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On Estimation of the Stress – Strength Reliability Based on Lomax Distribution

Abbas Najim Salman[1], Alaa M. Hamad[2]

[1, 2] Department of Mathematics, Ibn-Al-Haitham College of Education, University of Baghdad
abbasnajim66@yahoo.com , alaa_073@yahoo.com

Abstract: The present paper concerns with the problem of estimating the reliability system in the stress – strength model under the consideration non identical and independent of stress and strength and follows Lomax Distribution. Various shrinkage estimation methods were employed in this context depend on Maximum likelihood, Moment Method and shrinkage weight factors based on Monte Carlo Simulation. Comparisons among the suggested estimation methods have been made using the mean absolute percentage error criteria depend on MATLAB program.

1. Introduction

In the last decades, Lomax in (1954) was introduced an important and most widely used lifetime model used for stochastic modeling for reducing the failure rate called Lomax distribution. It has been applied in several studies of income such as size of cities, wealth inequality, several engineering applications, beside queuing theory and biological analysis [1].

Many Studies and literatures about Lomax distribution have been presented and discussed by several authors. Some of these studies deal with properties and moments of Lomax distribution and discussed the rate of changing the optimal times of stress life time following Lomax distribution. It is also has a point of significance in a playing field of reliability and life testing through it uses to fit business failure data [2]. And in (2017) Abbas and Fatima, they estimated the reliability of multicomponent system in stress–strength model for Exponentiated Weibull distribution, using; ML, MOM and the conclude results approved that the Shrinkage estimator using Shrinkage weight function was the best [3].

Assume a random variable x follows Lomax distribution
x~LD (λ, α), then the P.D.F of (x) will be:-

$$f(x) = \frac{\alpha\lambda}{(1+\lambda x)^{\alpha+1}} \qquad \lambda, \alpha, x > 0 \tag{1}$$

Everywhere, α and λ are respectively represented to shape and scale parameters.
The accumulative distribution functions of x become:

$$F(x) = P(X \leq x) = 1 - (1 + \lambda x)^{-\alpha} \tag{2}$$

As a special case, when $\lambda = 1$, the probability density function will be as below:

$$f(x) = \frac{\alpha}{(1+x)^{\alpha+1}} \qquad , x > 0 \tag{3}$$

The stress (Y) and the strength (X) in stress-strength (S-S) model will be considered as random variables follow the Lomax distribution. The strength in the distribution which will exceed the stress leads to the reliability system R=P(Y<X) in the stress-strength model [2].

Suppose that the random variables X and Y follow the Lomax distribution (LD) with parameter $(1, \alpha_1)$ and $(1, \alpha_2)$ respectively, then the (S-S) reliability can be calculated as follows; [4], [5] and [6].
i.e., when x~ LD $(1, \alpha_1)$ and y~ LD $(1, \alpha_2)$, then the (s-s) reliability is defined as below

$R = P(Y<X) = \int_0^\infty \int_0^x \mathrm{f(x)f(y)\ dydx}$

Hence,

$\mathrm{R} = \int_0^\infty \alpha_1(1+x)^{-(\alpha_1+1)}dx \int_0^x \alpha_2(1+y)^{-(\alpha_2+1)}\,dy$

$= \int_0^\infty(-\alpha_1(1+x)^{-(\alpha_1+1)-\alpha_2} + \alpha_1(1+x)^{-(\alpha_1+1)})dx$

$$R = \frac{\alpha_2}{\alpha_1+\alpha_2} \tag{4}$$

The aim of this paper is to estimate R when the stress and the strength are not identically independent follows the Lomax distribution (LD) using different shrinkage estimation methods and make comparisons between the proposed estimation methods through Mont Carlo simulation depend on minimum mean absolute percentage error (MAPE) criterion.

