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Intuitionistic fuzzy semi d-ideal spectrum

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Abstract. The main purpose of this paper is to study the spectrum of intuitionistic fuzzy semi d-ideal in d-algebra, and the relationship between the topological properties and the algebraic properties of the Spectrum of d-algebra X with respecting to connectedness and separation axioms .

1. Introduction

BCK-algebra is a classe of abstract algebras introduced by Y. Imai and K. Iseki [9,15] . A d-algebra is a useful generalization of BCK-algebra was introduced by J. Negger and H. S. Kim [7]. J. Negger , Y. B. Jun and H. S. Kim [8] discussed ideal theory in d-algebra. After the introduction of intuitionistic fuzzy set by Atanassov in 1986 [10], there was a number of generalizations of this concept . This concept was generalizations for fuzzy set concept which was introduced by Zadeh in 1965 [11]. In [14] Y. B. Jun, J. Neggers and H. S. Kim apply the ideal theory in fuzzy d-ideals of d-algebras . H. K. Abdullah and A. K. Hasan introduce the notation of semi d-ideal of d-algebra in [5]. Y. B. Jun , H. S. Kim and D.S. Yoo in [13] introduced the notion of intuitionistic fuzzy d-algebra. A. K. Hasan introduce the notion of intuitionistic fuzzy semi d-ideals of d-algebra in [1] . Ali K. Hasan and Osamah A. Shaheed introduce the notion of intuitionistic fuzzy prime semi d-ideals of d-algebra in [2], and in this paper we study the spectrum of intuitionistic fuzzy semi d-ideal in d-algebra, and the relationship between the topological properties and the algebraic properties of the d-algebra X . Also we consider strongly connected and separated properties .

2. Background

This section contains some basic about intuitionistic fuzzy set and the ordinary and intuitionistic fuzzy concepts about semi d-ideal and prime semi d-ideal in d-algebra, with some theorems and propositions.

Definition (2.1) : [7] A d-algebra is any non-empty set X with a binary operation $*$ and a constant 0 which satisfies that:

I. $a * a = 0$
II. $0 * a = 0$
III. If $a * b = b * a = 0$ then $a = b \ \forall\, a, b \in X$.

We will refer to $a * b$ by ab, and it is said to be commutative if $a(ab) = b(ba)$ for all $a, b \in X$, and $b(ba)$ is denoted by $(a \wedge b)$.Every set X in the following is a d-algebra

Definition (2.2) :[5] A semi d-ideal of a d-algebra X is a non empty subset J of X satisfies i) $a, b \in J$ imply $ab \in J$,
ii) $ab \in J$ and $b \in J$ imply $a \in J$, for all $a, b \in X$
Definition(2.3) : [4] In a commutative d-algebra X , a semi d-ideal I is said to be prime if $a \wedge b \in I$ implies $a \in I$ or $b \in I$, for all $a, b \in X$.
Definition (2.4) [10] : An IFS " intuitionistic fuzzy set " A in a set X is an object having the form $A = \{< a, \alpha_A(a), \beta_A(a) >: a \in X\}$, such that $\alpha_A: X \to [0,1]$ and $\beta_A: X \to [0,1]$ denoted the degree of

membership (namely $\alpha_A(a)$) and the degree of non membership (namely $\beta_A(a)$) for any elements $a \in X$ to the set A, and $0 \le \alpha_A(a) + \beta_A(a) \le 1$, for all $a \in X$. To simplicity, we shall use $A =< \alpha_A, \beta_A >$ instead of $A = \{< a, \alpha_A(a), \beta_A(a) >: a \in X\}$.

Definition (2.5) : [3] Let X, Y be d-algebra and let $f: X \to Y$ be a homomorphism mapping, and C be IFS in X we define IFS, f(C) in Y, by

$f(C)_{(y)} =< \alpha_{f(C)}(b), \beta_{f(C)}(b) >$, where $\alpha_{f(C)}(b) = \begin{cases} \sup \alpha_C(a) & a \in X, f(a) = b \quad \text{if } f^{-1}(b) \ne \emptyset \\ 0 & \text{otherwais} \end{cases}$, and

$\beta_{f(C)}(b) = \begin{cases} \inf \beta_C(a) & a \in X, f(a) = b \quad \text{if } f^{-1}(b) \ne \emptyset \\ 0 & \text{otherwais} \end{cases}$ for each $b \in Y$.

