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Some Estimation Methods for New Mixture Distribution with Simulation and Application[c]

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Abstract . In this paper, we investigate the New Mixture Distribution with discussion its studied mathematical and statistical properties. The classical methods (maximum likelihood, ordinary least square , and rank set sampling methods) are used to estimate the parameters of New Mixture with comparison between them using the mean square error for different sizes of simulation samples generated. Finally, we illustrate the usefulness of proposed by application of complete real data.

1. Introduction

In order to keep abreast of developments in the medical and engineering fields and other scientific areas related to data analysis and extract information that will help to take appropriate action based on this information. An important technique in analyzing data, especially in the field of reliability and survival, which was used of statistical distributions. These distributions which play important roles in the processing and analysis of data. In view of the developments in modern medical and technological fields, needing to develop statistical methods in processing and extracting data. The methods that the researchers developed is the style of a combination of statistical distributions to generate new distributions comparable to the characteristics known distributions. One method developed by the researchers is a combination of one of these ideas, which is based on the tail distribution in two or more distributional combinations. New mixture distribution is mixed between the exponential Weibull and exponential Rayleigh. Firstly, mixing between exponential and Weibull distributions. Secondly, mixing between exponential and Rayleigh distributions. Gauss (2013) introduced a new method to mix the distributions by using a tail of these distributions, and represented this new method to mix between exponential of one parameter and Weibull of two parameters[4]. Suleman, N., (2016); found a new distribution called "Serial Weibull Rayleigh distribution: theory and application[10]. Maysaa Jalil and Iden Hasan (2018a)[7] used the tail technical to introduce a new mixture distribution Besides that, Gauss M. Cordeiro and et al(2016), introduced a new distribution. It depending on exponential Weibull distribution said" The Kumaraswamy exponential-Weibull distribution: theory and applications[5]. Faton Merovci and Ibrahim Elbatal (2015)[3], represented a new distribution called



"Weibull Rayleigh Distribution". It depending on another method to mix between the distribution (integral method). Marcelo et al.(2014)[2], represented Weibull – G distribution based on odds $\frac{G(y)}{1-G(y)}$ where $\vartheta(y, \vartheta)$ is the (CDF) of random variable such that:

$$F(y, a, b, \vartheta) = \int_0^{\frac{\vartheta(y)}{1-\vartheta(y)}} abz^{b-1}e^{-az^b} dz = 1 - e^{\left\{-a\left[\frac{\vartheta(y, \vartheta)}{1-\vartheta(y)}\right]^b\right\}} \quad y \in \mathbb{R}; a, b > 0.$$

Ahmad Mahir Razali et al.(2008) introduced "On Simulation Study of Mixture of Two Weibull Distributions". Which is mixed between two Weibull Distributions[1]. Rodrigues & Silve, (2015), represented "The Exponentiated Kumaraswamy- Exponential Distribution". That is extended the exponential distribution[6].

In this article, discussion the definition of New Mixture Distribution. The researcher introduces the structure of mixed distribution with some preliminaries and properties of this distribution in section two, the estimation the three parameters of New Mixture Distribution using Maximum likelihood, Ordinary least square, and rank set sampling methods) in section three, simulation study to generate different sizes of samples and comparison between the methods above by using mean square error for the parameters estimation in section four, finally ,an application using real complete data in section five.

2. Preliminaries

2.1 New Mixture Distribution

In this section, present the new distribution with three parameters. It is mixed between exponential Weibull and exponential Rayleigh. The pdf of the new mixture distribution is:

$$f(x) = (2Y + \beta x + \alpha x^{\alpha-1}) e^{-\left(2Yx + \frac{\beta}{2}x^2 + x^\alpha\right)}, \quad x > 0 \quad (1)$$

