_{32} &= \tfrac{1}{2}\partial_{32}\partial_{21}\partial_{21} - \partial_{21}\partial_{31} \\
&= \tfrac{1}{2}\partial_{32}\partial_{21}\partial_{21} - \partial_{31}\partial_{21} \\
&= \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\partial_{21}
\end{aligned}
$$

So we get $g_1(v) = \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(v)$; $v \in \mathcal{D}_{9}\mathcal{F} \otimes \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{1}\mathcal{F}$

IOP Conf. Series: Materials Science and Engineering **571** (2019) 012039 doi:10.1088/1757-899X/571/1/012039

Now if we use the mapping Cone to the following diagram

$$\begin{tikzcd} 0 \arrow[r] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{7}\mathcal{F} \otimes \mathcal{D}_{1}\mathcal{F} \arrow[r, "h_1"] \arrow[d, "f_1"'] & \mathcal{D}_{9}\mathcal{F} \otimes \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{1}\mathcal{F} \arrow[d, "g_1"] \\ & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{6}\mathcal{F} \otimes \mathcal{D}_{2}\mathcal{F} \arrow[r, "h_2"] & \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{2}\mathcal{F} \end{tikzcd}$$

We get the subcomplex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{7}\mathcal{F} \otimes \mathcal{D}_{1}\mathcal{F} \xrightarrow{\varphi_3} \begin{array}{c} \mathcal{D}_{9}\mathcal{F} \otimes \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{1}\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_{6}\mathcal{F} \otimes \mathcal{D}_{2}\mathcal{F} \end{array} \xrightarrow{\delta_1} \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{8}\mathcal{F} \otimes \mathcal{D}_{2}\mathcal{F} \qquad (1)$$

Where $\varphi_3(x) = (-\partial_{21}(x), \partial_{32}(x))$ and

$$\delta_1(x_1, x_2) = (\ \partial_{21}^{(2)}(x_2) + \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(x_1)$$

Proposition (2.1):

$$\delta_1 \circ \varphi_3 = 0$$

Proof:

$$\begin{aligned} \delta_1 \circ \varphi_3(b) &= \delta_1(-\partial_{21}(b), \partial_{32}(b)) \\ &= \partial_{21}^{(2)}(\partial_{32}(b)) + \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(-\partial_{21}(b)) \\ &= \left(\partial_{21}^{(2)}\partial_{32}\right)(b) - \left(\tfrac{1}{2}\partial_{32}\partial_{21}\partial_{21}\right)(b) + (\partial_{31}\partial_{21})(b) \\ &= \left(\partial_{21}^{(2)}\partial_{32}\right)(b) - \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) + (\partial_{31}\partial_{21})(b) \end{aligned}$$

But from Capelli identities we have

$$\partial_{21}^{(2)}\,\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\,\partial_{31} \quad \text{and} \quad \partial_{31}\,\partial_{21} = \partial_{21}\,\partial_{31}$$

Then

$$\begin{aligned} \delta_1 \circ \varphi_3(b) &= \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) - (\partial_{21}\partial_{31})(b) - \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) + (\partial_{21}\partial_{31})(b) \\ &= 0 \quad \blacksquare \end{aligned}$$

By employing a mapping Cone again on the subcomplex (1) and the rest of diagram (2.1) we have

$$
\begin{tikzcd}
0 \arrow[r] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "\varphi_3"] & \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \arrow[r, "\delta_1"] \arrow[d, "\delta_2"'] \arrow[dr, phantom, "\mathrm{C}"] & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[d, "g_2"] \\
 & & \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[r, "h_3"] & \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d] \\
 & & & \mathcal{K}_{(8,7,3)}(\mathcal{F}) \arrow[d] \\
 & & & 0
\end{tikzcd}
$$

Diagram (2.2)

Now we define

$$
\delta_2 : \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \longrightarrow \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \quad \text{by}
$$

$$
\delta_2(a, b) = \partial_{32}^{(2)}(a) + \left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(b)
$$

Proposition (2.2):

The diagram C in diagram (2.2) is commute.

