IOP Conference Series: Materials Science and Engineering

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IOP Conf. Series: Materials Science and Engineering **571** (2019) 012017 doi:10.1088/1757-899X/571/1/012017

Some Common Random Fixed Points Theorems Of Generalized $\phi-$ Weakly Contractive Random Operators in Metric Spaces and Random Well-Posed

Yusra Jarallah Ajeel
Department of Mathematics, College of Science, Al-Mustansiriyah University, Baghdad-Iraq.

Ysrajara@yahoo.com

Abstract: In this paper, firstly, we prove the existence of random coincidence points for general $\phi-$ weakly contraction condition under two pairs of random operators in metric spaces X , where ϕ is continuous monotone real function. As applications related common random fixed point results are proved and the well-posed random fixed point problem is studied.

Keywords: Random coincidence points, Common random fixed points , random Well-posed.

1. Introduction and Preliminaries: Random coincidence point theorems are stochastic generalizations of classical coincidence point theorems. The study of random fixed point theorems was initiated by the Prague school of probability in the 1950. Random fixed point theorems for contraction mappings in polish space were proved by Špaček [1] and Hanš [2,3].

In 2014, Rashwan and Albaqeri [4] proved a fixed point theorem via contraction mappings of a pair of weakly increasing mappings using an altering function in a partially ordered complete separable metric spaces.
In 2015, Alsaidy el.at.[5] proved random common point theorem for pair of commuting mapping defined on separable weakly compact subset of complete p-normed space.
In 2016, Rashwan and Hamnad [6] proved a uniqe common random fixed point theorem in the framework of cone random metric spaces for four weakly random compatible mappings under strict contraction condition.
In 2017, Abed el.at.[7] proved common random fixed point theorem for two random operators under general quasi contraction condition in a complete p-normed space.
The aim of this article is to obtain random coincidence points results for two pairs of self random mappings, when one of these pairs is generalized $\phi-$weakly contractivity w.r.t the other and study the will posed-ness of their random fixed point problem.
Throughout this article X will be metric space, $\phi \neq A \subseteq X$ be a closed , (Ω, Σ) be the measurable space with Σ a sigma algebra of subsets of Ω, 2^X is the classes of all sub sets of $X$and CB(X) is the classes of all non-empty bounded closed subsets of X, $RF(S,T)$ the set of common random fixed points of S and T and $RC(S,T)$ the set of random coincidence points of S and T.

Definition (1.1)[8]: A mapping $F:\Omega \to 2^X$ is called measurable (respectively, weakly measurable) if, for any closed (respectively, open) subset B of X ,

$$F^{-1}(B)=\{\gamma \in \Omega : F(\gamma) \cap B \neq \phi\} \in \Sigma .$$

Definition (1.2)[9]: A mapping $\delta : \Omega \to X$ is called a measurable selector of a measurable mapping $F:\Omega \to 2^X$ if δ is measurable and $\delta(\gamma) \in F(\gamma)$ for each $\gamma \in \Omega$.

Definition (1.3)[10]: A mapping $h:\Omega\times X\to X$ (or $G:\Omega\times X\to CB(X)$) is called a random operator if for any $x\in X$, $h(.,x)$ (respectively $G:\Omega\times X\to CB(X)$)

is measurable .

Definition (1.4)[11]: A measurable mapping $\delta:\Omega\to A$ is called random fixed point of a random operator $h:\Omega\times X\to X$ (or $G:\Omega\times X\to CB(X)$ if for every $\gamma\in\Omega$, $\delta(\gamma)=h(\gamma,\delta(\gamma))$ (respectively $\delta(\gamma)\in G(\gamma,\delta(\gamma))$).

Definition (1.5)[12]: A measurable mapping $\delta:\Omega\to A$ is called random coincidence point of a random operator $h:\Omega\times X\to X$ and $G:\Omega\times X\to X$ if for every $\gamma\in\Omega$, $h(\gamma,\delta(\gamma))=G(\gamma,\delta(\gamma))$

Definition (1.6)[12]: A measurable mapping $\delta:\Omega\to A$ is called common random fixed point of a random operator $h:\Omega\times X\to X$ and $G:\Omega\times X\to X$ if for every $\gamma\in\Omega$, $\delta(\gamma)=h(\gamma,\delta(\gamma))=G(\gamma,\delta(\gamma))$ Now, we define a new type of random operators

Definition (1.7): Let $h,G,S,T:\Omega\times X\to X$ be four random operators. (h,G,ϕ) is called generalized $\phi-$ weakly contractive with respect to the pair (S,T) if for all $x,y\in X$,

$$d\left(S(\gamma,x),T(\gamma,y)\right)\le K\left[M(x,y)-\phi\left(M(x,y)\right)\right] \ldots\ldots(1.1)$$

Where

$$M(x,y)=max\begin{Bmatrix} d\left(h(\gamma,x),g(\gamma,y)\right),d\left(h(\gamma,x),S(\gamma,x)\right),d\left(g(\gamma,y),T(\gamma,y)\right) \\ ,d\left(h(\gamma,x),T(\gamma,y)\right),d\left(g(\gamma,y),S(\gamma,x)\right), \end{Bmatrix}$$

for each $\gamma\in\Omega, 0\le k<1$ and $\phi:[0,\infty)\to[0,\infty)$ is continuous and non-decreasing map such that , $\phi(0)=0$ and $\lim_{t\to\infty}\phi(t)=\infty$.