2. Estimation methods of R =P(Y<X)

2.1 Maximum Likelihood Estimator (MLE)

Assume that $x_{1,}x_{2,}\dots,.x_n$ to be a random sample of LD $(1,\alpha_1)$ and $y_{1,}y_{2,}\dots,.y_m$ to be a random sample of LD $(1,\alpha_2)$ then, the likelihood function $\mathrm{L}(\alpha_i,x_i,y_i)$ of the mentioned sample can be obtained as in below:

$l = \mathrm{L}(\alpha_i,x_i,y)$ i=1,2,…,n , j=1,2,…,m

$= \prod_{i=1}^n f(x_i) \prod_{i=1}^m g(y_i)$

$$= \prod_{i=1}^n \alpha_1^n\,(1+x_i)^{-(\alpha_1+1)} \prod_{j=1}^m \alpha_2^m(1+y_i)^{-(\alpha_2+1)} \tag{5}$$

Taking the logarithm of the both sides of the equation (5), then implies

$Ln(l) = nln(\alpha_1) - (\alpha_1+1)\sum_{i=1}^{n_i}\ln(1+x_i) + m\ln(\alpha_2) - (\alpha_2+1)\sum_{j=1}^m \ln(1+x_i)$

Derive the above equation w.r.t. $\alpha_i(i=1,2)$, and equating the result to zero, we conclude

$$\hat{\alpha}_{1_{mle}} = \frac{n}{\sum_{i=1}^n \ln(1+x_i)} \tag{6}$$

$$\hat{\alpha}_{2_{mle}} = \frac{m}{\sum_{j=1}^m \ln(1+x_j)} \tag{7}$$

By substituting $\hat{\alpha}_{i_{mle}}$ in equation (4), we get the reliability of estimation for (S-S) model using the Maximum Likelihood method as in the following:-

$$\hat{\mathrm{R}}_{mle} = \frac{\hat{\alpha}_{2mle}}{\hat{\alpha}_{1mle}+\hat{\alpha}_{2mle}} \tag{8}$$

2.2 Moment Method (MOM)

The moment method (MOM) will be considered in this subsection to estimate the parameter α_i , and we need the first order population moments for X and Y in LD, which is given below: [7]

$\mathrm{E(X)} = \frac{1}{\alpha_1-1}$ and $\mathrm{E(Y)} = \frac{1}{\alpha_2-1}$

Let $x_{1,}x_{2,}\dots,.x_n$ be a random sample of LD $(1,\alpha_1)$ and $y_{1,}y_{2,}\dots,.y_m$ be a random samples of LD $(1,\alpha_2)$.

Equating the 1st sample moment with the corresponding 1st population moment, we get

$$\bar{\mathrm{x}} = \frac{\sum_{i=1}^n x_i}{n} = \frac{1}{\alpha_1-1} \tag{9}$$

$$\bar{\mathrm{y}} = \frac{\sum_{i=1}^m y_i}{m} = \frac{1}{\alpha_2-1} \tag{10}$$

Using simplification of equations (9) and (10), we obtain the estimation of the unknown shape parameters α_1, α_2 using moment method as follows:-

$$\hat{\alpha}_{1_{mom}} = \frac{n+\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i} \tag{11}$$

$$\hat{\alpha}_{2_{mom}} = \frac{m+\sum_{i=1}^{m} y_i}{\sum_{i=1}^{m} y_i} \tag{12}$$

Substitution the equations (11) and (12) in the equation (4), we get the estimation of (S-S) reliability using moment method as in the below:-

$$\hat{R}_{mom} = \frac{\hat{\alpha}_{2mom}}{\hat{\alpha}_{1mom}+\hat{\alpha}_{2mom}} \tag{13}$$

2.3 Shrinkage Estimation Method (Sh)

Thompson in 1968, introduced the important reasons to use prior estimate , and he suggested the shrinkage estimation method which will be considered in this section ,it is depending on prior information about α as initial value α_0 from the past and usual estimator $\hat{\alpha}_{mle}$ through combine them by shrinkage weight factor as below:-

$\hat{\alpha}_{sh}= (\hat{\alpha}_i)\ \hat{\alpha}_{mle}+ (1-\varphi\ (\hat{\alpha}_i))\ \alpha_0$ i=1, 2 (14)

Where $\varphi\ (\hat{\alpha}_i)$, $0\leq (\hat{\alpha}_i) \leq 1$ represent shrinkage weight factor.
$(\hat{\alpha}_i)$ denotes to the trust of $\hat{\alpha}_{mle}$, and $(1- (\hat{\alpha}_i))$ denotes to trust of α_0, which may be constant or a function of sample size see [8], [9], [10] and [11].

Our assumption in this paper, takings the moment method instated of α_0 as initial value.