Definition (2.6) [6] : Let $\mu, v \in [0,1]$ such that $\mu + v \le 1$. An intuitionistic fuzzy point $x_{(\mu,v)}$ is defined to be an IFS in X, define by $x_{(\mu,v)}(y) = \begin{cases} (\mu, v) & \text{if } y = x \\ (0,1) & \text{if } y \ne x \end{cases}$ for all y in X, and $x_{(\mu,v)} \in A$ if and only if $\alpha \le \mu_A(x)$ and $\beta \ge v_A(x)$.

Notation (2.7): Let A be an IFS of a d-algebra X. We denote a level cut set A_* by $A_* = \{x \in X : \alpha_A(x) = \alpha_A(0), \beta_A(x) = \beta_A(0)\}$.

Definition (2.8) [3] : The IFS $\tilde{0}$ and $\tilde{1}$ in X are define as $\tilde{0} = \{\langle x, 0,1\rangle, x \in X\}$ and $\tilde{1} = \{\langle x, 1,0\rangle, x \in X\}$, where 1 and 0 represent the constant maps sending every element of X to 1 and 0, respectively.

Definition (2.9) [1] :An intuitionistic fuzzy semi d-ideal of X, "shortly IFSd − ideal" , is an IFS $D =< \alpha_D, \beta_D >$ in X satisfies the following inequalities :
$(IFSd_1)$ $\alpha_D(a) \ge \min\{\alpha_D(ab), \alpha_D(b)\}$, $(IFSd_2)$ $\beta_D(a) \le \max\{\beta_D(ab), \beta_D(b)\}$
$(IFSd_3)$ $\alpha_D(ab) \ge \min\{\alpha_D(a), \alpha_D(b)\}$, and $(IFSd_4)$ $\beta_D(ab) \le \max\{\beta_D(a), \beta_D(b)\}$, for all $a, b \in X$.

Definition(2.10) [2] : An IFSd − ideal $D =< \alpha_D, \beta_D >$ of X is an intuitionistic fuzzy prime semi d-ideal " shortly IFPSd − ideal " in X if it is satisfies $(IFPSd_1)$ $\alpha_D(a \wedge b) \le \max\{\alpha_D(a), \alpha_D(b)\}$
$(IFPSd_2)$ $\beta_D(a \wedge b) \ge \min\{\beta_D(a), \beta_D(b)\}$, for all $a, b \in X$

Theorem (2.11) [2] : If $D =< \alpha_D, \beta_D >$ is an IFPSd − ideal, then the set $A_* = \{a \in X: \alpha_D(a) = \alpha_D(0)$ and $\beta_D(a) = \beta_D(0)\}$ is a prime semi d-ideals.

Definition (2.12):[2] A non-constant intuitionistic fuzzy ideal A of a d-algebra X is called an intuitionistic fuzzy maximal semi d-ideal if for any intuitionistic fuzzy semi d-ideal B of X, if $A \subseteq B$, then either $B_* = A_*$ or $B_* = X$.

Theorem (2.13) : [2] Let A is an intuitionistic fuzzy maximal semi d-ideal of a d-algebra X, then A_* is a maximal semi d-ideal of X.

Definition (2.14): [2] Let A be an IFS of X. Then the least IFSd − ideal of X containing A is called the IFSd − ideal of X generated by A and is denoted by $\langle A\rangle$.

3. Topological spectrum

In this section we introduce the spectrum of d-algebra and we discuss the relationship between some algebraic and topological properties of d-algebra .

Notation (3.1) :
(i) $\chi = \{P, P\ is\ IFPSd - ideal\ of\ X\}$
(ii) $V(A) = \{P \in \chi, A \subseteq P, whrer\ A\ is\ an\ IFSd - ideal\ of\ X\}$.
(iii) $\chi(A) = \chi \setminus V(A)$ the complement of $V(A)$ in X , $\chi(A) = \{P \in \chi, A \nsubseteq P\}$.