Proof:

To prove the diagram is commute it is sufficient to prove that

$$
(g_2 \circ \delta_1)(a, b) = (h_3 \circ \delta_2)(a, b)
$$

$$
\begin{aligned}
(g_2 \circ \delta_1)(a, b) &= g_2\left(\partial_{21}^{(2)}(b)\right) + \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(a) \\
&= \partial_{32}\left(\partial_{21}^{(2)}(b)\right) + \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(a) \\
&= \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) + \left(\tfrac{1}{2}\partial_{32}\partial_{32}\partial_{21} - \partial_{32}\partial_{31}\right)(a) \\
&= \left(\partial_{32}\,\partial_{21}^{(2)}\right)(b) + \left(\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31}\right)(a)
\end{aligned}
$$

IOP Conf. Series: Materials Science and Engineering **571** (2019) 012039 doi:10.1088/1757-899X/571/1/012039

But from Capelli identities we have

$\partial_{32}^{(2)}\partial_{21} = \partial_{21}\partial_{32}^{(2)} + \partial_{32}\partial_{31}$ and $\partial_{32}\partial_{21}^{(2)} = \partial_{21}^{(2)}\partial_{32} - \partial_{21}\partial_{31}$

So we get

$$\begin{aligned}(\mathscr{g}_2 \circ \delta_1)(a, b) &= \left(\partial_{21}^{(2)}\partial_{32} + \partial_{21}\partial_{31}\right)(b) + \left(\partial_{21}\partial_{32}^{(2)} + \partial_{32}\partial_{31} - \partial_{32}\partial_{31}\right)(a) \\ &= \left(\tfrac{1}{2}\partial_{21}\partial_{21}\partial_{32} + \partial_{21}\partial_{31}\right)(b) + \left(\partial_{21}\partial_{32}^{(2)}\right)(a) \\ &= \partial_{21}\left[\left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(b) + \partial_{32}^{(2)}(a)\right] \\ &= (\hbar_3 \circ \delta_2)(a, b) \quad \blacksquare\end{aligned}$$

Hence from the mapping Cone, we have the following complex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{matrix}\mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}\end{matrix} \xrightarrow{\varphi_2} \begin{matrix}\mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}\end{matrix}$$

$$\xrightarrow{\varphi_1} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \xrightarrow{d'_{(8,7,3)}(\mathcal{F})} \mathcal{K}_{(8,7,3)}(\mathcal{F}) \longrightarrow 0$$

where

$$\varphi_2(a, b) = (-\delta_1(a, b), \delta_2(a, b))$$

$$= \left(-\partial_{21}^{(2)}(b) - \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(a), \partial_{32}^{(2)}(a) + \left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(b)\right)$$

$$\varphi_1(a, b) = \partial_{32}(a) + \partial_{21}(b)$$

Proposition (2.3):

$$\varphi_2 \circ \varphi_3 = 0$$

Proof:

$(\varphi_2 \circ \varphi_3)(a) = \varphi_2(-\partial_{21}(a), \partial_{32}(a))$; $a \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$

$$= \left(\left(-\partial_{21}^{(2)}\partial_{32}\right)(a) + \left(\tfrac{1}{2}\partial_{32}\partial_{21}\partial_{21} - \partial_{31}\partial_{21}\right)(a),\right.$$

$$\left(-\partial_{32}^{(2)}\partial_{21}\right)(a)+\left(\tfrac{1}{2}\partial_{21}\partial_{32}\partial_{32}+\partial_{31}\partial_{32}\right)(a)\Big)$$

$$=\left(\left(-\partial_{21}^{(2)}\partial_{32}\right)(a)+\left(\partial_{32}\partial_{21}^{(2)}-\partial_{31}\partial_{21}\right)(a),\left(-\partial_{32}^{(2)}\partial_{21}\right)(a)+\right.$$
$$\left.\left(\partial_{21}\partial_{32}^{(2)}+\partial_{31}\partial_{32}\right)(a)\right)$$

