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To cite this article: R N Majeed and S A El-Sheikh 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **571** 012026

IOP Conf. Series: Materials Science and Engineering **571** (2019) 012026 doi:10.1088/1757-899X/571/1/012026

Fuzzy orbit topological spaces

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Abstract. The concept of fuzzy orbit open sets under the mapping $f: X \to X$ in a fuzzy topological space (X, τ) was introduced by Malathi and Uma (2017). In this paper, we introduce some conditions on the mapping f, to obtain some properties of these sets. Then we employ these properties to show that the family of all fuzzy orbit open sets construct a new fuzzy topology, which we denoted by τ_{FO} coarser than τ. As a result, a new fuzzy topological space (X, τ_{FO}) is obtained. We refer to this topological space as a fuzzy orbit topological space. In addition, we define the notion of fuzzy orbit interior (closure) and study some of their properties. Finally, the category of fuzzy orbit topological spaces $\mathbb{F}\mathbb{O}\mathbf{TOP}$ is defined, and we prove it can be embedded in the category of fuzzy topological spaces $\mathbb{F}\mathbf{TOP}$.

1. Introduction

The theory of fuzzy sets are a generalization of conventional set theory that were introduced by Zadeh [17] in 1965 as a mathematical direction to represent waif and vagueness and to supply formalized tools for dealing with the problems in everyday life. The fuzzy set theory applied in many directions and fields such as information [12], control [13] and decision making [3, 10]. The modern theory of fuzzy topology was introduced by Chang [6] in 1968, as a generalization to the basic concepts of classical topology. After that, a lot of contributions to the evolution of fuzzy topology have been published (cf.[1, 2, 5, 8, 14, 15]). The orbit in mathematics has an important role in the study of dynamical systems, an orbit is a collection of points associated by the evolution function of the dynamical system. One of the objectives of the modern theory of dynamical systems is using topological methods to understanding the properties of dynamical systems [9]. The concept of the fuzzy orbit set was introduced by Malathi and Uma [11] in 2017, as a generalization to the concept of the orbit point in general metric space [7]. Also, Malathi and Uma [11] introduced the concepts of fuzzy orbit open sets and fuzzy orbit continuous mappings. The purpose of this paper, is to study the collection of fuzzy orbit open sets under the mapping $\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{X}$. In Section 3, we give the necessary conditions on the mapping $\boldsymbol{f}$ in order to obtain a fixed orbit of a fuzzy set (i.e., $\boldsymbol{f}(\boldsymbol{\mu}) = \boldsymbol{\mu}$) for any fuzzy orbit open set $\boldsymbol{\mu}$ under the mapping $\boldsymbol{f}$. Also, some properties of fuzzy orbit open sets related with union (intersection) of these sets are introduced. In Section 4, we prove the family of all fuzzy orbit open sets constructs a fuzzy topological space. This new space is called fuzzy orbit topological space $(\boldsymbol{X}, \boldsymbol{\tau_{FO}})$. Furthermore, the concept of fuzzy orbit interior (closure) is defined and study some of their properties. Finally, in Section 5, the category of fuzzy orbit topological spaces and fuzzy continuous mappings $\mathbb{F}\mathbb{O}\mathbf{TOP}$ is defined. And we show this category is isomorphic to a subcategory of the category of fuzzy topological spaces.

2. Preliminaries

Throughout this paper, let X be a nonempty countable set, $I = [0, 1]$. A fuzzy set μ of X is a mapping from X to I, the family of all fuzzy sets of X is denoted by I^X. By $\overline{0}$ and $\overline{1}$ we denote constant maps on

X with value 0 and 1, respectively. For any fuzzy set $\mu \in I^X$the complement of μ, denoted by $\overline{1} - \mu$. For the principle terminology of category theory, see [4].

Definition 2.1 [11] Let X be a nonempty set and let $f: X \to X$ be any mapping. Let λ be any fuzzy set of X. The fuzzy orbit $O_f(\lambda)$ of λ under the mapping f is defined as $O_f(\lambda) = \{\lambda, f(\lambda), f^2(\lambda), \dots\}$.

Definition 2.2 [11] Let X be a nonempty set and let $f: X \to X$ be any mapping. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge$ .... the intersection of all members of $O_f(\lambda)$.

Definition 2.3 [11] Let (X, τ) be a fuzzy topological space or *fts*. Let $f: X \to X$ be any mapping. The fuzzy orbit set under the mapping f which is in the fuzzy topology τ is called fuzzy orbit open set under the mapping f. Its complement is called fuzzy orbit closed set under the mapping f.

The following example explain the concept of fuzzy orbit open set.

Example 2.1 Let $X = \{a_1, a_2, a_3\}$. Define $\tau = \{\overline{0}, \overline{1}, \lambda, \mu\}$ where $\lambda, \mu \in I^X$ defined as

$$\lambda = \{(a_1, 0.5), (a_2, 0.4), (a_3, 0.6)\} \text{ and } \mu = \{(a_1, 0.4), (a_2, 0.4), (a_3, 0.4)\}.$$

Define $f: X \to X$ as $f(a_1) = a_3$, $f(a_2) = a_1$, $f(a_3) = a_2$. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge \dots = \mu$. Therefore, μ is a fuzzy orbit open set under the mapping f.

From the Definition 2.3, its clear that every fuzzy orbit open set under the mapping f is an open fuzzy set in X. But the converse is not true, in Example 2.1, the fuzzy set λ is an open fuzzy set, however it is not fuzzy orbit open set under the mapping f, because there is not exists a fuzzy set $\nu \in I^X$ such that $FO_f(\nu) = \lambda$.

Definition 2.4 [6] A mapping f from a *fts* (X, τ) to a *fts* (Y, τ^*) is fuzzy continuous iff the inverse image of each open fuzzy set in Y is an open fuzzy set in X.

Theorem 2.1 [16] Let $\mathbb{F}$**TOP** be the family of all *fts's* $(X, \tau), (Y, \tau'), \dots$. For any two pair of objects $(X, \tau), (Y, \tau')$ of $\mathbb{F}$**TOP**, define $Mor((X, \tau), (Y, \tau'))$ to be the set of all fuzzy continuous mappings f with respect to τ and τ'. Then, $\mathbb{F}$**TOP** is a category.