2.3.1 Shrinkage weight function (sh1)

In this subsection, the shrinkage weight factor: as a function of n and m respectively, will be considered in the equation (14) as in below:-

$\varphi(\hat{\alpha}_1) = K_1 = \left|\frac{\sin(n)}{n}\right|$

$\varphi(\hat{\alpha}2) = K_2 = \left|\frac{\sin(m)}{m}\right|$

$\hat{\alpha}_{ish1} = K_i\ \hat{\alpha}_{imle}+ (1-K_i)\ \hat{\alpha}_{imom}$, for i=1, 2 (15)

The corresponding (S-S) reliability using above shrinkage weight factor method sh_1 will be

$$\hat{R}_{sh1} = \frac{\hat{\alpha}_{2sh1}}{\hat{\alpha}_{1sh1}+\hat{\alpha}_{2sh1}} \tag{16}$$

2.3.2 Constant shrinkage factor (sh_2)

In this subsection the constant shrinkage weight factor will be assumed as $(\hat{\alpha}_1) = K_3=0.3$, and $(\hat{\alpha}_1) = K_4=0.3$and this implies to the following shrinkage estimators.

$\hat{\alpha}_{ish2} = Ki\ \hat{\alpha}_{imle}+ (1-Ki)\ \hat{\alpha}_{imom}$ for i=3, 4 (17)

When substitution the equation (17) in the equation (4), lead to the estimation of (S-S) reliability using constant shrinkage factor estimator sh_2 as below:

$$\hat{R}_{sh2} = \frac{\hat{\alpha}_{2sh2}}{\hat{\alpha}_{1sh2}+\hat{\alpha}_{2sh2}} \tag{18}$$

2.3.3 Beta shrinkage factor (sh_3)

In this subsection the Beta shrinkage weight factor will be assumed as
$(\hat{\alpha}_1) = K_5$=Beta (n, m),
And $\varphi\ (\hat{\alpha}_1) = K_6$= Beta (n, m),
Where n and m represent sample size and this implies to the following shrinkage estimators:-

$\hat{\alpha}_{ish3} = Ki\ \hat{\alpha}_{imle}+ (1-Ki)\ \hat{\alpha}_{imom}$ for i=5, 6 (19)

When substitution the equation (17) in the equation (4), this will lead to the estimation of (S-S) reliability using beta shrinkage factor estimator sh_2 as in the following:-

$$\widehat{R}_{sh3} = \frac{\hat{\alpha}_{2sh3}}{\hat{\alpha}_{1sh3}+\hat{\alpha}_{2sh3}} \tag{20}$$

3. Simulation Study

In this section, Monte Carlo simulation method has been used and the obtained results are compared to the numerical results that previously obtained in the section2.The simulation process were done using unlike sample size = (30, 50 and 100) and built on 1000 replications via MAPE measures to check the performance as in the following:-

Step1: the random sample generated for (x) according to the uniform distribution over the interval (0, 1) as $u_1, u_2 \dots u_n$.

Step2: the random sample generated for (y) according to the uniform distribution over the interval (0, 1) as $w_1, w_2 \dots w_m$

Step3: transforming the above Lomax distribution (LD) with using (c.d.f.) as in the following:-

$F(x) = 1 - (1 + \lambda xi)^{-\alpha}$

$x_i = [(1 - Ui)^{-1/\alpha 1} - 1]$, i=1, 2, 3… n

and, by applying the same way, we get

$y_j = (1 - Ui)^{-1/\alpha 2} - 1]$, j = 1, 2, 3… m

Step4: calculate $\hat{\alpha}_{1_{mle}}$ and $\hat{\alpha}_{2_{mle}}$ via equations (6) and (7) respectively.

Step5: calculate $\hat{\alpha}_{1_{mom}}$ and $\hat{\alpha}_{2_{mom}}$ by equations (11) and (12) respectively.

Step6: calculate $\widehat{R}_{sh1}$, $\widehat{R}_{sh2}$ and $\widehat{R}_{sh3}$ through equations (16), (18) and (20) respectively.

Step7: using (L=1000) replication, the MAPE will be calculated as in the following:-

$$\text{MAPE} = \frac{1}{L}\sum_{i=1}^{L}\left|\frac{\widehat{R}i - Ri}{Ri}\right|$$

Where $\widehat{R}$ is mention to suggested estimators of the reliability R .The outcomes are put as shown in the tables (1), (2), (3),(4) (5) and (6) respectively.