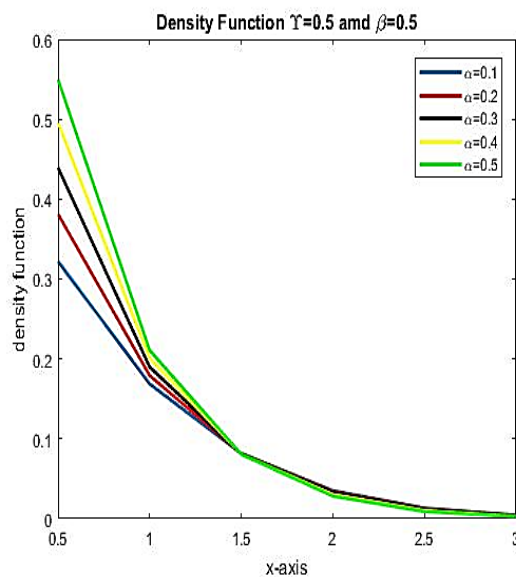
Where, $Y > 0$, $\beta > 0$ are two scale parameters and $\alpha > 0$, is the shape parameter of a new mixture distribution. The cdf function is

$$G(x) = 1 - e^{-\left(2Yx + \frac{\beta}{2}x^2 + x^\alpha\right)}, \quad x > 0 \quad (2)$$

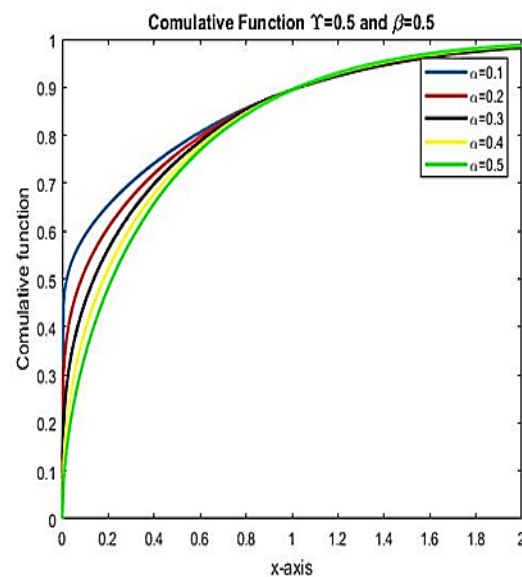
The survival and hazard functions respectively of new mixture distribution are:

$$S(x) = e^{-\left(2Yx + \frac{\beta}{2}x^2 + x^\alpha\right)}, \quad x > 0 \quad (3)$$

$$h(x) = 2Y + \beta x + \alpha x^{\alpha-1}, \quad x > 0 \quad (4)$$



Figure(1): The density function of New Mixture Distribution the scale parameters are constants with different shape parameter values



Figure(2): The cumulative function of New Mixture Distribution the scale parameters are constants with different shape parameter values

2.2 The Statistical Properties

The r th moment about origin of New Mixture Distribution is:

$$M'_r = E(X^r) = 2YK(r, Y, \beta, \alpha) + \beta K(r+1, Y, \beta, \alpha) + \alpha K(r+\alpha-1, Y, \beta, \alpha) \quad (5)$$

Where,

$$K(r, Y, \beta, \alpha) = \frac{1}{\alpha} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(-2Y)^n}{n!} \frac{(-\beta)^m}{2^m m!} \Gamma\left(\frac{r+n+2m+1}{\alpha}\right)$$

Then, the mean is:

$$E(X) = 2YK(1, Y, \beta, \alpha) + \beta K(2, Y, \beta, \alpha) + \alpha K(\alpha, Y, \beta, \alpha).$$

And the variance of the New Mixture Distribution is:

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= 2YK(2, Y, \beta, \alpha) + \beta K(3, Y, \beta, \alpha) + \alpha K(\alpha+1, Y, \beta, \alpha) \\ &\quad - [2YK(1, Y, \beta, \alpha) + \beta K(2, Y, \beta, \alpha) + \alpha K(\alpha, Y, \beta, \alpha)]^2 \end{aligned}$$