3. Some properties of fuzzy orbit open sets

In our work we consider X as a nonempty countable set, we give the conditions on a mapping $f: X \to X$, to obtain a fixed fuzzy orbit open set (i.e., $f(\mu) = \mu$) for any fuzzy orbit open set μ, and study some properties of these sets.

Theorem 3.1 Let (X, τ) be a fts and $f: X \to X$ be any bijective mapping. Then $f(\mu) = \mu$ for any fuzzy orbit open set μ under the mapping f.

Proof. Let (X, τ) be a fts and $f: X \to X$ be a bijective mapping. Then we have three cases:
Case 1: If $f(a_i) = a_j$; $a_i, a_j \in X$ and $i \neq j$ for all $i, j \in \Lambda$.
Suppose $X = \{a_1, a_2\}$ and $f: X \to X$ defined as $f(a_1) = a_2$, $f(a_2) = a_1$. Let μ be a fuzzy orbit open set under the mapping f. Then there exists a fuzzy set $\lambda \in I^X$ such that $FO_f(\lambda) = \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge \dots = \mu$.
Let $\lambda = \{(a_1, t_1), (a_2, t_2);\ a_1, a_2 \in X, t_1, t_2 \in I\}$. This implies

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$f(\lambda) = \{(a_1, t_2);\ (a_2, t_1)\}$, $f^2(\lambda) = \{(a_1, t_1);\ (a_2, t_2), \ldots$. Therefore,

$$\begin{aligned} FO_f(\lambda) &= \{(a_1, \inf\{t_1, t_2, t_1, \ldots\}), (a_2, \inf\{t_2, t_1, t_2, \ldots\})\} \\ &= \{(a_1, \min\{t_1, t_2\}), (a_2, \min\{t_1, t_2\})\} \\ &= \mu\,. \end{aligned}$$

In general, if $X = \{a_1, a_2, \ldots\}$ and μ be a fuzzy orbit open set under the mapping f, then there exists a fuzzy set $\lambda = \{(a_1, t_1), (a_2, t_2), (a_3, t_3), \ldots\} = \{(a_i, t_i);\ a_i \in X,\ t_i \in I, i \in \Lambda\}$ such that $FO_f(\lambda) = \mu$. That means

$$\begin{aligned} FO_f(\lambda) &= \{(a_i, \inf\{t_i\}); a_i \in X,\ t_i \in I, i \in \Lambda\} \\ &= \{(a_i, s); a_i \in X, s = \inf\{t_i;\ t_i \in I\}, i \in \Lambda\} \\ &= \mu\,. \end{aligned}$$

Now, for each $a_j \in X$, we have

$$f(\mu)(a_j) = \begin{cases} \bigvee_{f(a_i)=a_j} \mu(a_i) & if\ f^{-1}(a_j) \neq \emptyset, \\ 0 & if\ f^{-1}(a_j) = \emptyset. \end{cases}$$

From the hypothesis and the definition of f, we get $f(\mu)(a_j) = \mu(a_i) = s$ for all $a_j \in X$. Hence $f(\mu) = \mu$.

Case 2: If $f(a_i) = a_j$; $a_i, a_j \in X$ and $i = j$ for some $i, j \in \Lambda$. In this case the least number of elements in X must be three elements. So, suppose that if $X = \{a_1, a_2, a_3\}$, then from the hypothesis and the definition of f, the mapping f can be defined as $f(a_1) = a_1$, $f(a_2) = a_3$ and $f(a_3) = a_2$ (i.e., $f(a_i) = a_j$ when $i = j = 1$ and $f(a_i) = a_j$, $i \neq j$ when $i, j \in \{2,3\}$).

Let μ be a fuzzy orbit open set under the mapping f. Then there exists a fuzzy set $\lambda \in I^X$ such that $FO_f(\lambda) = \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge \ldots = \mu$.

Let $\lambda = \{(a_1, t_1), (a_2, t_2), (a_3, t_3);\ a_i \in X,\ t_i \in I, i = 1,2,3\}$. Then from the definition of f, we get $f(\lambda) = \{(a_1, t_1);\ (a_2, t_3), (a_3, t_2)\}$, $f^2(\lambda) = \{(a_1, t_1);\ (a_2, t_2), (a_3, t_3), \ldots$.

Therefore,

$$\begin{aligned} FO_f(\lambda) &= \{(a_1, t_1), (a_2, \inf\{t_2, t_3, t_2, \ldots\}), (a_3, \inf\{t_3, t_2, t_3, \ldots\})\} \\ &= \{(a_1, t_1), (a_2, \min\{t_2, t_3\}), (a_3, \min\{t_2, t_3\})\} \\ &= \mu\,. \end{aligned}$$

In general, if $X = \{a_1, a_2, \ldots\}$ and μ be a fuzzy orbit open set under the mapping f, then there exists a fuzzy set $\lambda = \{(a_1, t_1), (a_2, t_2), (a_3, t_3), \ldots\} = \{(a_i, t_i);\ a_i \in X,\ t_i \in I, i \in \Lambda\}$ such that $FO_f(\lambda) = \mu$. This implies

$$\begin{aligned} FO_f(\lambda) &= \{(a_i, t_i); f(a_i) = a_j, i = j, (a_i, \inf\{t_i, i \in \Lambda\}); f(a_i) = a_j, i \neq j\} \\ &= \{(a_i, t_i); f(a_i) = a_j, i = j, (a_i, s); s = \inf\{t_i, i \in \Lambda\}; f(a_i) = a_j, i \neq j\} \\ &= \mu\,. \end{aligned}$$

Now, for each $a_j \in X$, we have

$$f(\mu)(a_j) = \mu(a_i) = \begin{cases} \bigvee_{f(a_i)=a_j} \mu(a_i) & if\ f^{-1}(a_j) \neq \emptyset, \\ 0 & if\ f^{-1}(a_j) = \emptyset. \end{cases}$$

From the hypothesis and the definition of f, we get for all $a_j \in X$

$$f(\mu)(a_j) = \begin{cases} t_i & if\ i = j, \\ s & if\ i \neq j. \end{cases}$$

Hence $f(\mu) = \mu$.

Case 3: If f is the identity mapping. In this case, every open fuzzy set in X is fuzzy orbit open set under the mapping f and $f(\mu) = \mu$ for every fuzzy set $\mu \in I^X$. Thus, the proof is obtained.