Lemma (3.2) : Let A and B be $IFSd - ideal$. If $A \subseteq B$, then $V(B) \subseteq V(A)$.
proof : Let $P \in V(B)$ that implies $B \subseteq P$, and so $A \subseteq B \subseteq P$ that mean $P \in V(A)$.

proposition (3.3) : If P is a smallest $IFPSd - ideal$ containing A , then $V(A) = V(P)$.
proof : It is clear that $V(P) \subseteq V(A)$ by lemma (3.2) . Now let $P_1 \in V(A)$, so $A \subseteq P_1$, but P is a smallest $IFPSd - ideal$ containing A , so $P \subseteq P_1$, then $P_1 \in V(A)$. Thus $V(A) = V(P)$.

Proposition (3.4) : Let A be an $IFSd - ideal$, then $V(\langle A\rangle) = V(A)$.
proof : Let $P \in V(A)$ that implies $A \subseteq P$, and so $\langle A\rangle \subseteq P$. Hence $P \in V(\langle A\rangle)$.

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Conversely , let $P \in V(\langle A\rangle)$, then $\langle A\rangle \subseteq P$, note that so $A \subseteq \langle A\rangle \subseteq P$, we get $P \in V(A)$.
Therefore $V(\langle A\rangle) = V(A)$.
Proposition (3.5) : Let A and B be two $IFSd-ideal$, then $V(A\cup B) \subseteq V(A) \cup V(B)$

Proof :Since $A \subseteq A\cup B$ and $B \subseteq A \cup B$, so $V(A\cup B) \subseteq V(A)$ and $V(A\cup B) \subseteq V(B)$. Thus $V(A\cup B) \subseteq V(A)\cup V(B)$.

Definition (3.6) : For an $IFPSd-ideal\ A$ of X . The prime radical $rad(A)$ of A is the intersection of all
$IFSd-ideals$ of X containing A . In case there is no such $IFSPd-ideal$, then $rad(A) = \tilde{1}$.
Proposition (3.7) : Let A be an $IFSd-ideal$, then
i) $A \subseteq rad(A)$
ii) $rad\big(rad(A)\big) = rad(A)$
iii) If $A\ IFPSd-ideal$, then $rad(A) = A$
iv) If $A \subseteq B$, then $rad(A) \subseteq rad(B)$
proof :
i) It is clear that $A \subseteq \bigcap\{P_i, P\in A, \forall i\in\Lambda\}$.
ii) We can easily show that $\bigcap\{P_i, rad(A) \subseteq P\} = \bigcap\{P_i, \bigcap[\acute{P_i}, A\subseteq \acute{P}], \bigcap\{\acute{P_i}, A \subseteq \acute{P}\} \subseteq P\}$ for all $i\in\Lambda$, so $A \subseteq P$, then $rad\big(rad(A)\big) = rad(A)$.
iii) Since A is an $IFPSd-ideal$, then $\bigcap P_i = A$ for all $i \in \Lambda$ this mean $rad(A) = A$.
iv) It is clear .
Proposition (3.8) : For any $IFSd-ideal\ A\ and\ B$ the following are hold
i) $V(A) = V(rad(A))$
ii) $V(A) = V(B)$ if and only if $rad(A) = rad(B)$.
proof :
i) Since $A \subseteq rad(A)$, then $V(rad(A)) \subseteq V(A)$. Now let $P \in V(A)$, thus $A \subseteq P$,
so $rad(A) = \bigcap\{\acute{P_i} \in Spec(X): A \subseteq \acute{P}\}$, this imply that $rad(A) \subseteq P$. Thus $P \in V(rad(A))$, then $V(A) \subseteq V(rad(A))$. Hence $V(A) = V(rad(A))$.
ii) It is clear .
proposition (3.9): If f is a d-morphisim from X to $\acute{X}$, then $f\big(x_{(\mu,v)}\big) = (f(x))_{(\mu,v)}$, for all $x\in X$ and for all $\mu, v \in (0,1]$ such that $\mu + v \le 1$.
proof : Let $y \in \acute{X}$ be any element, then $f\big(x_{(\mu,v)}\big)(y) = \langle \alpha_{f(x_{(\mu,v)})}(y), \beta_{f(x_{(\mu,v)})}(y)\rangle$, where