Definition (1.8): Let A be a nonempty subset of a metric space X and let S and T be self-mappings of A The pair (S,T) is said to be:

1) weakly compatible [13] , if they commute at their coincidence points, i.e., $STx=TSx$ for all x satisfying $S(x)=T(x)$.
2) R-weakly commuting maps [14] if for all $x\in A$ there exists $R>0$ such that $d(STx,TSx)<Rd(Sx,Tx)$, if R =1, then the maps are called weakly commuting.

The following definition appears in [5] and [13] respectively:

Definition (1.9): A random operators $h,G:\Omega\times X\longrightarrow X$ are said to be R-weakly commute (or Weakly Compatible) if $h(\gamma,.)$ and $G(\gamma,.)$ are R-weakly commute (respectively weakly compatible) for each $\gamma\in\Omega$.

2. Random Coincidence Theorems

Theorem (2.1): Let $\phi\ne A\subseteq X$, with $h,G,S,T:\Omega\times A\to A$ such that for all $x,y\in A$, the pair (S,T) is generalized $\phi-$ weakly contractive w.r.t the pair (h,G). If

$cl\left(S\left(\gamma,A\right)\right)\subseteq g\left(\gamma,A\right), cl\left(T\left(\gamma,A\right)\right)\subseteq h\left(\gamma,A\right)$ and one of the subsets $cl\left(S\left(\gamma,A\right)\right), cl\left(T\left(\gamma,A\right)\right), cl\left(h\left(\gamma,A\right)\right)$ or $cl\left(g\left(\gamma,A\right)\right)$ is separable complete subspace of A. Then $RC(S,h)\neq\phi$ and $RC\left(T,g\right)\neq\phi$.

Proof: Let $\delta_\circ:\Omega\to A$ be arbitrary measurable mapping.

We constrict a sequence of measurable maps $\delta_n:\Omega\to A$.

Since $cl\left(S\left(\gamma,A\right)\right)\subseteq g\left(\gamma,A\right), cl\left(T\left(\gamma,A\right)\right)\subseteq h\left(\gamma,A\right)$ then we can find $\delta_1:\Omega\to A$ such that $S\left(\gamma,\delta_\circ(\gamma)\right)=g\left(\gamma,\delta_1(\gamma)\right)$ for $\gamma\in\Omega$ and for this function $\delta_1:\Omega\to A$,

we can choose another function $\delta_2:\Omega\to A$ such that $T\left(\gamma,\delta_1(\gamma)\right)=h\left(\gamma,\delta_2(\gamma)\right)$ for $\gamma\in\Omega$. By induction, we construct sequence of measurable mappings $\delta_n:\Omega\to A$, such that $S\left(\gamma,\delta_{2n}(\gamma)\right)=g\left(\gamma,\delta_{2n+1}(\gamma)\right)$ and $T\left(\gamma,\delta_{2n+1}(\gamma)\right)=h\left(\gamma,\delta_{2n+2}(\gamma)\right)$ …..(2.1)

We can define sequence of functions for $\gamma\in\Omega$, $\{y_n(\gamma)\}$ such that

$y_{2n}(\gamma)=S(\gamma,\delta_{2n}(\gamma))=g(\gamma,\delta_{2n+1}(\gamma))$ and $y_{2n+1}(\gamma)=T(\gamma,\delta_{2n+1}(\gamma))=h(\gamma,\delta_{2n+2}(\gamma))$ and $n=0,1,2,....$ ……(2.2)

$$d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right)=d\left(S(\gamma,\delta_{2n}(\gamma),T(\gamma,\delta_{2n+1}(\gamma)\right)$$

$$\leq k\left[M\left(\delta_{2n}(\gamma),\delta_{2n+1}(\gamma)\right)-\phi\left(M\left(\delta_{2n}(\gamma),\delta_{2n+1}(\gamma)\right)\right)\right]$$

$$=k[\max\left\{d\left(h(\gamma,\delta_{2n}(\gamma),g(\gamma,\delta_{2n+1}(\gamma)\right),d\left(h(\gamma,\delta_{2n}(\gamma),S(\gamma,\delta_{2n}(\gamma)\right),\right.$$