Theorem 3.2 Let (X, τ) be a fts and $f: X \to X$ be any constant mapping. Then $f(\mu) = \mu$ for any fuzzy orbit open set μ under the mapping f.

Proof. Let (X, τ) be a *fts* and let μ be a fuzzy orbit open set under the mapping f. Then, from Definition 2.3, there exists a fuzzy set $\lambda = \{(a_i, t_i);\ a_i \in X,\ t_i \in I, i \in \Lambda\}$ such that $FO_f(\lambda) = \mu$. Since f is constant mapping, this implies there exists a fixed element $a_k \in X$ such that $f(a_i) = a_k$ for all $a_i \in X$ and $i \in \Lambda$.
Now, from the definition of $f(\lambda)$ for all $a_j \in X$ we have,

$$f(\lambda)(a_j) = \begin{cases} \vee_{f(a_i)=a_j} \{\lambda(a_i)\} & if\ f^{-1}(a_j) \neq \emptyset, \\ 0 & otherwise. \end{cases}$$

Thus,

$$f(\lambda)(a_j) = \begin{cases} sup_{i\in\Lambda} \{\lambda(a_i)\} & if\ a_j = a_k\ , \\ 0 & if\ a_j \neq a_k\ . \end{cases}$$

Therefore, $f(\lambda) = \{(a_k, sup_{i\in\Lambda} \{\lambda(a_i)\})\}$. This means $f(\lambda)$ is a fuzzy point in X with support a_k and degree $sup_{i\in\Lambda} \{\lambda(a_i)\}$. By the same way, we have $f^2(\lambda) = \{(a_k, sup_{i\in\Lambda} \{\lambda(a_i)\})\}$, $f^3(\lambda) = \{(a_k, sup_{i\in\Lambda} \{\lambda(a_i)\})\}$, …. For more clearing, we have the following:
$\lambda = \{(a_1, t_1), (a_2, t_2), \dots, (a_k, t_k), \dots\}$
$f(\lambda) = \{(a_1, 0), (a_2, 0), \dots, (a_k, sup_{i\in\Lambda} \{\lambda(a_i)\}), \dots\}$
$f^2(\lambda) = \{(a_1, 0), (a_2, 0), \dots, (a_k, sup_{i\in\Lambda} \{\lambda(a_i)\}), \dots\}$
$f^3(\lambda) = \{(a_1, 0), (a_2, 0), \dots, (a_k, sup_{i\in\Lambda} \{\lambda(a_i)\}), \dots\}$
.
.
.
Thus,

$$\begin{aligned} FO_f(\lambda) &= \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge \dots \\ &= \{(a_1, 0), (a_2, 0), \dots, (a_k,\ \min\{t_k, sup_{i\in\Lambda} \{\lambda(a_i)\}\}), \dots\} \\ &= \begin{cases} (a_i, 0) & if\ i \neq k, \\ (a_k,\ \min\{t_k, sup_{i\in\Lambda} \{\lambda(a_i)\}\}) & if\ \ i = k. \end{cases} \\ &= \mu. \end{aligned}$$

This yield $FO_f(\lambda) = \mu$ is a fuzzy point in X with support a_k and degree $\min\{t_k, sup_{i\in\Lambda} \{\lambda(a_i)\}\}$. Hence, from the definition of f, we get $f(\mu) = \mu$.

Remark 3.1 The condition to be $f: X \to X$ is bijective or constant is necessary condition to obtain fixed fuzzy orbit open sets for any fuzzy orbit open set μ under the mapping f. For more explain, we give an example for a fts (X, τ) and $f: X \to X$ not bijective, we show that $f(\mu) \neq \mu$ for some fuzzy orbit open set μ under the mapping f.

Example 3.1 Let $X = \{a_1, a_2, a_3, a_4, a_5\}$. Define $\tau = \{\overline{0}, \overline{1}, \mu\}$ where $\mu \in I^X$ defined as

$$\mu = \{(a_1, 0), (a_2, 0.1), (a_3, 0), (a_4, 0.6), (a_5, 0.6)\}.$$

Define $f: X \to X$ as $f(a_1) = f(a_3) = a_2, f(a_2) = a_3, f(a_4) = a_5$ and $f(a_5) = a_4$. It is clear that f is not bijective mapping (i.e., f is not one to one and not onto). Let $\lambda \in I^X$ defined as follows:

$$\lambda = \{(a_1, 0.1), (a_2, 0.2), (a_3, 0), (a_4, 0.6), (a_5, 0.7)\}.$$

Then, the fuzzy orbit of λ are $O_f(\lambda) = \{\lambda, f(\lambda), f^2(\lambda), \dots\}$. Which is

$$f(\lambda) = \{(a_1, 0), (a_2, 0.1), (a_3, 0.2), (a_4, 0.7), (a_5, 0.6)\}$$
$$f^2(\lambda) = \{(a_1, 0), (a_2, 0.2), (a_3, 0.1), (a_4, 0.6), (a_5, 0.7)\}$$

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$$f^3(\lambda) = \{(a_1, 0), (a_2, 0.1), (a_3, 0.2), (a_4, 0.7), (a_5, 0.6)\}$$

Therefore, the fuzzy orbit set of λ is

$$\begin{aligned} FO_f(\lambda) &= \lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge \dots \\ &= \{(a_1, \inf\{0.1, 0,0, \dots\}), \\ &\quad (a_2, \inf\{0.2, 0.1,0.2, 0.1, \dots\}), \\ &\quad (a_3, \inf\{0, 0.2,0.1, 0.2, \dots\}), \\ &\quad (a_4, \inf\{0.6, 0.7,0.6, 0.7, \dots\}), \\ &\quad (a_5, \inf\{0.7, 0.6,0.7, 0.6, \dots\})\} \\ &= \{(a_1, 0), (a_2, 0.1), (a_3, 0), (a_4, 0.6), (a_5, 0.6)\} \\ &= \mu. \end{aligned}$$

Thus, the open fuzzy set μ is fuzzy orbit open set under the mapping f. But $f(\mu) = \{(a_1, 0), (a_2, 0), (a_3, 0.1), (a_4, 0.6), (a_5, 0.6)\} \neq \mu$.

From Theorems 3.1 and 3.2, we obtain the following result.