But from Capelli identities we have

$\partial_{32}\partial_{21}^{(2)}=\partial_{21}^{(2)}\partial_{32}+\partial_{21}\partial_{31}$, $\partial_{21}\partial_{32}^{(2)}=\partial_{32}^{(2)}\partial_{21}-\partial_{32}\partial_{31}$,

$\partial_{21}\partial_{31}=\partial_{31}\partial_{21}$ and $\partial_{32}\partial_{31}=\partial_{31}\partial_{32}$

Which implies that

$(\varphi_2\circ\varphi_3)(a)$

$$=\left(\left(-\partial_{21}^{(2)}\partial_{32}\right)(a)+\left(\partial_{21}^{(2)}\partial_{32}\right)(a)+(\partial_{21}\partial_{31})(a)-(\partial_{21}\partial_{31})_{31}(a),\right.$$
$$\left.\left(-\partial_{32}^{(2)}\partial_{21}\right)(a)+\left(\partial_{32}^{(2)}\partial_{21}\right)(a)-(\partial_{32}\partial_{31})(a)+(\partial_{32}\partial_{31})(a)\right)$$

= (0,0) ■

Proposition (2.4):

$$\varphi_1\circ\varphi_2=0$$

Proof:

$$(\varphi_1\circ\varphi_2)(a,b)=\varphi_1\Big(-\partial_{21}^{(2)}(b)-\left(\tfrac{1}{2}\partial_{32}\partial_{21}-\partial_{31}\right)(a),\partial_{32}^{(2)}(a)+$$
$$\left(\tfrac{1}{2}\partial_{21}\partial_{32}+\partial_{31}\right)(b)\Big)$$

$$=\left(-\partial_{32}\partial_{21}^{(2)}\right)(b)-\left(\tfrac{1}{2}\partial_{32}\partial_{32}\partial_{21}\right)(a)-(\partial_{31}\partial_{32})(a)+$$
$$\left(\partial_{21}\partial_{32}^{(2)}\right)(a)+\left(\tfrac{1}{2}\partial_{21}\partial_{21}\partial_{32}\right)(b)+(\partial_{21}\partial_{31})(b)$$

$$=\left(-\partial_{32}\partial_{21}^{(2)}\right)(b)-\left(\partial_{32}^{(2)}\partial_{21}\right)(a)-(\partial_{31}\partial_{32})(a)+$$
$$\left(\partial_{21}\partial_{32}^{(2)}\right)(a)+\left(\partial_{21}^{(2)}\partial_{32}\right)(b)+(\partial_{21}\partial_{31})(b)$$

Again from Capelli identities we get

$(\varphi_1 \circ \varphi_2)(a, b) =$
$\left(-\partial_{21}^{(2)}\partial_{32}\right)(b) - (\partial_{21}\partial_{31})(b) - \left(\partial_{21}\partial_{32}^{(2)}\right)(a) - (\partial_{32}\partial_{31})(a) +$
$(\partial_{32}\partial_{31})(a) + \left(\partial_{21}\partial_{32}^{(2)}\right)(a) + \left(\partial_{21}^{(2)}\partial_{32}\right)(b) + (\partial_{21}\partial_{31})(b)$
$= 0$ ■

Finally, we present the following theorem which shows that the complex of Lascoux in the case of partition (8,7,3) is exact.

Theorem (2.5):
The complex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \xrightarrow{\varphi_2} \begin{matrix} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{matrix}$$

$$\xrightarrow{\varphi_1} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \xrightarrow{d'_{(8,7,3)}(\mathcal{F})} \mathcal{K}_{(8,7,3)}(\mathcal{F}) \longrightarrow 0$$

Is exact.