The Moments Generating function is:

$$M(t) = 2YK(t, 2Y-t, \beta, \alpha) + \beta K(t+1, 2Y-t, \beta, \alpha) + \alpha K(t+\alpha-1, 2Y-t, \beta, \alpha) \quad (6)$$

The Factorial Moments Generating function is:

$$M_x(t) = 2YK(t, 2Y - \ln t, \beta, \alpha) + \beta K(t+1, 2Y - \ln t, \beta, \alpha) + \alpha K(t+\alpha-1, 2Y - \ln t, \beta, \alpha) \quad (7)$$

Finally, the Characteristic function is:

$$\varphi_x(it) = 2YK(t, 2Y - it, \beta, \alpha) + \beta K(t+1, 2Y - it, \beta, \alpha) + \alpha K(t+\alpha-1, 2Y - it, \beta, \alpha) \quad (8)$$

We can see another properties of the New Mixture Distribution in [7].

3. Estimation parameters of New Mixture Distribution

In this section, determine classical estimation methods to estimate the parameters of New Mixture Distribution (Y, β, α).

3.1 Maximum likelihood Estimation method

Using (1), $f(x) = (2Y + \beta x + \alpha x^{\alpha-1})e^{-(2Yx + \frac{\beta}{2}x^2 + x^\alpha)}$, $x > 0$

$$L(x_1, x_2, \dots, x_n, Y, \beta, \alpha) = \prod_{i=1}^n f(x_i, Y, \beta, \alpha)$$

$$L(Y, \beta, \alpha, x_i) = \prod_{i=1}^n f(x_i, Y, \beta, \alpha)$$

$$L(Y, \beta, \alpha, x_i) = \prod_{i=1}^n [(2Y + \beta x_i + \alpha x_i^{\alpha-1}) e^{-\sum_{i=1}^n (2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha)}]$$

The log-likelihood function is:

$$\ln L(Y, \beta, \alpha, x_i) = \sum_{i=1}^n \ln(2Y + \beta x_i + \alpha x_i^{\alpha-1}) - \sum_{i=1}^n \left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)$$

Assume that:

$$g(Y^\wedge) = \frac{\partial \ln L}{\partial Y} = 2 \left[\sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-1} - \sum_{i=1}^n x_i \right] \quad (9)$$

$$w(\beta^\wedge) = \frac{\partial \ln L}{\partial \beta} = \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-1} (x_i) - \frac{1}{2} \sum_{i=1}^n x_i^2 \quad (10)$$

$$d(\alpha^\wedge) = \frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-1} x_i^{\alpha-1} [\alpha \ln(x_i) + 1] - \sum_{i=1}^n x_i^\alpha \ln(x_i) \quad (11)$$

When $\frac{\partial \ln L}{\partial Y} = \frac{\partial \ln L}{\partial \beta} = \frac{\partial \ln L}{\partial \alpha} = 0$, there is no closed solution of equations(9),(10),(11). Therefore, numerical technique(Newton –Raphson method) should be applied[9].

$$\begin{bmatrix} Y_{i+1}^\wedge \\ \beta_{i+1}^\wedge \\ \alpha_{i+1}^\wedge \end{bmatrix} = \begin{bmatrix} Y_i^\wedge \\ \beta_i^\wedge \\ \alpha_i^\wedge \end{bmatrix} - J^{-1} \begin{bmatrix} g(Y^\wedge) \\ w(\beta^\wedge) \\ d(\alpha^\wedge) \end{bmatrix} \quad (12)$$