Result 3.1 Let (X, τ) be a fts, $f: X \to X$ be any mapping such that either f is bijective mapping or f is constant mapping and μ is a fuzzy orbit open set under the mapping f, then $f(\mu) = \mu$.

In our work, we consider the mapping $f: X \to X$ that satisfies the conditions in Result 3.1.

Proposition 3.1 Let (X, τ) be a fts and $f: X \to X$ be any mapping. If μ is a fuzzy orbit open set under the mapping f, then $FO_f(\mu) = \mu$.

Proof. The proof follows directly from the definition of $FO_f(\mu)$and Result 3.1. i.e., $FO_f(\mu) = \mu \wedge f(\mu) \wedge f^2(\mu) \wedge \dots$. From Result 3.1, we have $f(\mu) = \mu$, this implies $f^2(\mu) = f(f(\mu)) = \mu$, $f^3(\mu) = f(f^2(\mu)) = \mu$,.... Hence, $FO_f(\mu) = \mu$.

Theorem 3.3 Let (X, τ) be a fts and $f: X \to X$ be a mapping. If μ_1 and μ_2 are fuzzy orbit open sets under the mapping f, then $FO_f(\mu_1 \wedge \mu_2) = FO_f(\mu_1) \wedge FO_f(\mu_2)$.

Proof. First we prove the theorem if f is bijective mapping. From Theorem 3.1, we have three cases. We prove the theorem in case 1. The proof of theorem in case 2 is similar to case 1, and the prove of theorem in case 3 is easy.

Case 1: Suppose that f is bijective mapping and $f(a_i) = a_j; a_i, a_j \in X$ and $i \neq j$ for all $i, j \in \Lambda$. Let μ_1 and μ_2 are fuzzy orbit open sets under the mapping f. Then, there exist $\lambda_1, \lambda_2 \in I^X$ defined as $\lambda_1 = \{(a_i, t_i);\ a_i \in X,\ t_i \in I, i \in \Lambda\}$ and $\lambda_2 = \{(a_i, s_i);\ a_i \in X,\ s_i \in I, i \in \Lambda\}$such that $FO_f(\lambda_1) = \mu_1$and $FO_f(\lambda_2) = \mu_2$. From Theorem 3.1 case 1, we have $FO_f(\lambda_1) = \{(a_i, t); t = \inf\{t_i, i \in \Lambda\}\} = \mu_1$ and $FO_f(\lambda_2) = \{(a_i, s); s = \inf\{s_i, i \in \Lambda\}\} = \mu_2$. Thus, $\mu_1 \wedge \mu_2 = \{(a_i, \min\{t, s\}); a_i \in X, i \in \Lambda\}$. Let $k = \min\{t, s\}$. Now, for all $a_j \in X, j \in \Lambda$

$$\begin{aligned} f(\mu_1 \wedge \mu_2)(a_j) &= \begin{cases} \bigvee_{f(a_i)=a_j} (\mu_1 \wedge \mu_2)(a_i) & if\ f^{-1}(a_j) \neq \emptyset, \\ 0 & if\ f^{-1}(a_j) = \emptyset. \end{cases} \\ &= k \end{aligned}$$

Hence, $f(\mu_1 \wedge \mu_2)= \mu_1 \wedge \mu_2$. This implies, $f^2(\mu_1 \wedge \mu_2)= \mu_1 \wedge \mu_2$, $f^3(\mu_1 \wedge \mu_2)= \mu_1 \wedge \mu_2$,.... Therefore, from the definition of $FO_f(\mu_1 \wedge \mu_2)$ and Theorem 3.1, we get $FO_f(\mu_1 \wedge \mu_2) = \mu_1 \wedge \mu_2 = FO_f(\mu_1) \wedge FO_f(\mu_2)$.

Case 2: Suppose that f is bijective mapping and $f(a_i) = a_j$; $a_i, a_j \in X$ and $i = j$ for some $i, j \in \Lambda$. Let μ_1 and μ_2 are fuzzy orbit open sets under the mapping f. Then, there exist $\lambda_1, \lambda_2 \in I^X$ defined as $\lambda_1 = \{(a_i, t_i);\ a_i \in X,\ t_i \in I, i \in \Lambda\}$ and $\lambda_2 = \{(a_i, s_i);\ a_i \in X,\ s_i \in I, i \in \Lambda\}$such that $FO_f(\lambda_1) = \mu_1$and $FO_f(\lambda_2) = \mu_2$. From Theorem 3.1 case 2, we have
$FO_f(\lambda_1) = \{(a_i, t_i); f(a_i) = a_j, i = j, (a_i, t = \inf\{t_i, i \in \Lambda\}); f(a_i) = a_j, i \neq j\} = \mu_1$ and
$FO_f(\lambda_2) = \{(a_i, s_i); f(a_i) = a_j, i = j, (a_i, s = \inf\{s_i, i \in \Lambda\}); f(a_i) = a_j, i \neq j\} = \mu_2$.
Thus, $\mu_1 \wedge \mu_2 = \{(a_i, \min\{t_i, s_i\}); f(a_i) = a_j, i = j, (a_i, \min\{t, s\}); f(a_i) = a_j, i \neq j\}$. Now, for all $a_j \in X, j \in \Lambda$

$$f(\mu_1 \wedge \mu_2)(a_j) = \begin{cases} \bigvee_{f(a_i)=a_j} (\mu_1 \wedge \mu_2)(a_i) & if\ f^{-1}(a_j) \neq \emptyset, \\ 0 & if\ f^{-1}(a_j) = \emptyset. \end{cases}$$
$$= \begin{cases} \min\{t_i, s_i\} & if\ f(a_i) = a_j, i = j, \\ \min\{t, s\} & if\ f(a_i) = a_j, i \neq j. \end{cases}$$

Hence, $f(\mu_1 \wedge \mu_2)= \mu_1 \wedge \mu_2$. This implies, $f^2(\mu_1 \wedge \mu_2)= \mu_1 \wedge \mu_2$, $f^3(\mu_1 \wedge \mu_2)= \mu_1 \wedge \mu_2$,.... Therefore, from the definition of $FO_f(\mu_1 \wedge \mu_2)$ and Theorem 3.1, we get $FO_f(\mu_1 \wedge \mu_2) = \mu_1 \wedge \mu_2 = FO_f(\mu_1) \wedge FO_f(\mu_2)$.