$$\alpha_{f(x_{(\mu,v)})}(y) = \sup\left\{\alpha_{x_{(\mu,v)}}(p), f(p) = y\right\} = \begin{cases}\mu\,; & if\ p = x\ , y = f(x)\\ 0\,; & otherwaise\end{cases} = \alpha_{(f(x))_{(\mu,v)}}(y)$$, and

$$\beta_{f(x_{(\mu,v)})}(y) = inf\left\{\beta_{x_{(\mu,v)}}(p), f(p) = y\right\} = \begin{cases} v\,; & if\ p = x\ , y = f(x)\\ 0\,; & otherwaise\end{cases} = \beta_{(f(x))_{(\mu,v)}}(y) .$$

Hence $f\big(x_{(\mu,v)}\big) = (f(x))_{(\mu,v)}$.

Definition (3.10) : Let $A\ and\ B\ \ are\ IFS$ we will define $A.B = \{< a, \alpha_{A.B}(a), \beta_{A.B}(a) >: a \in X\} =< \alpha_A.\alpha_B, \beta_A.\beta_B >$
Theorem (3.11) : Let $T = \{\chi(A), A\ is\ IFSd-ideal\ in\ X\}$. Then T is a topology on χ .
proof : Since $V\big(\tilde{0}\big) = X$ and $V\big(\tilde{1}\big) = \emptyset$, so that $\chi\big(\tilde{0}\big) = \emptyset$ and $\chi\big(\tilde{1}\big) = X$, and that implies $\emptyset, X \in T$.
Next let A_1 and A_2 be any two $IFSd-ideal$. Then let $B \in V(A_1) \cup V(A_2)$ that mean $A_1 \subseteq B$ or $A_2 \subseteq B$ then $A_1 \cap A_2 \subseteq B$, so $B \in V(A_1 \cap A_2)$, and if
$B \in V(A_1 \cap A_2)$ we get that $A_1 \cap A_2 \subseteq B$ and that's mean $A_1.A_2 \subseteq B$ then $A_1 \subseteq B$ or $A_2 \subseteq B$ and thus $B \in V(A_1) \cup V(A_2)$. Hence $V(A_1) \cup V(A_2) = V(A_1 \cap A_2)$, so $\chi(A_1) \cap \chi(A_2) = \chi(A_1 \cap A_2)$,

and that mean
$\chi(A_1) \cap \chi(A_2) = \chi(A_1 \cap A_2)$. This show that T closed under finite intersection .
Finally, let $\{A_i, i \in \Lambda\}$ be any family of $IFSd-ideal$ of X it can be easily confirm that $\cup \{V(A_i), i \in \Lambda\} = V(\langle \cup \{A_i, i \in \Lambda\}\rangle)$. In other words , $\bigcup_{i\in\Lambda} \chi(A_i) = \chi(\langle \bigcup_{i\in\Lambda} A_i\rangle)$. Hence T is closed under arbitrary unions . Thus T is a topology on X .
Remark (3.12) : The topological space (X,T) defined in theorem (3.11) is called the intuitionistic fuzzy prime semi d-ideal spectrum of d-algebra and is denoted by $IFPSd-Spec(X)$ or for convenience χ .
Notations (3.13) :
1- We will denoted for all $x \notin \bigcap\{J: J \text{ is prime semi } d-ideal \text{ in } X\}$ by $\tilde{x}$, and for all $x \in \bigcap\{J: J \text{ is prime semi } d-ideal \text{ in } X\}$ by $\breve{x}$.
2- Let A be an IFS of the X. Put $\wedge(A) = \{(\alpha_0,\beta_0),(\alpha_1,\beta_1), \dots\dots,(\alpha_n,\beta_n)\}$, where $\alpha_i, \beta_i \in [0,1]$ such that $\alpha_i + \beta_i \leq 1$ for all $i = 0,1,2 \dots, n$.