$$\left.d\left(g(\gamma,\delta_{2n+1}(\gamma),T(\gamma,\delta_{2n+1}(\gamma)\right),d\left(h(\gamma,\delta_{2n}(\gamma),T(\gamma,\delta_{2n+1}(\gamma)\right),d\left(g(\gamma,\delta_{2n+1}(\gamma),S(\gamma,\delta_{2n}(\gamma)\right)\right\}$$
$$-\phi\left(\left\{d\left(h(\gamma,\delta_{2n}(\gamma),g(\gamma,\delta_{2n+1}(\gamma)\right),d\left(h(\gamma,\delta_{2n}(\gamma),S(\gamma,\delta_{2n}(\gamma)\right),\right.\right.$$

$$\left.\left.d\left(g(\gamma,\delta_{2n+1}(\gamma),T(\gamma,\delta_{2n+1}(\gamma)\right),d\left(h(\gamma,\delta_{2n}(\gamma),T(\gamma,\delta_{2n+1}(\gamma)\right),d\left(g(\gamma,\delta_{2n+1}(\gamma),S(\gamma,\delta_{2n}(\gamma)\right)\right\}\right)\right]$$

$$=k[\max\left\{d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right),d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right),d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right),\right.$$

$$\left.d\left(y_{2n-1}(\gamma),y_{2n+1}(\gamma)\right),d\left(y_{2n}(\gamma),y_{2n}(\gamma)\right)\right\}$$

$$-\phi\left(d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right),d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right),d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right),\right.$$

$$\left.d\left(y_{2n-1}(\gamma),y_{2n+1}(\gamma)\right),d\left(y_{2n}(\gamma),y_{2n}(\gamma)\right)\right)]$$

Using triangle inequality, we get

$$=k[\max\left\{d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right),d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right),d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right),\right.$$

$d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right)+d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right),d\left(y_{2n}(\gamma),y_{2n}(\gamma)\right)\}$

$-\phi\Big(d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right),d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right),d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right),$

$d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right)+d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right),d\left(y_{2n}(\gamma),y_{2n}(\gamma)\right)\Big)]$

Hence, $d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right)\le k\left[d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right)+d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right)-\right.$

$\left.\phi\Big(d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right)+d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right)\Big)\right]$

$\le k\left[d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right)+d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right)\right]$

In general, $d\left(y_{2n}(\gamma),y_{2n+1}(\gamma)\right)\le \lambda d\left(y_{2n-1}(\gamma),y_{2n}(\gamma)\right)$, where $\lambda=\frac{k}{1-k}<1$

Therefore, $d\left(y_{n}(\gamma),y_{n+1}(\gamma)\right)\le \lambda d\left(y_{n}(\gamma),y_{n-1}(\gamma)\right)$

$$\le \lambda^2 d\left(y_{n-1}(\gamma),y_{n-2}(\gamma)\right)$$

. $d\left(y_{n}(\gamma),y_{n+1}(\gamma)\right)\le \lambda^n d\left(y_{0}(\gamma),y_{1}(\gamma)\right)$ for all $\gamma\in\Omega$.

Now, we shall prove for $\gamma\in\Omega$, $\{y_n(\gamma)\}$ is a Cauchy sequence. For this for every positive integer p we have, for $\gamma\in\Omega$

$d\left(y_{n}(\gamma),y_{n+p}(\gamma)\right)\le d\left(y_{n}(\gamma),y_{n+1}(\gamma)\right)+d\left(y_{n+1}(\gamma),y_{n+2}(\gamma)\right)+\ldots+d\left(y_{n+p-1}(\gamma),y_{n+p}(\gamma)\right)$

$\le\left(\lambda^n+\lambda^{n+1}+\ldots+\lambda^{n+p-1}\right)d\left(y_{\circ}(\gamma),y_1(\gamma)\right)$

$$=\lambda^n(1+\lambda+\ldots+\lambda^{p-1})d\left(y_{\circ}(\gamma),y_1(\gamma)\right)$$

$$\le\left(\tfrac{\lambda^n}{1-\lambda}\right)d\left(y_{\circ}(\gamma),y_1(\gamma)\right)\text{ for all }\gamma\in\Omega.$$

$d\left(y_{n}(\gamma),y_{n+p}(\gamma)\right)\to 0$ as $n\to\infty$ for $\gamma\in\Omega$. ….(2.3)

It follows that for $\gamma\in\Omega$, $\{y_n(\gamma)\}$ is a Cauchy sequence .

Suppose that $cl\left(S\left(\gamma,A\right)\right)$ is complete subspace of A , this implies the sequence $\{y_n(\gamma)\}$ has a limit $t:\Omega\to A$. Such that $y_n(\gamma)\to t(\gamma)$ as $n\to\infty$.