Now, if f is constant mapping, let μ_1 and μ_2 are fuzzy orbit open sets under the mapping f. Then, there exist $\lambda_1, \lambda_2 \in I^X$ defined as $\lambda_1 = \{(a_i, t_i);\ a_i \in X,\ t_i \in I, i \in \Lambda\}$ and $\lambda_2 = \{(a_i, s_i);\ a_i \in X,\ s_i \in I, i \in \Lambda\}$such that $FO_f(\lambda_1) = \mu_1$and $FO_f(\lambda_2) = \mu_2$. From Theorem 3.2, we have

$$FO_f(\lambda_1) = \begin{cases} (a_i, 0) & if\ i \neq k, \\ (a_k, \min\{t_k, sup_{i\in\Lambda}\{\lambda_1(a_i)\}\}) & if\ i = k. \end{cases}$$
$$= \mu_1$$

And,

$$FO_f(\lambda_2) = \begin{cases} (a_i, 0) & if\ i \neq k, \\ (a_k, \min\{s_k, sup_{i\in\Lambda}\{\lambda_2(a_i)\}\}) & if\ i = k. \end{cases}$$
$$= \mu_2.$$

Thus,

$$\mu_1 \wedge \mu_2 = \begin{cases} (a_i, 0) & if\ i \neq k, \\ (a_k, \min\{\min\{t_k, sup_{i\in\Lambda}\{\lambda_1(a_i)\}\}, \min\{s_k, sup_{i\in\Lambda}\{\lambda_2(a_i)\}\}\}) & if\ i = k. \end{cases}$$

This means $\mu_1 \wedge \mu_2$ is a fuzzy point in X with support a_k and degree $\min\{\min\{t_k, sup_{i\in\Lambda}\{\lambda_1(a_i)\}\}, \min\{s_k, sup_{i\in\Lambda}\{\lambda_2(a_i)\}\}\}$. Hence, from the definition of f, we get $f(\mu_1 \wedge \mu_2)= \mu_1 \wedge \mu_2$. This implies, $f^2(\mu_1 \wedge \mu_2)= \mu_1 \wedge \mu_2$, $f^3(\mu_1 \wedge \mu_2)= \mu_1 \wedge \mu_2$,.... Therefore, from the definition of $FO_f(\mu_1 \wedge \mu_2)$ and Theorem 3.2, we get $FO_f(\mu_1 \wedge \mu_2) = \mu_1 \wedge \mu_2 = FO_f(\mu_1) \wedge FO_f(\mu_2)$.

Theorem 3.4 Let (X, τ) be a fts and $f: X \to X$ be a mapping. Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of fuzzy orbit open sets under the mapping f, then $FO_f(\bigvee_{\alpha\in\Delta} \mu_\alpha) = \bigvee_{\alpha\in\Delta} FO_f(\mu_\alpha)$.

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Proof. The outline of proofing this theorem is proceeds in a way similar to Theorem 3.3. As in Theorem 3.3, we consider three cases:

Case 1 : Suppose that f is bijective mapping and $f(a_i) = a_j$, ; $a_i, a_j \in X$ and $i \neq j$ for all $i, j \in \Lambda$. Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of fuzzy orbit open sets under the mapping f. Then, there exist $\lambda_\alpha \in I^X$, $\alpha \in \Delta$ defined as $\lambda_\alpha = \{(a_i, t_{i_\alpha});\ a_i \in X, t_{i_\alpha} \in I, i \in \Lambda\}$ such that $FO_f(\lambda_\alpha) = \mu_\alpha$ for all $\alpha \in \Delta$. From Theorem 3.1 Case 1, we have $FO_f(\lambda_\alpha) = \{(a_i, t_\alpha); t_\alpha = \inf\{t_{i_\alpha}, i \in \Lambda\}\} = \mu_\alpha$.

Thus, $\vee_{\alpha\in\Delta}\mu_\alpha = \{(a_i, \sup_{\alpha\in\Delta}\{t_\alpha\}); a_i \in X, i \in \Lambda\}$. Let $k = \sup_{\alpha\in\Delta}\{t_\alpha\}$. Now, for all $a_j \in X, j \in \Lambda$

$$f(\vee_{\alpha\in\Delta}\mu_\alpha)(a_j) = \begin{cases} \vee_{f(a_i)=a_j}(\vee_{\alpha\in\Delta}\mu_\alpha)(a_i) & if\ f^{-1}(a_j) \neq \emptyset, \\ 0 & if\ f^{-1}(a_j) = \emptyset. \end{cases}$$
$$= k$$

Hence, $f(\vee_{\alpha\in\Delta}\mu_\alpha)= \vee_{\alpha\in\Delta}\mu_\alpha$. This implies, $f^2(\vee_{\alpha\in\Delta}\mu_\alpha)= \vee_{\alpha\in\Delta}\mu_\alpha$, $f^3(\vee_{\alpha\in\Delta}\mu_\alpha)= \vee_{\alpha\in\Delta}\mu_\alpha$,.... Therefore, from the definition of $FO_f(\vee_{\alpha\in\Delta}\mu_\alpha)$ and Theorem 3.1, we get $FO_f(\vee_{\alpha\in\Delta}\mu_\alpha) = \vee_{\alpha\in\Delta}\mu_\alpha = \vee_{\alpha\in\Delta}FO_f(\mu_\alpha)$.

Case 2: Suppose that f is bijective mapping and $f(a_i) = a_j$; $a_i, a_j \in X$ and $i = j$ for some $i, j \in \Lambda$. Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of fuzzy orbit open sets under the mapping f. Then, there exist $\lambda_\alpha \in I^X$, $\alpha \in \Delta$ defined as $\lambda_\alpha = \{(a_i, t_{i_\alpha});\ a_i \in X, t_{i_\alpha} \in I, i \in \Lambda\}$ such that $FO_f(\lambda_\alpha) = \mu_\alpha$ for all $\alpha \in \Delta$. From Theorem 3.1 case 2, we have

$FO_f(\lambda_\alpha) = \{(a_i, t_{i_\alpha}); f(a_i) = a_j, i = j, (a_i, \inf\{t_{i_\alpha}, i \in \Lambda\}); f(a_i) = a_j, i \neq j\} = \mu_\alpha$. Put $t_\alpha = \inf\{t_{i_\alpha}, i \in \Lambda\}$, it follows:

$\vee_{\alpha\in\Delta}\mu_\alpha = \{(a_i, sup_{\alpha\in\Delta}\{t_{i_\alpha}\}); f(a_i) = a_j, i = j, (a_i, sup_{\alpha\in\Delta}\{t_\alpha\}); f(a_i) = a_j, i \neq j\}$.