Where the Jacobean matrix is define as: $J = \begin{bmatrix} \frac{\partial g(Y)}{\partial Y} & \frac{\partial g(Y)}{\partial \beta} & \frac{\partial g(Y)}{\partial \alpha} \\ \frac{\partial w(\beta)}{\partial Y} & \frac{\partial w(\beta)}{\partial \beta} & \frac{\partial w(\beta)}{\partial \alpha} \\ \frac{\partial d(\alpha)}{\partial Y} & \frac{\partial d(\alpha)}{\partial \beta} & \frac{\partial d(\alpha)}{\partial \alpha} \end{bmatrix}$

$$\frac{\partial g(Y)}{\partial Y} = -4 \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2}$$

$$\frac{\partial g(Y)}{\partial \beta} = -2 \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} (x_i),$$

$$\frac{\partial g(Y)}{\partial \alpha} = -2 \left[\sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} x_i^{\alpha-1} (\alpha \ln(x_i) + 1) \right]$$

$$\frac{\partial w(\beta)}{\partial \beta} = -1 \left[\sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} (x_i^2) (\alpha \ln(x_i) + 1) \right]$$

$$\frac{\partial w(\beta)}{\partial \alpha} = - \left[\sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} x_i^{\alpha} \right]$$

$$\begin{aligned} \frac{\partial d(\alpha)}{\partial \alpha} &= \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-1} x_i^{\alpha-1} \ln(x_i) \\ &\quad - \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} x_i^{2(\alpha-1)} [\alpha \ln(x_i) + 1]^2 - \sum_{i=1}^n x_i^{\alpha} (\ln(x_i))^2 \end{aligned}$$

Since the Jacobean matrix must be a non –singular symmetric matrix, then:

$$\frac{\partial g(Y)}{\partial \beta} = \frac{\partial w(\beta)}{\partial Y}, \quad \frac{\partial w(\beta)}{\partial \alpha} = \frac{\partial d(\alpha)}{\partial \beta}, \quad \text{and} \quad \frac{\partial g(Y)}{\partial \alpha} = \frac{\partial d(\alpha)}{\partial Y}$$

3.2 Ordinary Least Square Estimation Method

Minimization the sum of squared differences between the sample value (J_i) and the expected values $E(P_i)$ is the main idea of this method by the formula:

$$\begin{aligned} \sum_{i=1}^n \epsilon_i^2 &= \sum_{i=1}^n (P_i - E(P_i))^2 \\ \sum_{i=1}^n \epsilon_i^2 &= \sum_{i=1}^n (P_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_{\alpha} x_i^{\alpha})^2 \end{aligned}$$

By taking the c. d. f. of three parameters New Mixture Distribution:

$$G(x_i) = 1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)}, \quad x > 0$$

$$1 - G(x_i) = e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)}$$

$$\ln(1 - G(x_i)) = -2Yx_i - \frac{\beta}{2}x_i^2 - x_i^\alpha$$

$$P_i = \ln(1 - G(x_i)), \beta_0 = 0, \beta_1 = -2Y, \beta_2 = -\frac{\beta}{2}, \text{ and } \beta_\alpha = -1$$

$$\epsilon_i = \ln(1 - G(x_i)) + 2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha$$

$$\sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n \left[\ln(1 - G(x_i)) + 2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right]^2 \quad (13)$$

$$\frac{\partial \sum_{i=1}^n \epsilon_i^2}{\partial Y} = g(Y) = 4 \sum_{i=1}^n [\ln(1 - G(x_i)) + 2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha] (x_i) \quad (14)$$

$$\frac{\partial \sum_{i=1}^n \epsilon_i^2}{\partial \beta} = w(\beta) = \sum_{i=1}^n [\ln(1 - G(x_i)) + 2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha] (x_i^2) \quad (15)$$

$$\frac{\partial \sum_{i=1}^n \epsilon_i^2}{\partial \alpha} = d(\alpha) = 2 \sum_{i=1}^n [\ln(1 - G(x_i)) + 2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha] (x_i^\alpha \ln(x_i)) \quad (16)$$

J is the Jacobean matrix is define as follows:

$$J = \begin{bmatrix} \frac{\partial g(Y)}{\partial Y} & \frac{\partial g(Y)}{\partial \beta} & \frac{\partial g(Y)}{\partial \alpha} \\ \frac{\partial w(\beta)}{\partial Y} & \frac{\partial w(\beta)}{\partial \beta} & \frac{\partial w(\beta)}{\partial \alpha} \\ \frac{\partial d(\alpha)}{\partial Y} & \frac{\partial d(\alpha)}{\partial \beta} & \frac{\partial d(\alpha)}{\partial \alpha} \end{bmatrix}$$