Theorem (3.14) : Let $x, y \in X$ and $\mu, v \in [0,1]$ such that $\mu + v \leq 1$, then

i) $\chi(x_{(\mu,v)}) \cap \chi(y_{(\mu,v)}) = \chi(xy_{(\mu,v)})$

ii) $\chi(x_{(\mu,v)}) = \emptyset$ if and only if x is $\breve{x}$.

iii) $\chi(x_{(\mu,v)}) = \chi$ if and only if is x is $\tilde{x}$ in X.

proof :

i) If $P \in \chi(x_{(\mu,v)}) \cap \chi(y_{(\mu,v)})$, then $P \in \chi(x_{(\mu,v)})$ and $P \in \chi(y_{(\mu,v)})$, that means $x_{(\mu,v)} \not\subseteq P$ and $y_{(\mu,v)} \not\subseteq P$, and that implies $\alpha_P(x) < \mu$, $\beta_P(x) > v$ and $\alpha_P(y) < \mu$, $\beta_P(y) > v$. Thus $\mu > \alpha_P(x) = \alpha_P(y) = \alpha_P(xy)$, and $v < \beta_P(x) = \beta_P(y) = \beta_P(xy)$, since $P_* = \{x \in X : \alpha_P(x) = 1 , \beta_P(x) = 0\}$ is a prime semi d-ideal of X and $\wedge(P) = \{(0,1),(\mu,v)\}$ implies that $\alpha_P(a) = \alpha_P(b)$ and $\beta_P(a) = \beta_P(b)$ for all $a, b \in X \setminus P_*$ and $x, y, xy \notin P_*$. Then $xy_{(\mu,v)} \not\subseteq P$, which means that $P \in \chi(xy_{(\mu,v)})$. The proof of (i) is complete, since all the implication can be reversed.

ii) Let J be any prim semi d-ideal of d-algebra X and let X_J be the intuitionistic fuzzy characteristic function of J. It is follows that $X_J \in X$. Next if $\chi(x_{(\mu,v)}) = \emptyset$, then $V(x_{(\mu,v)}) = X$, which implies that $x_{(\mu,v)} \subseteq X_J$, and therefore $\alpha_{X_J}(x) = 1$ and $\beta_{X_J}(x) = 0$, so $x \in J$. Thus $x \in \bigcap\{J: J \text{ is prime semi } d-ideal \text{ in } X\}$. Hence x is $\breve{x}$. Conversely, assume that x is $\breve{x}$. Let $A \in X$, then A_* is prim semi d-ideal of X, and $x \in A_*$, therefore $\alpha_A(x) = 1$, $\beta_A(x) = 0$. Hence $\mu = \alpha_{x_{(\mu,v)}}(x) \leq \alpha_A(x)$ and $v = \beta_{x_{(\mu,v)}}(x) \geq \beta_A(x)$, where $x_{(\mu,v)} \subseteq A$ for all $A \in X$. Thus $V(x_{(\mu,v)}) = X$, i.e. $\chi(x_{(\mu,v)}) = \emptyset$.

iii) Let J be any prim semi d-ideal of d-algebra X and let X_J be the intuitionistic fuzzy characteristic function of J. Now if $\chi(x_{(\mu,v)}) = X$, then $V(x_{(\mu,v)}) = \emptyset$, which implies that $x_{(\mu,v)} \not\subseteq X_J$, and therefore $\alpha_{X_J}(x) < \mu$ and $\beta_{X_J}(x) > v$, so $x \notin J$. Thus $x \notin \bigcap\{J: J \text{ is prime semi } d-ideal \text{ in } X\}$. Hence x is $\breve{x}$. The converse in the converse way .

Theorem (3.15) : The sub-family $\{\chi(x_{(\mu,v)}), x \in X$ and $\mu, v \in (0,1]$ such that $\mu + v \leq 1\}$ of χ is a base for T.