Now, for all $a_j \in X, j \in \Lambda$

$$f(\vee_{\alpha\in\Delta}\mu_\alpha)(a_j) = \begin{cases} \vee_{f(a_i)=a_j}(\vee_{\alpha\in\Delta}\mu_\alpha)(a_i) & if\ f^{-1}(a_j) \neq \emptyset, \\ 0 & if\ f^{-1}(a_j) = \emptyset. \end{cases}$$

$$= \begin{cases} sup_{\alpha\in\Delta}\{t_{i_\alpha}\} & if\ f(a_i) = a_j, i = j, \\ sup_{\alpha\in\Delta}\{t_\alpha\} & if\ f(a_i) = a_j, i \neq j. \end{cases}$$

Hence, $f(\vee_{\alpha\in\Delta}\mu_\alpha)= \vee_{\alpha\in\Delta}\mu_\alpha$. This implies, $f^2(\vee_{\alpha\in\Delta}\mu_\alpha)= \vee_{\alpha\in\Delta}\mu_\alpha$, $f^3(\vee_{\alpha\in\Delta}\mu_\alpha)= \vee_{\alpha\in\Delta}\mu_\alpha$,.... Therefore, from the definition of $FO_f(\vee_{\alpha\in\Delta}\mu_\alpha)$ and Theorem 3.1, we get $FO_f(\vee_{\alpha\in\Delta}\mu_\alpha) = \vee_{\alpha\in\Delta}\mu_\alpha = \vee_{\alpha\in\Delta}FO_f(\mu_\alpha)$.

Now, if f is constant mapping, let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of fuzzy orbit open sets under the mapping f. Then, there exist $\lambda_\alpha \in I^X$, $\alpha \in \Delta$ defined as $\lambda_\alpha = \{(a_i, t_{i_\alpha});\ a_i \in X, t_{i_\alpha} \in I, i \in \Lambda\}$ such that $FO_f(\lambda_\alpha) = \mu_\alpha$ for all $\alpha \in \Delta$. From Theorem 3.2, we get

$$FO_f(\lambda_\alpha) = \begin{cases} (a_i, 0) & if\ i \neq k, \\ \left(a_k, \min\left\{t_{k_\alpha}, sup_{i\in\Lambda}\{\lambda_\alpha(a_i)\}\right\}\right) & if\ i = k. \end{cases}$$
$$= \mu_\alpha.$$

Thus,

$$\vee_{\alpha\in\Delta}\mu_\alpha = \begin{cases} (a_i, 0) & if\ i \neq k, \\ \left(a_k, sup_{\alpha\in\Delta}\{\min\left\{t_{k_\alpha}, sup_{i\in\Lambda}\{\lambda_\alpha(a_i)\}\right\}\right) & if\ i = k. \end{cases}$$

This means $\vee_{\alpha\in\Delta}\mu_\alpha$ is a fuzzy point in X with support a_k and degree $sup_{\alpha\in\Delta}\{\min\left\{t_{k_\alpha}, sup_{i\in\Lambda}\{\lambda_\alpha(a_i)\}\right\}$. Hence, from the definition of f, we get $f(\vee_{\alpha\in\Delta}\mu_\alpha)= \vee_{\alpha\in\Delta}\mu_\alpha$. This implies, $f^2(\vee_{\alpha\in\Delta}\mu_\alpha)= \vee_{\alpha\in\Delta}\mu_\alpha$, $f^3(\vee_{\alpha\in\Delta}\mu_\alpha)= \vee_{\alpha\in\Delta}\mu_\alpha$,.... Therefore, from the definition of $FO_f(\vee_{\alpha\in\Delta}\mu_\alpha)$ and Theorem 3.2, we get $FO_f(\vee_{\alpha\in\Delta}\mu_\alpha) = \vee_{\alpha\in\Delta}\mu_\alpha = \vee_{\alpha\in\Delta}FO_f(\mu_\alpha)$.

4. Fuzzy orbit topological spaces

In this section we show that the family of all fuzzy orbit open sets under the mapping f constrict a fuzzy topology on X, denoted by τ_{FO} which is coarser than τ.

Theorem 4.1 Let (X,τ) be a fts and $f: X \to X$ be a mapping. Let τ_{FO} denote to the family of all fuzzy orbit open sets under the mapping f. Then, τ_{FO} is a fuzzy topology on X coarser than τ.

Proof. We must show τ_{FO} satisfies the three axioms of the definition of fuzzy topology. It is clear that $\overline{0}$ and $\overline{1}$ are fuzzy orbit open sets, because there exist $\lambda = \overline{0}$ and $\nu = \overline{1}$ such that $FO_f(\lambda) = \overline{0} \in \tau$ and $FO_f(\nu) = \overline{1} \in \tau$. Thus, $\overline{0} \in \tau_{FO}$ and $\overline{1} \in \tau_{FO}$.

Let μ_1 and μ_2 be fuzzy orbit open sets under the mapping f. To show $\mu_1 \wedge \mu_2$ is a fuzzy orbit open set under the mapping f, we must find a fuzzy set $\lambda \in I^X$ such that $FO_f(\lambda) = \mu_1 \wedge \mu_2 \in \tau$.
If we choose $\lambda = \mu_1 \wedge \mu_2$, then from Theorem 3.3 and Proposition 3.1 we have,
$FO_f(\lambda) = FOf\ (\mu_1 \wedge \mu_2) = FOf\ (\mu_1)\ \wedge\ FOf\ (\mu_2) =\ \mu_1 \wedge \mu_2$. On the other hand since every fuzzy orbit open set is an open fuzzy set in X, then $\mu_1 \wedge \mu_2 \in \tau$. Hence, the result.