Proof:

Since the diagrams, A and B in a diagram (2.1) are commutes and each of the maps
$h_1 : \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \longrightarrow \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$; where $h_1(v) = \partial_{21}(v)$,
and
$h_2 : \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$; where $h_2(v) = \partial_{21}^{(2)}(v)$,
are injective [9] and [11], then we have a commuting diagram with an exact row. But from proposition (2.1) we have $\delta_1 \circ \varphi_3 = 0$ which implies that the mapping Cone conditions are satisfied and the complex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \xrightarrow{\delta_1} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$$

Is exact.

Now consider the diagram (2.2), since diagram C is commute and $\hbar_3 : \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \longrightarrow \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$; where $\hbar_3(v) = \partial_{21}(v)$ is injective [9] and [11], so we have diagram (2.2) commute with exact rows. But $\varphi_2 \circ \varphi_3 = 0$ (proposition (2.3)) and $\varphi_1 \circ \varphi_2 = 0$ (proposition (2.4)) then again the mapping Cone conditions are satisfied, so the complex

$$0 \longrightarrow \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{matrix} \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix} \xrightarrow{\varphi_2} \begin{matrix} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \end{matrix}$$

$$\xrightarrow{\varphi_1} \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \xrightarrow{d'_{(8,7,3)}(\mathcal{F})} \mathcal{K}_{(8,7,3)}(\mathcal{F}) \longrightarrow 0$$

Is exact. ■

Conclusions

By using mapping Cone we can find the resolution of Weyl module for characteristic zero in the case of partition (8,7,3) without depending on the resolution of Weyl module for characteristic free for the same partition.

Acknowledgments

The authors thank Mustansiriyah University / College of Science / Department of Mathematics, University of Baghdad / College of Education for Pure Science – Ibn Al-Haitham / Department of Mathematics and University of Kerbala / College of Education for Pure Science / Department of Mathematics for their supported this work.

References

[1] Haytham R.H. 2013 Complex of Lascoux in Partition (4,4,4), *Iraqi Journal of Science*, Vol. 54, No. 1, pp. 170-173,(.
[2] Haytham R.H. and Alaa O.A. 2015 Complex of Lascoux in Partition (3,3,2), *Ibn Al-Haitham J. for Pure & Appl. Sci.*, Vol 28, No. 1, pp.171-178.
[3] Haytham R.H., Mays M.M. 2015 Complex of Lascoux in Partition (6,6,3), *International Journal of Engineering and Applied Sciences (IJEAS)*, Vol. 2, No. 9, pp.15-17.
[4] Haytham R.H. and Noor T.A. 2016 Complex of Lascoux in Partition (6,5,3), *International Journal of Engineering Research And Management (IJERM)*, Vol. 3, No. 3, pp. 197-199.
[5] Haytham R.H. and Najah M.M. 2016 Complex of Lascoux in the Case of Partition (7,6,3), Australian Journal of Basic and Appl. Sci., Vol.10, No. 18, pp.89-93.

[6] Haytham R.H., Niran S.J. 2018 A Complex of Characteristic Zero in the Case of the Partition (8,7,3), *Science International (Lahore)*, Vol. 30, No. 4, pp.639-641.
[7] Haytham R.H., Niran S.J. 2018 Application of Weyl Module in the Case of Two Rows, *J. Phys.:Conf.Ser*., Vol. 1003 (012051), pp.1-15.
[8] Haytham R.H., Niran S.J. 2018 On Free Resolution of Weyl Module and Zero Characteristic Resolution In The Case of Partition (8,7,3), *Baghdad Science Journal*, Vol. 15, No. 4, pp. 455-465.
[9] Buchsbaum D.A. and Rota G.C. 2001 Approaches to resolution of Weyl modules, *Adv. In Applied Math.*, Vol.27, pp.82-191.
[10] Vermani L.R. 2003 An elementary approach to homogical algebra, Chapman and Hall/CRC, *Monographs and Surveys in pure and Applied mathematics*, Vol.130.
[11] D.A. Buchsbaum 2004 A characteristic-free example of Lascoux resolution, and letter place methods for intertwining numbers, *European Journal of Gombinatorics*, Vol.25, pp.1169-1179.