Obtained a mapping $u:\Omega\to A$ such that $g\left(\gamma,u(\gamma)\right)=t(\gamma)$ .Thus we have

$g\left(\gamma,u(\gamma)\right)=t(\gamma)$.

$t(\gamma)=\lim_{n\to\infty}y_{2n}(\gamma)=\lim_{n\to\infty}S\left(\gamma,\delta_{2n}(\gamma)\right)=\lim_{n\to\infty}T\left(\gamma,\delta_{2n}(\gamma)\right)=\lim_{n\to\infty}h\left(\gamma,\delta_{2n+1}(\gamma)\right)=\lim_{n\to\infty}g\left(\gamma,\delta_{2n+1}(\gamma)\right)$ Using (2.2) and (1.1), we have

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$d\left(y_{2n}(\gamma),T(\gamma,u(\gamma)\right)=d\left(S(\gamma,\delta_{2n}(\gamma),T(\gamma,u(\gamma)\right)$

$\leq k\,[\max\{d\left(h\left(\gamma,\delta_{2n}(\gamma)\right),g\left(\gamma,u(\gamma)\right)\right),d\left(h\left(\gamma,\delta_{2n}(\gamma)\right),S\left(\gamma,\delta_{2n}(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),T\left(\gamma,u(\gamma)\right)\right),$
$d\left(h\left(\gamma,\delta_{2n}(\gamma)\right),T\left(\gamma,u(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),S\left(\gamma,\delta_{2n}(\gamma)\right)\right)\}-$

$\phi(\max\{d\left(h\left(\gamma,\delta_{2n}(\gamma)\right),g\left(\gamma,u(\gamma)\right)\right),d\left(h\left(\gamma,\delta_{2n}(\gamma)\right),S\left(\gamma,\delta_{2n}(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),T\left(\gamma,u(\gamma)\right)\right),$
$d\left(h\left(\gamma,\delta_{2n}(\gamma)\right),T\left(\gamma,u(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),S\left(\gamma,\delta_{2n}(\gamma)\right)\right)\}]$

taking limit as $n\to\infty$, we get
$d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right)\leq k\;[\max\{d\left(t(\gamma),g(\gamma,u(\gamma))\right),d\left(t(\gamma),t(\gamma)\right),d\left(g(\gamma,u(\gamma)),T(\gamma,u(\gamma))\right),$
$d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),t(\gamma)\right)\}-\phi(\max\{d\left(t(\gamma),g\left(\gamma,u(\gamma)\right)\right),d\left(t(\gamma),t(\gamma)\right),$
$d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),t(\gamma)\right)\})]$

From $g\left(\gamma,u(\gamma)\right)=t(\gamma)$,we have
$d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right)\leq k\,[d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right)-\phi\left(d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right)\right)]$

$\leq k\,[d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right)$

this implies, $(1-k)\,d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right)\leq 0$

hence $d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right)=0\Rightarrow t(\gamma)=T\left(\gamma,u(\gamma)\right)=g\left(\gamma,u(\gamma)\right)\ldots\ldots(2.4)$
Therefore $RC(T,I)\neq\phi.$
Since $cl\left(T\left(\gamma,A\right)\right)\subseteq h\left(\gamma,A\right)$, then $t(\gamma)\in h(\gamma,A)$. We obtained a mapping $v:\Omega\to A$ such that $h\left(\gamma,v(\gamma)\right)=t(\gamma)$......(2.5)

By using (2.4), (1.1) and (2.5), we have
$d\left(S\left(\gamma,v(\gamma)\right),t(\gamma)\right)=d\left(S\left(\gamma,v(\gamma)\right),T\left(\gamma,u(\gamma)\right)\right)$

$\leq k\,[max\;\{d\left(h\left(\gamma,v(\gamma)\right),g\left(\gamma,u(\gamma)\right)\right),d\left(h\left(\gamma,v(\gamma)\right),S\left(\gamma,v(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),T\left(\gamma,u(\gamma)\right)\right),$
$d\left(h\left(\gamma,v(\gamma)\right),T\left(\gamma,u(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),S\left(\gamma,v(\gamma)\right)\right)\}-$

$\phi(max\;[\{d\left(h\left(\gamma,v(\gamma)\right),g\left(\gamma,u(\gamma)\right)\right),d\left(h\left(\gamma,v(\gamma)\right),S\left(\gamma,v(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),T\left(\gamma,u(\gamma)\right)\right),$
$d\left(h\left(\gamma,v(\gamma)\right),T\left(\gamma,u(\gamma)\right)\right),d\left(g\left(\gamma,u(\gamma)\right),S\left(\gamma,v(\gamma)\right)\right)\}]$