Table1. Estimation when R = 0.4, alpha1= 3, alpha2= 2

n	m	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$
30	30	0.385893	0.386283	0.385701
	50	0.392433	0.392245	0.392465
	100	0.381133	0.382161	0.380904
50	30	0.416617	0.417307	0.416514
	50	0.396927	0.397935	0.396871
	100	0.404119	0.404509	0.404097
100	30	0.418895	0.418318	0.419202
	50	0.395553	0.394748	0.395598
	100	0.368436	0.370115	0.368347

Table2. MAPE values when R = 0.4, alpha1= 3alpha2=2

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	Best
30	30	0.044947	0.044102	0.045363	$\hat{R}_{sh2}$
	50	0.020811	0.020655	0.020911	$\hat{R}_{sh2}$
	100	0.051487	0.048985	0.051927	$\hat{R}_{sh2}$
50	30	0.041543	0.043268	0.041285	$\hat{R}_{sh3}$
	50	0.017198	0.017193	0.017197	$\hat{R}_{sh2}$
	100	0.019439	0.019081	0.019458	$\hat{R}_{sh2}$
100	30	0.047238	0.045796	0.048004	$\hat{R}_{sh2}$
	50	0.025208	0.025552	0.025189	$\hat{R}_{sh3}$
	100	0.07891	0.074714	0.079133	$\hat{R}_{sh2}$

Table3. Estimation when R = 0.555, alpha1= 2.4, alpha2= 3

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$
30	30	0.536588	0.537365	0.536205
	50	0.546132	0.546208	0.546113
	100	0.535303	0.536512	0.534965
50	30	0.569186	0.570227	0.569009
	50	0.547642	0.549181	0.547557
	100	0.554986	0.555889	0.554936
100	30	0.570917	0.570701	0.571066
	50	0.545849	0.545536	0.545867
	100	0.528219	0.529510	0.528151

Table4.MAPE values when R= 0.555,
alpha1= 2.4, alpha2= 3

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	Best
30	30	0.037440	0.036468	0.037918	$\hat{R}_{sh2}$
	50	0.018353	0.018157	0.018524	$\hat{R}_{sh2}$
	100	0.037319	0.035398	0.037778	$\hat{R}_{sh2}$
50	30	0.024989	0.026578	0.024701	$\hat{R}_{sh3}$
	50	0.016783	0.015363	0.016881	$\hat{R}_{sh2}$
	100	0.012952	0.012829	0.012959	$\hat{R}_{sh2}$
100	30	0.028431	0.027892	0.028624	$\hat{R}_{sh2}$
	50	0.020871	0.021245	0.020850	$\hat{R}_{sh3}$
	100	0.049205	0.046882	0.049329	$\hat{R}_{sh2}$

Table5. Estimation when R = 0.444, alpha1= 3, alpha2= 2.4

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$
30	30	0.428050	0.428602	0.427780
	50	0.435787	0.435715	0.435816
	100	0.425170	0.426258	0.424930
50	30	0.460467	0.461258	0.460315
	50	0.440129	0.441295	0.440065
	100	0.447401	0.447938	0.447371
100	30	0.461593	0.461178	0.461817
	50	0.436921	0.436383	0.436951
	100	0.413592	0.415176	0.413508

Table6. MAPE values when R = 0.444, alpha1= 3, alpha2= 2.4

n	m	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	Best
30	30	0.043884	0.042896	0.044370	$\hat{R}_{sh2}$
	50	0.020489	0.020475	0.020590	$\hat{R}_{sh2}$
	100	0.046489	0.044191	0.046901	$\hat{R}_{sh2}$
50	30	0.036050	0.037830	0.035709	$\hat{R}_{sh3}$
	50	0.016860	0.016316	0.016901	$\hat{R}_{sh2}$
	100	0.017113	0.016864	0.017126	$\hat{R}_{sh2}$
100	30	0.038583	0.037651	0.039089	$\hat{R}_{sh2}$
	50	0.025026	0.025497	0.025000	$\hat{R}_{sh3}$
	100	0.069418	0.065854	0.069608	$\hat{R}_{sh2}$

4. Numerical Results

i- when n =30, the minimum mean absolute percentage error (MAPE) for the (S-S) reliability estimators of the Lomax distribution is holds using the shrinkage estimator based on constant shrinkage weight function (R_{sh2}) for all m= (30, 50,100) and each $\alpha 1$ and $\alpha 2$, this result indicates that, the shrinkage estimator of (S-S) reliability (sh2) is the best and follows by shrinkage estimator (sh1) and shrinkage estimator (sh3).