Then, from the equations(14,15,16):

$$\frac{\partial g(Y)}{\partial Y} = 8 \sum_{i=1}^n x_i^2$$

$$\frac{\partial g(Y)}{\partial \beta} = 2 \sum_{i=1}^n x_i^3$$

$$\begin{aligned}\frac{\partial g(Y)}{\partial \alpha} &= 4 \sum_{i=1}^n x_i^{\alpha+1} \ln(x_i) \\ \frac{\partial w(\beta)}{\partial \beta} &= 1/2 \sum_{i=1}^n x_i^4 \\ \frac{\partial w(\beta)}{\partial \alpha} &= \sum_{i=1}^n x_i^{\alpha+2} \ln(x_i) \\ \frac{\partial d(\alpha)}{\partial \alpha} &= 2 \sum_{i=1}^n [\ln(1 - G(x_i)) x_i^\alpha (\ln(x_i))^2 + 2Y x_i^{\alpha+1} \ln(x_i) + \frac{\beta}{2} x_i^{\alpha+2} \ln(x_i) \\ &\quad + 2x_i^{2\alpha} (\ln(x_i))^2] \\ \begin{bmatrix} Y_{i+1}^{\wedge} \\ \beta_{i+1}^{\wedge} \\ \alpha_{i+1}^{\wedge} \end{bmatrix} &= \begin{bmatrix} Y_i^{\wedge} \\ \beta_i^{\wedge} \\ \alpha_i^{\wedge} \end{bmatrix} - J^{-1} \begin{bmatrix} g(Y^{\wedge}) \\ w(\beta^{\wedge}) \\ d(\alpha^{\wedge}) \end{bmatrix}\end{aligned}\quad (17)$$

It is numerical(Newton –Raphson method). Since Jacobean matrix is non-singular matrix, then

$$\frac{\partial g(Y)}{\partial \beta} = \frac{\partial w(\beta)}{\partial Y}, \quad \frac{\partial w(\beta)}{\partial \alpha} = \frac{\partial d(\alpha)}{\partial \beta}, \text{ and } \frac{\partial g(Y)}{\partial \alpha} = \frac{\partial d(\alpha)}{\partial Y}$$

3.3 Rank Set Sampling Estimation Method

The p.d.f of the new mixture distribution which obtained by increasing ordering random sampling $(X_1, X_2, X_3, \dots, \dots, X_n)$ is:

$$\begin{aligned}f(x_i) &= \frac{n!}{(i-1)!(n-i)!} [G(x_i)]^{i-1} [1 - G(x_i)]^{n-i} f(x_i) \\ f(x_i) &= \frac{n!}{(i-1)!(n-i)!} \left[1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right]^{i-1} \left[1 - 1 + e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right]^{n-i} \left[(2Y + \beta x \right. \\ &\quad \left. + \alpha x^{\alpha-1}) e^{-\left(2Yx + \frac{\beta}{2}x^2 + x^\alpha\right)} \right]\end{aligned}\quad (18)$$

let $M = \frac{n!}{(i-1)!(n-i)!}$ then,

$$f(x_i) = M(2Y + \beta x + \alpha x^{\alpha-1}) \left[1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right]^{i-1} \left[e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right]^{n-i+1}$$

The likelihood function is,

$$\begin{aligned}L(Y, \beta, \alpha, x_i) &= M^n \prod_{i=1}^n \left[(2Y + \beta x_i \right. \\ &\quad \left. + \alpha x_i^{\alpha-1}) \prod_{i=1}^n \left[1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right]^{i-1} \prod_{i=1}^n \left[e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right]^{n-i+1} \right]\end{aligned}$$