Let $\{\mu_\alpha\}_{\alpha\in\Delta}$ be any family of fuzzy orbit open sets under the mapping f. Let $\lambda = \vee_{\alpha\in\Delta}\mu_\alpha$. Then from Theorem 3.4, $FO_f(\lambda) = FO_f(\vee_{\alpha\in\Delta}\mu_\alpha) = \vee_{\alpha\in\Delta}FO_f(\mu_\alpha) = \vee_{\alpha\in\Delta}\mu_\alpha$. And $\vee_{\alpha\in\Delta}\mu_\alpha \in \tau$. Thus, τ_{FO} is a fuzzy topology on X. Furthermore, $\tau_{FO} \subset \tau$ since every fuzzy orbit open set is an open fuzzy set in X.

Definition 4.1 Let (X,τ) be a fts and $f: X \to X$ be a mapping. The pair (X,τ_{FO}) is called fuzzy orbit topological space associated with (X,τ).

Example 4.1
1. For any nonempty countable set X, ${\tau^0}_{FO}=\{\overline{0}, \overline{1}\}$ is a fuzzy orbit topology on X, and is called the indiscrete fuzzy orbit topology.
2. For any nonempty countable set X, if $f: X \to X$ is the identity mapping, then $\tau_{FO} = \tau$.

Next the notion of fuzzy orbit closure (resp. interior) of a fuzzy set is introduced.

Definition 4.2 Let (X,τ_{FO}) be a fuzzy orbit topological space and $\lambda \in I^X$. The fuzzy orbit closure of λ, denoted by $cl_{FO}(\lambda)$, is the intersection of all fuzzy orbit closed supersets under the mapping f of λ. i.e.,

$$cl_{FO}(\lambda) = \wedge\{\rho \in I^X | \rho \geq \lambda, \overline{1} - \rho \in \tau_{FO}\}.$$

And, the fuzzy orbit interior of λ, denoted by $Int_{FO}(\lambda)$, is the union of all fuzzy orbit open subsets under the mapping f of λ. i.e.,

$$Int_{FO}(\lambda) = \vee\{\rho \in I^X | \rho \leq \lambda, \rho \in \tau_{FO}\}.$$

Clearly, $cl_{FO}(\lambda)$ (resp., $Int_{FO}(\lambda)$) is the smallest (resp., largest) fuzzy orbit closed (resp., open) set under the mapping f which contains (resp., contained in) λ.

Proposition 4.1 Let (X,τ_{FO}) be a fuzzy orbit topological space and $\lambda \in I^X$. Then

$$Int_{FO}(\lambda) \leq \ Int(\lambda) \leq \lambda \leq cl(\lambda) \ \leq cl_{FO}(\lambda).$$

Proof. The proof follows directly from the fact that every fuzzy orbit closed (resp., open) set under the mapping f is closed (resp., open) fuzzy set.

Proposition 4.2 Let $(X, \tau_{FO}\ )$ be a fuzzy orbit topological space and $\lambda, \mu \in I^X$. Then,
1. $cl_{FO}(\overline{0})$ = $\overline{0}$ and $cl_{FO}(\overline{1})$ = $\overline{1}$.
2. $\lambda \leq cl_{FO}(\lambda)$.
3. $cl_{FO}(\lambda \vee \mu)$ = $cl_{FO}(\lambda) \vee cl_{FO}(\mu)$.
4. If $\lambda \leq\ \mu$, then $cl_{FO}(\lambda) \leq\ cl_{FO}(\mu)$.
5. $cl_{FO}(cl_{FO}(\lambda)) = cl_{FO}(\lambda)$.
6. λ is fuzzy orbit closed set under the mapping f iff λ = $cl_{FO}(\lambda)$.

Proof. Straightforward.

Proposition 4.3 Let $(X, \tau_{FO}\ )$ be a fuzzy orbit topological space and $\lambda, \mu \in I^X$. Then,
1. $Int_{FO}(\overline{0}) = \overline{0}$ and $Int_{FO}(\overline{1}) = \overline{1}$.
2. $Int_{FO}(\lambda) \leq\ \lambda$.
3. $Int_{FO}(\lambda\ \wedge \mu)\ =\ Int_{FO}(\lambda)\ \wedge\ Int_{FO}(\mu)$.
4. If $\leq \mu$, then $Int_{FO}(\lambda)\ \leq\ Int_{FO}(\mu)$.
5. $Int_{FO}(Int_{FO}(\lambda))\ =\ Int_{FO}(\lambda)$.
6. λ is fuzzy orbit open set under the mapping f iff λ = $Int_{FO}\ (\lambda)$.

Proof. Straightforward.

Theorem 4.2 Let $(X, \tau_{FO}\ )$ be a fuzzy orbit topological space and $\lambda \in I^X$. Then,
1. $\overline{1} - Int_{FO}\ (\lambda)$ = $cl_{FO}(\overline{1} - \lambda)$.
2. $\overline{1} - cl_{FO}(\lambda)$= $Int_{FO}\ (\overline{1} - \lambda)$.

Proof. We prove part 1 and by the similar way one can prove part 2. From Proposition 4.3 part 2, $Int_{FO}\ (\lambda) \leq \lambda$ so by taking the complement we have, $\overline{1} - \lambda \leq \overline{1} - Int_{FO}\ (\lambda)$. Since $\overline{1} - Int_{FO}\ (\lambda)$ is a fuzzy orbit closed set and by Proposition 4.2 part 4, $cl_{FO}(\overline{1} - \lambda) \leq cl_{FO}\ (\overline{1} - Int_{FO}\ (\lambda))$ = $\overline{1} - Int_{FO}(\lambda)$. Hence, $cl_{FO}(\overline{1} - \lambda) \leq \overline{1} - Int_{FO}\ (\lambda)$.
Conversely, by Proposition 4.2 part 2, $\overline{1} - \lambda \leq cl_{FO}(\overline{1} -\ \lambda)$. By taking the complement, $\overline{1} -\ cl_{FO}$ $(\overline{1} -\ \lambda) \leq \lambda$. Since $cl_{FO}\ (\overline{1} -\ \lambda)$ is a fuzzy orbit closed set. Then $\overline{1} - cl_{FO}\ (\overline{1} -\ \lambda)$ is a fuzzy orbit open set and by Proposition 4.3 part 6, we have $\overline{1} - cl_{FO}\ (\overline{1} -\ \lambda) \leq Int_{FO}\ (\lambda)$, again by taking the complement we obtain, $\overline{1} - Int_{FO}\ (\lambda) \leq cl_{FO}\ (\overline{1} -\ \lambda)$.