proof : Let $\chi(A) \in T$, and let $B \in \chi(A)$, then $\alpha_B(x) < \alpha_A(x)$ and $\beta_B(x) > \beta_A(x)$ for some $x \in X$. Let $\alpha_A(x) = \mu$ and $\beta_A(x) = v$, then $x_{(\mu,v)} \nsubseteq A$ and so $A \in \chi\left(x_{(\mu,v)}\right)$. Now $V(A) \subseteq V\left(x_{(\mu,v)}\right)$, because if $P \in V(A)$, then $\alpha_P(x) \geq \alpha_A(x) = \mu = \alpha_{x_{(\mu,v)}}(x)$, and $\beta_P(x) \leq \beta_A(x) = v = \beta_{x_{(\mu,v)}}(x)$. So that $x_{(\mu,v)} \subseteq P$ and thus $P \in V\left(x_{(\mu,v)}\right)$. Hence $\chi\left(x_{(\mu,v)}\right) \subseteq \chi(A)$. Thus $B \in \chi\left(x_{(\mu,v)}\right) \subseteq \chi(A)$. And this complete the proof .

Theorem (3.16) : Spec(X) is disconnected if and only if there exist two $IFSd-ideal$ A, B such that $\mathrm{rad}(\mathrm{A} \cup \mathrm{B}) = \mathrm{rad}(1)$ and $\mathrm{rad}(\mathrm{A} \cap \mathrm{B}) = \mathrm{rad}(0)$.

proof : Let spec(X) be disconnected, then there exist two $IFSd-ideal$ A,B in X such that $\chi(\mathrm{A}) \neq \emptyset$, $\chi(\mathrm{B}) \neq \emptyset$, $\chi(\mathrm{A}) \cap \chi(\mathrm{B}) = \emptyset$, $\chi(\mathrm{A}) \cup \chi(\mathrm{B}) = \mathrm{spec(X)}$. That is mean $\chi(\mathrm{A}) \cap \chi(\mathrm{B}) = \chi(\tilde{0})$ and $\chi(\mathrm{A}) \cup \chi(\mathrm{B}) = \chi(\tilde{1})$. Thus $\chi(\mathrm{A} \cap \mathrm{B}) = \chi(\tilde{0})$ and $\chi(\mathrm{A} \cup \mathrm{B}) = \chi(\tilde{1})$. So by proposition (3.10)(ii) " we get $\mathrm{rad}(\mathrm{A} \cap \mathrm{B}) = \mathrm{rad}(\tilde{0})$ and $\mathrm{rad}(\mathrm{A} \cup \mathrm{B}) = \mathrm{rad}(\tilde{1})$. and in the converse way the proof will complete .

Recall that a subset A of a topological space X is called strongly connected (s-connected) when we get for any open subset U and V of X , if $A \subseteq U \cup V$, then $A \subseteq U$ or $A \subseteq V$. **[12]**

Theorem (3.17) : Any subset of spec(X) is S-connected .

proof : Let $\wp$ be a collection of an $IFPSd-ideal$ of spec(X), and let C, D be an $IFSd-ideal$ in X. Since $\wp \subseteq \chi(\mathrm{C}) \cup \chi(\mathrm{D}) \subseteq \chi(\mathrm{C} \cup \mathrm{D})$. Then by proposition (3.5) we get that $\wp \subseteq \chi(\mathrm{C})$ or $\wp \subseteq \chi(\mathrm{D})$ and this complete the proof .

Theorem (3.18) : Spec(X) is a $\mathrm{T}_0-\mathrm{space}$.

Proof : Let $\mathrm{A}, \mathrm{B} \in \chi$ and $\mathrm{A} \neq \mathrm{B}$. Then either $\mathrm{A} \nsubseteq \mathrm{B}$ or $\mathrm{B} \nsubseteq \mathrm{A}$. Let $\mathrm{A} \nsubseteq \mathrm{B}$ then $\mathrm{B} \notin \mathrm{V(A)}$, but $\mathrm{A} \in \mathrm{V(A)}$, Then $\mathrm{B} \in \mathrm{X(A)}$, and $\mathrm{A} \notin \mathrm{X(A)}$. Now let $\mathrm{B} \nsubseteq \mathrm{A}$ similarly we can get $\mathrm{A} \in \mathrm{X(B)}$ but $\mathrm{B} \notin \mathrm{X(B)}$. It follow that spec(X) is a $\mathrm{T}_0-\mathrm{space}$.