ii- when n=50, the minimum mean absolute percentage error (MAPE) for the (S-S) reliability estimators of the Lomax distribution is holds using the shrinkage estimator based on constant shrinkage weight function (R_{sh2}) for all m= (50,100) and each $\alpha 1$ and $\alpha 2$, and the best estimator was a beta shrinkage estimator(sh3) when m= 30, this result indicates that, the shrinkage estimator of (S-S) reliability (sh2) was at most the best and follows by shrinkage estimator (sh3) and shrinkage estimator (sh1).

iii- when n=100, the minimum mean absolute percentage error (MAPE) for the (S-S) reliability estimator of the Lomax distribution is holds using the shrinkage estimator based on a constant shrinkage estimator (sh2) for all m = (30,100) and each $\alpha 1$ and $\alpha 2$,and the best estimator was a beta shrinkage estimator(sh3) when m =50,this result indicates that, the shrinkage estimator of (S-S) reliability (sh2) was at most the best and follows by shrinkage estimator (sh3) according to the obtained simulation results.

5. Conclusion

The suggested shrinkage estimation methods have good performance and it is considered since it will have minimum MAPE. These estimators are depending on assumption of combining the Maximum likelihood estimation (MLE) method and the moment (MOM) method as prior information using a different shrinkage weight factor, and the shrinkage estimator using constant shrinkage weight factor (sh_1) has a minimum statistical indicator (MAPE) in at most cases.

References

[1] A.S.Hassan, M. A. Abdelghafar, “Exponentiated Lomax Geometric Distribution: Properties and Applications,” *Pakistan Journal of Statistics and Operation Research*, vol. 13 no. 3, 2017.

[2] S. K. Ashour, A. M.M.Abd-Elfattah, R. Mndouh,“ Bayesian and Non-Bayesian Estimation of Pr {Y<X} in Two Parameters Lomax Distribution,” Department of Mathematical Statistics, Institute of Statistical Studies & Research Cairo University, Egypt ,Discover scientific knowledge at ResearchGate,. May 03, 2015.

[3] Abdulateef, E. A.; Abbas. N.S. On Shrinkage Estimation for R (s, k) in Case of Exponentiated Pareto Distribution. *Ibn AL-Haitham Journal For Pure and Applied Science* 32.1 (**2019**): 152-162.

[4] N.A.Amiri, A.R.Yaghmaei& M.Babanezhad, “Estimation of Stress – Strength parameter for Tow- parameter Weibull distribution,” *International Journal of Advanced Statistics and probability,* vol. 1, no.1, pp. 4-8,(2013).

[5] N.S. Karam, & H.G.Ramk, “Reliability Estimation for a Component Exposed Two, Three Independent Stresses Based on Weibull and Invers Lindly Distribution,” *Mathematical Theory and Modeling*, vol.6, no. 8, ISSN 2225-0522, (2001).

[6] N.S. Karam,One, “Two and Multi –Component Gompertz Stress –Strength Reliability Estimation,” *Mathematical Theory and Modeling*, vol.6, no. 3, ISSN 2225-0522, (2016).

[7] H.M.Ali, “Comparison of some methods to estimate some of stress –strength Lomax models,” M.Sc. Thesis, Al -Mustansiriya University, (2013).

[8] Z.N. Al-Hemyari, A.N.Al-Joboori, “On Thompson Type Estimators for the Mean of Normal Distribution,” *Revista Investigación Operacional*, vol. 30, no. 2, pp.109-116, (2009).

[9] Z.A. AL-Hemyari, I.H. Hassan and A.N. AL-Jobori, “A class of efficient and modified estimator for the mean of normal distribution using complete data,”*International Journal of data Analysis Techniques and Strategies*, vol.3, no. 4, pp.406-425,(2011).

[10] A.N. AL-Jobori, et al, “Single and Double Stage Shrinkage Estimators for the Normal Mean with the Variance Cases,” *International Journal of statistic*, vol. 38, no. 2, pp.1127-1134, (2014).

[11] J.R.Thompson, “Some shrinkage Techniques for Estimating the mean,” *Journal of the American Statistical Association (JASA),* vol. 63, pp.113-122, (1968).