The log- likelihood function is,

$$\begin{aligned}
\ln L &= n \ln M + \sum_{i=1}^n \ln(2Y + \beta x_i + \alpha x_i^{\alpha-1}) + \sum_{i=1}^n (i-1) \ln \left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right) \\
&\quad - \sum_{i=1}^n (n-i+1) \left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right) \\
g(Y^\wedge) &= \frac{\partial \ln L}{\partial Y} = 2 \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-1} \\
&\quad + \sum_{i=1}^n (i-1) \left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right)^{-1} e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} (2x_i) \\
&\quad - \sum_{i=1}^n (n-i+1) (2x_i)
\end{aligned} \tag{19}$$

$$\begin{aligned}
w(\beta^\wedge) &= \frac{\partial \ln L}{\partial \beta} = \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-1} (x_i) \\
&\quad + \sum_{i=1}^n (i-1) \left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right)^{-1} e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \left(\frac{1}{2} x_i^2 \right) \\
&\quad - \sum_{i=1}^n (n-i+1) \left(\frac{1}{2} x_i^2 \right)
\end{aligned} \tag{20}$$

$$\begin{aligned}
d(\alpha^\wedge) &= \frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-1} x_i^{\alpha-1} (\alpha \ln(x_i) + 1) \\
&\quad + \sum_{i=1}^n (i-1) \left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} \right)^{-1} e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha\right)} (x_i^\alpha \ln(x_i)) \\
&\quad - \sum_{i=1}^n (n-i+1) (x_i^\alpha \ln(x_i)).
\end{aligned} \tag{21}$$

Assume that $\frac{\partial \ln L}{\partial Y} = \frac{\partial \ln L}{\partial \beta} = \frac{\partial \ln L}{\partial \alpha} = 0$, by using the numerical method (Newton–Raphson) to find the solution of these equations, therefore, and the Jacobean matrix is define as follows:

$$J = \begin{bmatrix} \frac{\partial g(Y)}{\partial Y} & \frac{\partial g(Y)}{\partial \beta} & \frac{\partial g(Y)}{\partial \alpha} \\ \frac{\partial w(\beta)}{\partial Y} & \frac{\partial w(\beta)}{\partial \beta} & \frac{\partial w(\beta)}{\partial \alpha} \\ \frac{\partial d(\alpha)}{\partial Y} & \frac{\partial d(\alpha)}{\partial \beta} & \frac{\partial d(\alpha)}{\partial \alpha} \end{bmatrix}$$

Where:

$$\begin{aligned} \frac{\partial g(Y)}{\partial Y} = & -4 \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} \\ & - \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-2} \right] \left[e^{-2\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (4x_i^2) \right] \\ & - \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-1} \right] \left[e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (4x_i^2) \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial g(Y)}{\partial \beta} = & -2 \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} (x_i) \\ & - \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-2} \right] \left[e^{-2\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (x_i^3) \right] \\ & - \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-1} \right] \left[e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (x_i^3) \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial g(Y)}{\partial \alpha} = & -2 \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} (x_i^{\alpha-1}) (\alpha \ln(x_i) + 1) \\ & - 2 \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-2} \right] \left[e^{-2\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (x_i^{\alpha+1}) (\ln(x_i)) \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial w(\beta)}{\partial \beta} = & -1 \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} (x_i^2) \\ & - \frac{1}{4} \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-2} \right] \left[e^{-2\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (x_i^4) \right] \\ & - \frac{1}{4} \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-1} \right] \left[e^{-2\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (x_i^4) \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial w(\beta)}{\partial \beta} = & -1 \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} (x_i^\alpha) (\alpha \ln(x_i) + 1) \\ & - \frac{1}{2} \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-2} \right] \left[e^{-2\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (x_i^{\alpha+2}) (\ln(x_i)) \right] \\ & - \frac{1}{2} \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-1} \right] \left[e^{-2\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (x_i^{\alpha+2}) (\ln(x_i)) \right] \end{aligned}$$