$=k\;[\max\{d\left(t(\gamma),g\left(\gamma,u(\gamma)\right)\right),d\left(t(\gamma),S\left(\gamma,v(\gamma)\right)\right),d\left(t(\gamma),T\left(\gamma,u(\gamma)\right)\right)\,,$

$d\left(t(\gamma),T(\gamma,u(\gamma))\right),d\left(t(\gamma),S(\gamma,v(\gamma))\right)\}-$

$\phi\,(\max\{d\left(t(\gamma),g(\gamma,u(\gamma))\right),d\left(t(\gamma),S(\gamma,v(\gamma))\right),d\left(t(\gamma),T(\gamma,u(\gamma))\right),$

$d\left(t(\gamma),T(\gamma,u(\gamma))\right),d\left(t(\gamma),S(\gamma,v(\gamma))\right)\})]$

$=k\,[d\left(t(\gamma),S(\gamma,v(\gamma))\right)-\phi\left(d\left(t(\gamma),S(\gamma,v(\gamma))\right)\right)]$

$\leq \mathrm{k}d\left(t(\gamma),S(\gamma,v(\gamma))\right)$

this implies, $(1-k)d\left(t(\gamma),S(\gamma,v(\gamma))\right)\leq 0$

$d\left(t(\gamma),S(\gamma,v(\gamma))\right)=0\Rightarrow t(\gamma)=S(\gamma,v(\gamma))=h(\gamma,v(\gamma))$ …….(2.6)

Therefore $RC(S,h)\neq\phi$.

Theorem(2.2): Let $X,A,S,T,h,g,cl\left(S(\gamma,A)\right),cl\left(T(\gamma,A)\right),cl\left(h(\gamma,A)\right),cl\left(g(\gamma,A)\right)$ as in theorem (3.1.1) .If the pairs $\{T,g\}$ and $\{S,h\}$ are weakly compatible(or R-weakly commuting), then $RF(S)\cap RF(T)\cap RF(h)\cap RF(g)$ is a unique singleton element.

Proof: By theorem (2.1), there exists random coincidence point $u:\Omega\rightarrow A$ of T and g such that $T\left(\gamma,u(\gamma)\right)=g\left(\gamma,u(\gamma)\right)$ and random coincidence point $v:\Omega\rightarrow A$ of S and h such that $S\left(\gamma,v(\gamma)\right)=h\left(\gamma,v(\gamma)\right)$ for all $\gamma\in\Omega$.

If the pairs $\{S,h\}$ and $\{T,g\}$ are weakly compatible, then $S\left(\gamma,h(\gamma,v(\gamma))\right)=h\left(\gamma,S(\gamma,v(\gamma))\right)$ and $T\left(\gamma,g(\gamma,u(\gamma))\right)=g\left(\gamma,T(\gamma,u(\gamma))\right)$ from (2.6) and (2.4), we have

$S\left(\gamma,t(\gamma)\right)=h\left(\gamma,t(\gamma)\right)$ and $T(\gamma,t(\gamma))=g(\gamma,t(\gamma))$ ……..(2.7)

From (2.6) , (1.1) and (2.7), we have

$\left(t(\gamma),T(\gamma,t(\gamma))\right)=d\left(S(\gamma,v(\gamma)),T(\gamma,t(\gamma))\right)\leq$

$k\;[max\{d\left(h(\gamma,v(\gamma)),g(\gamma,t(\gamma))\right),d\left(h(\gamma,v(\gamma)),S(\gamma,v(\gamma))\right),d\left(g(\gamma,t(\gamma)),T(\gamma,t(\gamma))\right),$
$d\left(h(\gamma,v(\gamma)),T(\gamma,t(\gamma))\right),d\left(g(\gamma,t(\gamma)),S(\gamma,v(\gamma))\right)\}-$

$\phi(max\{d\left(h(\gamma,v(\gamma)),g(\gamma,t(\gamma))\right),d\left(h(\gamma,v(\gamma)),S(\gamma,v(\gamma))\right),d\left(g(\gamma,t(\gamma)),T(\gamma,t(\gamma))\right),$
$d\left(h(\gamma,v(\gamma)),T(\gamma,t(\gamma))\right),d\left(g(\gamma,t(\gamma)),S(\gamma,v(\gamma))\right)\}]$

$$= k\ [max\{d(t(\gamma),T(\gamma,t(\gamma))),d(t(\gamma),t(\gamma)),d(g(\gamma,t(\gamma)),T(\gamma,t(\gamma))),d(t(\gamma),T(\gamma,t(\gamma))),$$
$$d(T(\gamma,t(\gamma)),t(\gamma))\}]-$$

$$\phi\ (max\{d(t(\gamma),T(\gamma,t(\gamma))),d(t(\gamma),t(\gamma)),d(g(\gamma,t(\gamma)),T(\gamma,t(\gamma))),d(t(\gamma),T(\gamma,t(\gamma))),$$
$$d(T(\gamma,t(\gamma)),t(\gamma))\})]$$

$$= k\ [d(t(\gamma),T(\gamma,t(\gamma))) - \phi(d(t(\gamma),T(\gamma,t(\gamma))))]$$

$$\le k\ d(t(\gamma),T(\gamma,t(\gamma)))$$

Thus we have $(1-k)d(t(\gamma),T(\gamma,t(\gamma))) \le 0 \Rightarrow t(\gamma) = T(\gamma,t(\gamma))$