Theorem 4.3 Let $f\colon (X, \tau_{FO}\ ) \rightarrow\ (Y, \tau'_{FO})$ and $g\colon (Y, \tau'_{FO}) \rightarrow\ (Z, \tau''_{FO})$be two mappings. Then $g\ of$ is fuzzy continuous mapping if f and g are fuzzy continuous.

Proof. Clear.

5. Category of fuzzy orbit topological spaces

In this section, we will construct the category of fuzzy orbit topological spaces, and study its relation with the category of fts's.

Definition 5.1 Let $\mathbb{F}\mathbb{O}$**TOP** be the collection of all fuzzy orbit topological spaces $(X, \tau_{FO}\ )$, (Y, τ'_{FO}), … associated with $(X, \tau\ ), (Y, \tau')$, … respectively. For each pair of objects $(X, \tau_{FO}\ )$, (Y, τ'_{FO}) of $\mathbb{F}\mathbb{O}$**TOP**, define Mor($(X, \tau_{FO}\ )$, (Y, τ'_{FO})) to be the set of all fuzzy continuous mapping f with respect to τ_{FO} and τ'_{FO}. Composition of two morphisms $f\colon (X, \tau_{FO}) \to (Y, \tau'_{FO}), g\colon (Y, \tau'_{FO}) \to (Z, \tau''_{FO})$ is defined by $g\ of\ : (X, \tau_{FO}) \to (Z, \tau''_{FO})$.

Theorem 5.1 $\mathbb{F}\mathbb{O}$**TOP** is a category.

Proof. First Form Theorem 4.3, the composition of fuzzy continuous mappings between fuzzy orbit topological spaces is also fuzzy continuous, hence the composition of morphisms is well defined and associative. Second, to each object (X, τ_{FO}) in $\mathbb{F}\mathbb{O}$**TOP** define the identity morphism $1_{(X,\tau_{FO})}$: $(X, \tau_{FO}) \to (X, \tau_{FO})$, by the identity set mapping. Thus, we get to the required result.

Remark 5.1 $\mathbb{F}\mathbb{O}$**TOP** is not a subcategory of $\mathbb{F}$**TOP**, because if f is fuzzy continuous from (X, τ_{FO}) to (Y, τ'_{FO}), then f need not to be fuzzy continuous from $(X, \tau)$to (Y, τ'). That is mean Mor((X, τ_{FO}), (Y, τ'_{FO})) $\nsubseteq$ Mor($(X, \tau), (Y, \tau')$). We give an example to explain that.

Example 5.1 Let $X = \{a_1, a_2, a_3\}$ and $Y = \{b_1, b_2, b_3\}$. Define $\tau = \{\overline{0}, \overline{1}, \lambda\}$and $\tau' = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where $\lambda \in I^X$ and $\mu_1, \mu_2 \in I^Y$ such that $\lambda = \{(a_1, 0.2), (a_2, 0.3), (a_3, 0.3)\}$, $\mu_1 = \{(b_1, 0.2), (b_2, 0.3), (b_3, 0.3)\}$ and $\mu_2 = \{(b_1, 0.6), (b_2, 0.5), (b_3, 0.7)\}$. Clearly, (X, τ) and (Y, τ') are fts's.
Define $f\colon (X, \tau) \to (Y, \tau')$, $f_1\colon X \to X$ and $f_2\colon Y \to Y$ as $f(a_1) = b_1$, $f(a_2) = b_3$, $f(a_3) = b_2$, $f_1(a_1) = a_1$, $f_1(a_2) = a_3$, $f_1(a_3) = a_2$ and $f_2(b_1) = b_1$, $f_2(b_2) = b_3$, $f_2(b_3) = b_2$. Then, $\tau_{FO} = \{\overline{0}, \overline{1}, \lambda\}$ and $\tau'_{FO} = \{\overline{0}, \overline{1}, \mu_1\}$. It is clear that f is fuzzy continuous with respect to τ_{FO} and τ'_{FO}. But f is not fuzzy continuous with respect to τ and τ', since μ_2 is an open fuzzy set in Y, however $f^{-1}(\mu_2)$ is not an open fuzzy set in X.

Theorem 5.2 $\mathbb{F}\mathbb{O}$**TOP** isomorphic to a subcategory of $\mathbb{F}$**TOP**.

Proof. Let $\mathbb{F}\mathbf{TOP}_\omega$ be a collection $\{(X, I_X)\}$ of objects in $\mathbb{F}$**TOP**, such that I_X is the indiscrete fuzzy topology on X. For any pair of objects $(X, I_X), (Y, I_Y)$ of $\mathbb{F}\mathbf{TOP}_\omega$, take Mor($(X, I_X), (Y, I_Y)$) (in $\mathbb{F}$**TOP**) as the set of morphisms in $\mathbb{F}\mathbf{TOP}_\omega$. Then it is clear$\mathbb{F}\mathbf{TOP}_\omega$ is a subcategory of $\mathbb{F}$**TOP**. Now define $F\colon \mathbb{F}\mathbb{O}\mathbf{TOP} \to \mathbb{F}\mathbf{TOP}_\omega$ by $F((X, \tau_{FO})) = (X, I_X)$ and for each morphism $f\colon (X, \tau_{FO}) \to (Y, \tau'_{FO})$ define $F(f) = f\colon (X, I_X) \to (Y, I_Y)$. It can be verified that F is indeed a bijective functor. Thus $\mathbb{F}\mathbb{O}$**TOP** isomorphic to $\mathbb{F}\mathbf{TOP}_\omega$.

Remark 5.2 From the above theorem, we can say that $\mathbb{F}\mathbb{O}$**TOP** is embedded in $\mathbb{F}$**TOP** as a subcategory.

Conclusion In this paper, we study the collection of fuzzy orbit open sets under the mapping $f\colon X \to X$. We give the necessary conditions on the mapping f in order to obtain a fixed orbit of a fuzzy set for any fuzzy orbit open set under the mapping f. As a main result, we prove the family of all fuzzy orbit open sets constructs a fuzzy topological space. In addition, the category of fuzzy orbit topological spaces and fuzzy continuous mappings $\mathbb{F}\mathbb{O}$**TOP** is defined. And we show this category is isomorphic to a subcategory of the category of fuzzy topological spaces.

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