Theorem (3.19) : In Spec(X) , $\mathrm{V(A)} = \overline{\{\mathrm{A}\}}$ for all $IFSd-ideal$ in X

proof : It is clear that $\overline{\{\mathrm{A}\}} \subseteq V(A)$, since $V(A)$ is closed set containing A. Now let $\mathrm{B} \notin \overline{\{\mathrm{A}\}}$, then there exist an open set $\mathrm{X} \setminus \mathrm{V(C)}$ containing B but not A, therefore $\mathrm{C} \nsubseteq \mathrm{B}$, but $\mathrm{C} \subseteq A$ and so $\mathrm{B} \notin \mathrm{V(A)}$. Thus $V(A) \subseteq \overline{\{\mathrm{A}\}}$, and that complete the proof .

Corollary (3.20) : $\mathrm{B} \in \overline{\{\mathrm{A}\}}$ if and only if $\mathrm{A} \subseteq B$.

proof : it is follow directly from theorem (3.19).

Theorem (3.21) : Let $Y = \{P \in X : \wedge(P) = \{(0,1),(\mu,v)\} : \mu, v \in [0,1)$ such that $\mu + v \leq 1\}$, then Y is T_1 if and only if every singleton element of Y is an intuitionistic fuzzy maximal semi d-ideal of X .

proof : we need to show that the semi d-ideal $A_* = \{x \in X, \alpha_A(x) = 1, \beta_A(x) = 0\}$ is a maximal semi d-ideal . It is sufficient to show that there is no prime semi d-ideal of X containing A_*. Let J is a prim semi d-ideal containing A_*, consider an $IFSd-ideal$ B of X defined by $\alpha_B(x) = \begin{cases} 1 & if\ x \in J \\ \mu & if\ x \notin J \end{cases}$ and

$\beta_B(x) = \begin{cases} 0 & if\ x \in J \\ v & if\ x \notin J \end{cases}$, where $\mu + v \leq 1$. Then $B \in Y$ and A containing in B . This contradiction the fact that $V(A) \cap Y = \{A\}$.

Conversely, let A is an $IFMSd - ideal$ then the ideal $A_* = \{x \in X, \alpha_A(x) = 1, \beta_A(x) = 0\}$ is maximal, we claim that $V(A) \cap Y = \{A\}$. Clearly $\{A\} \subseteq V(A) \cap Y$. Now if $B \in V(A) \cap Y$, then $A \subseteq B$ and $A_* \subseteq B_*$. This mean that $A_* = B_*$, since A_* is a maximal semi d-ideal . Hence $B = A$, since $\wedge(A) = \wedge(B) = \{(1,0), (\mu, v)\}$, therefore $V(A) \cap Y = \{A\}$, consequently $\{A\}$ is closed subset of Y.

Theorem (3.22) : If every prime semi d-ideal in X is maximal , Then the space $IFPSd - Spec(X)$ is not Hausdorff.

proof : Let J be a prim semi d-ideal of X, consider two $IFPSd - ideals\ A, B$ of X defined by $\alpha_A(x) = \begin{cases} 1 & if\ x \in J \\ 0.1 & if\ x \notin J \end{cases}$ and $\beta_A(x) = \begin{cases} 0 & if\ x \in J \\ 0.2 & if\ x \notin J \end{cases}$, $\alpha_B(x) = \begin{cases} 1 & if\ x \in J \\ 0.3 & if\ x \notin J \end{cases}$ $\beta_B(x) = \begin{cases} 0 & if\ x \in J \\ 0.4 & if\ x \notin J \end{cases}$ Let $X(x_{(\mu,v)})$ and $X(y_{(\mu,v)})$ be any two basic open set in X containing A and B respectively where $x, y \in X\ and\ \mu + v \leq 1$. Then $x_{(\mu,v)} \nsubseteq$ A and $y_{(\mu,v)} \nsubseteq$ B , and so $x \notin A_* = J$ and $y \notin B_* = J$. Since J is prime then $xy \notin J$, then xy is not nilpotent and so by theorem " (3.14) (i) and (ii) " we have $X(x_{(\mu,v)}) \cap X(y_{(\mu,v)}) = X(xy_{(\mu,v)}) \neq \emptyset$. Hence X is not Hausdorff .

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