From (2.7) we have $t(\gamma) = T(\gamma,t(\gamma)) = g(\gamma,t(\gamma))$ ....(2.8)

Thus $t(\gamma)$ is a common random fixed point of T and g .

Again, from (2.8), (1.1) and (2.7), we get

$$d(S(\gamma,t(\gamma)),t(\gamma)) = d(S(\gamma,t(\gamma)),T(\gamma,t(\gamma)))$$

$$\le k[M(t(\gamma),t(\gamma)) - \phi(M((t(\gamma),t(\gamma))))]$$

$$= k\ [max\{d(h(\gamma,t(\gamma)),g(\gamma,t(\gamma))),d(h(\gamma,t(\gamma)),S(\gamma,t(\gamma))),d(g(\gamma,t(\gamma)),T(\gamma,t(\gamma)))$$
$$,d(h(\gamma,t(\gamma)),T(\gamma,t(\gamma))),d(g(\gamma,t(\gamma)),S(\gamma,t(\gamma)))\}-$$

$$\phi\ (max\{d(h(\gamma,t(\gamma)),g(\gamma,t(\gamma))),d(h(\gamma,t(\gamma)),S(\gamma,t(\gamma))),d(g(\gamma,t(\gamma)),T(\gamma,t(\gamma)))$$
$$,d(h(\gamma,t(\gamma)),T(\gamma,t(\gamma))),d(g(\gamma,t(\gamma)),S(\gamma,t(\gamma)))\})]$$

$$= k\ [max\{d(S(\gamma,t(\gamma)),t(\gamma)),d(h(\gamma,t(\gamma)),S(\gamma,t(\gamma))),d(g(\gamma,t(\gamma)),T(\gamma,t(\gamma))),$$
$$d(S(\gamma,t(\gamma)),t(\gamma)),d(t(\gamma),S(\gamma,t(\gamma)))\}-$$

$$\phi\ (max\{d(S(\gamma,t(\gamma)),t(\gamma)),d(h(\gamma,t(\gamma)),S(\gamma,t(\gamma))),d(g(\gamma,t(\gamma)),T(\gamma,t(\gamma))),$$

$$d(S(\gamma,t(\gamma)),t(\gamma)),d(t(\gamma),S(\gamma,t(\gamma)))\})]$$

$$= k\ [d(S(\gamma,t(\gamma)),t(\gamma)) - \phi(d(S(\gamma,t(\gamma)),t(\gamma)))]$$

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$\leq k\, d\left(S\left(\gamma, t(\gamma)\right), t(\gamma)\right)$

Thus we have $(1-k)d\left(S\left(\gamma, t(\gamma)\right), t(\gamma)\right) \leq 0 \Rightarrow S\left(\gamma, t(\gamma)\right) = t(\gamma)$ ……(2.9)

Thus $t:\Omega \to G(A)$ is a common random fixed point of S, T, h and g.

Uniqueness:

Let $z(\gamma)$ be another common random fixed point of S, T, h and g, then by using (1.1), we have

$$d\left(t(\gamma), z(\gamma)\right) = d\left(S\left(\gamma, t(\gamma)\right), T\left(\gamma, z(\gamma)\right)\right)$$

$$\leq k\left[M\left(t(\gamma), z(\gamma)\right) - \phi\left(M\left(\left(t(\gamma), z(\gamma)\right)\right)\right)\right]$$

$$= k[max\{d\left(h\left(\gamma, t(\gamma)\right), g\left(\gamma, z(\gamma)\right)\right), d\left(h\left(\gamma, t(\gamma)\right), S\left(\gamma, t(\gamma)\right)\right), d\left(g\left(\gamma, z(\gamma)\right), T\left(\gamma, z(\gamma)\right)\right),$$
$$\left(h\left(\gamma, t(\gamma)\right), T\left(\gamma, z(\gamma)\right)\right), d\left(g\left(\gamma, z(\gamma)\right), S\left(\gamma, t(\gamma)\right)\right)\} -$$
$$\phi(max\{d\left(h\left(\gamma, t(\gamma)\right), g\left(\gamma, z(\gamma)\right)\right), d\left(h\left(\gamma, t(\gamma)\right), S\left(\gamma, t(\gamma)\right)\right), d\left(g\left(\gamma, z(\gamma)\right), T\left(\gamma, z(\gamma)\right)\right),$$
$$\left(h\left(\gamma, t(\gamma)\right), T\left(\gamma, z(\gamma)\right)\right), d\left(g\left(\gamma, z(\gamma)\right), S\left(\gamma, t(\gamma)\right)\right)\})]$$
$$= k[\max\left\{d\left(t(\gamma), z(\gamma)\right), d\left(t(\gamma), t(\gamma)\right), d\left(z(\gamma), z(\gamma)\right), d\left(t(\gamma), z(\gamma)\right), d\left(z(\gamma), t(\gamma)\right)\right\}$$
$$-\phi(\max\left\{d\left(t(\gamma), z(\gamma)\right), d\left(t(\gamma), t(\gamma)\right), d\left(z(\gamma), z(\gamma)\right), d\left(t(\gamma), z(\gamma)\right), d\left(z(\gamma), t(\gamma)\right)\right\})]$$

This implies $(1-k)d\left(t(\gamma), z(\gamma)\right) \leq 0 \Rightarrow t(\gamma) = z(\gamma)$.

Assume that $\{T, g\}$ and $\{S, h\}$ are R-weakly commuting, $u(\gamma)$ is a random coincidence point of T and g and $v(\gamma)$ is a random coincidence point of S and h it follows that

$d\left(T\left(\gamma, g(\gamma, u(\gamma)\right), g\left(\gamma, T(\gamma, u(\gamma)\right)\right) \leq Rd\left(T(\gamma, u(\gamma), g(\gamma, u(\gamma)\right) = 0$, thus

$$T\left(\gamma, g(\gamma, u(\gamma))\right) = g\left(\gamma, T(\gamma, u(\gamma))\right)$$

By similarly proof we can show that $S\left(\gamma, h(\gamma, v(\gamma))\right) = h\left(\gamma, h(\gamma, v(\gamma))\right)$

Hence the pairs $\{T, g\}$ and $\{S, h\}$ are weakly compatible. then the same steps above we can show that $t(\gamma)$ is a common random fixed point of S, T, h and g.

As a consequence, we get the following

Corollary (2.3): Let $X, A, T, h, cl\left(T(\gamma, A)\right), cl\left(h(\gamma, A)\right)$ as in theorem (2.1), and for all $\gamma \in \Omega$, the mappings $T, h(\gamma, .): A \longrightarrow A$ satisfies the following condition

$$d\left(T(\gamma, x), T(\gamma, y)\right) \leq k[M(x, y) - \phi\left(M(x, y)\right)]$$

where

$$M(x,y)=max\left\{\begin{matrix} d\left(h(\gamma,x),h(\gamma,y)\right),d\left(h(\gamma,x),T(\gamma,x)\right),d\left(h(\gamma,y),T(\gamma,y)\right) \\ ,d\left(h(\gamma,x),T(\gamma,y)\right),d\left(h(\gamma,y),T(\gamma,x)\right) \end{matrix}\right\}$$

…….(2.10)

, $0\le k<\frac{1}{2}$ for all $x,y\in A$. Then $RC(h\cap T)\ne\phi$.

Proof: Put $S=T$ and $h=g$ in theorem (2.1), then the corollary (2.3) follows from theorem (2.1).■

Corollary (2.4): Let $X,A,T,h,cl\left(T(\gamma,A)\right),cl\left(h(\gamma,A)\right)$ as in corollary (2.3). If the pair $\{T,h\}$ is weakly compatible or (R-weakly commuting), then $RF(h)\cap RF(T)$
is a unique singlton element.

3. Random Well-posed Problem

Definition (3.1): Let (X,d) be a metric space and $T:\Omega\times X\to X$ a random operator. the random fixed point problem of T is said to be well- posed if :

i. T has a unique random fixed point $\delta:\Omega\to X$;

ii. for any sequence $\{\delta_n(\gamma)\}$ of measurable mappings in X such that

$\lim_{n\to\infty} d(T(\gamma,\delta_n(\gamma)),\delta_n(\gamma))=0$, we have $\lim_{n\to\infty} d(\delta_n(\gamma),\delta(\gamma))=0$.

Definition (3.2): Let (X,d) be a metric space and let T be a set of a random operators in X. The random fixed point of T is said to be well-posed if :

i. T has a unique random fixed point $\delta:\Omega\to X$;

ii. for any sequence $\{\delta_n(\gamma)\}$ of measurable mappings in X such that $\lim_{n\to\infty} d(T(\gamma,\delta_n(\gamma)),\delta_n(\gamma))=0$
, $\forall T\in\mathrm{T}$. we have $\lim_{n\to\infty} d(\delta_n(\gamma),\delta(\gamma))=0$.

Theorem(3.3): If $X,A,S,T,h,g,cl\left(S(\gamma,A)\right),cl\left(T(\gamma,A)\right),cl\left(h(\gamma,A)\right),cl\left(g(\gamma,A)\right)$ as in theorem (2.2), then the common random fixed point for the set of random operators

$\{S,T,h,g\}$ is well-posed .

Proof: By theorem (2.2), the random operators S,T,h and g have a unique common random fixed point $t:\Omega\to A$. Let $\{\delta_n(\gamma)\}$ be a sequence of measurable mappings in A such that

$$\lim_{n\to\infty} d\left(S(\gamma,\delta_n(\gamma)),\delta_n(\gamma)\right)=\lim_{n\to\infty} d\left(T(\gamma,\delta_n(\gamma)),\delta_n(\gamma)\right)=\lim_{n\to\infty} d\left(h(\gamma,\delta_n(\gamma)),\delta_n(\gamma)\right)=$$
$$\lim_{n\to\infty} d\left(g(\gamma,\delta_n(\gamma)),\delta_n(\gamma)\right)=0$$

By the triangle inequality , (1.1) ,(2.7) and (2.8), we have

$$d\left(t(\gamma),\delta_n(\gamma)\right) \le d\left(S\left(\gamma,t(\gamma)\right),T\left(\gamma,\delta_n(\gamma)\right)\right)+d\left(T\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)$$

$$\le k\left[M\left(t(\gamma),\delta_n(\gamma)\right)-\phi\left(M\left(t(\gamma),\delta_n(\gamma)\right)\right)\right]+d\left(T\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)$$

$$= k[\max\{d\left(h\left(\gamma,t(\gamma)\right),g\left(\gamma,\delta_n(\gamma)\right)\right),d\left(h\left(\gamma,t(\gamma)\right),S\left(\gamma,t(\gamma)\right)\right),d\left(g\left(\gamma,\delta_n(\gamma)\right),T\left(\gamma,\delta_n(\gamma)\right)\right),$$
$$d\left(h\left(\gamma,t(\gamma)\right),T\left(\gamma,\delta_n(\gamma)\right)\right),d\left(g\left(\gamma,\delta_n(\gamma)\right),S\left(\gamma,t(\gamma)\right)\right)\}-$$

$$\phi(\max\{d\left(h\left(\gamma,t(\gamma)\right),g\left(\gamma,\delta_n(\gamma)\right)\right),d\left(h\left(\gamma,t(\gamma)\right),S\left(\gamma,t(\gamma)\right)\right),d\left(g\left(\gamma,\delta_n(\gamma)\right),T\left(\gamma,\delta_n(\gamma)\right)\right),$$
$$d\left(h\left(\gamma,t(\gamma)\right),T\left(\gamma,\delta_n(\gamma)\right)\right),d\left(g\left(\gamma,\delta_n(\gamma)\right),S\left(\gamma,t(\gamma)\right)\right)\})+d\left(T\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)$$

$$= k[d\left(g\left(\gamma,\delta_n(\gamma)\right),t(\gamma)\right)+d\left(t(\gamma),T\left(\gamma,\delta_n(\gamma)\right)\right)]-\phi(d\left(g\left(\gamma,\delta_n(\gamma)\right),t(\gamma)\right)+d\left(t(\gamma),T\left(\gamma,\delta_n(\gamma)\right)\right)]$$
$$+d\left(T\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)$$

$$\le k[d\left(g\left(\gamma,\delta_n(\gamma)\right),t(\gamma)\right)+d\left(t(\gamma),T\left(\gamma,\delta_n(\gamma)\right)\right)]+d\left(T\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)$$

$$\le k[d\left(g\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)+d\left(\delta_n(\gamma),t(\gamma)\right)+d\left(t(\gamma),\delta_n(\gamma)\right)+d\left(\delta_n(\gamma),T\left(\gamma,\delta_n(\gamma)\right)\right)]+$$
$$d\left(T\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)$$

$$= kd\left(g\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)+2k\,d\left(\delta_n(\gamma),t(\gamma)\right)+(1+k)d\left(T\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)$$

this implies

$$(1-2k)d\left(\delta_n(\gamma),t(\gamma)\right)\le kd\left(g\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)+(1+k)d\left(T\left(\gamma,\delta_n(\gamma)\right),\delta_n(\gamma)\right)$$

thus we have, $\lim_{n\to\infty} d(\delta_n(\gamma),\delta(\gamma))=0$, it follows that the common random fixed point for the set of random operators $\{S,T,h,g\}$ is well-posed.■

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