$$\begin{aligned}
\frac{\partial d(\alpha)}{\partial \alpha} = & \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-1} (x_i^{\alpha-1} \ln(x_i)) (\alpha \ln(x_i) + 2) \\
& - \sum_{i=1}^n (2Y + \beta x_i + \alpha x_i^{\alpha-1})^{-2} (x_i^{2(\alpha-1)}) (\alpha \ln(x_i) + 1)^2 \\
& - \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-2} \right] \left[e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (x_i^{2\alpha}) (\ln x_i)^2 \right] \\
& - \sum_{i=1}^n (i-1) \left[\left(1 - e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} \right)^{-1} \right] \left[e^{-\left(2Yx_i + \frac{\beta}{2}x_i^2 + x_i^\alpha \right)} (x_i^{2\alpha}) (\ln x_i)^2 \right] \\
& - \sum_{i=1}^n (n-i+1) (x_i^\alpha) (\ln(x_i))^2
\end{aligned}$$

Since, Jacobean is non-singular matrix, then, $\frac{\partial g(Y)}{\partial \beta} = \frac{\partial w(\beta)}{\partial Y}$, $\frac{\partial w(\beta)}{\partial \alpha} = \frac{\partial d(\alpha)}{\partial \beta}$ and $\frac{\partial g(Y)}{\partial \alpha} = \frac{\partial d(\alpha)}{\partial Y}$.

Therefore,

$$\begin{bmatrix} Y_{i+1}^{\wedge} \\ \beta_{i+1}^{\wedge} \\ \alpha_{i+1}^{\wedge} \end{bmatrix} = \begin{bmatrix} Y_i^{\wedge} \\ \beta_i^{\wedge} \\ \alpha_i^{\wedge} \end{bmatrix} - J^{-1} \begin{bmatrix} g(Y^{\wedge}) \\ w(\beta^{\wedge}) \\ d(\alpha^{\wedge}) \end{bmatrix} \quad (22)$$

It is numerical(Newton –Raphson method)to find the solution of these equations.

4. Simulation Study

In this section, provided the method to generate different sizes of the random samples. It depends on the cumulative distribution function of new mixture distribution. From the equation (2)the CDF of New Mixture Distribution is:

$$G(x) = 1 - e^{-\left(2Yx + \frac{\beta}{2}x^2 + x^\alpha \right)}, \quad x > 0, \text{ then } 1 - G(x) = e^{-\left(2Yx + \frac{\beta}{2}x^2 + x^\alpha \right)}$$

Take the Logarithm to both sides in the equation, besides that substitution $\alpha = 1$. As a result, getting the following equation:

$$x = \frac{-(2Y + 1) \mp ((2Y + 1)^2 - 2\beta \ln(1 - u))^{1/2}}{\beta} \quad (23)$$

Because (x) is positive, the negative values resulting from this generation are ignored. According to equation (23), generating different sizes of samples $n=10, 30, 50, 100$. And estimation the unknown parameters by using (MLE, OLS, RSS) methods. Therefore, calculating the value of mean square error to compare between the methods. $MSE = \sum_{i=1}^L \frac{(\hat{\theta} - \theta)^2}{L}$, where n is the size of sample and L which is the number of Repeating the experiments ($L=500$), $n=10, 30, 50, 100$.

4.1 Results and discussion

In this section, discussing the MSE of different sizes of samples that used to compare the estimation methods. Repeating the experiments 500 times for each sample.

Table (1): The MSE values where ($\gamma=0.0102$, $\beta=0.0291$ and $\alpha=1$)

n	MLE ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)	RSS ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)	OLS ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)
10	2.27E-9	1.69E-5	3.48E-33
	0.005731	3.12E-5	4.24E-32
	0.000124	9.85E-9	6.94E-31
30	2.09E-5	1.67E-6	6.64E-35
	0.051679	2.33E-5	2.46E-34
	0.001114	2.31E-8	1.89E-33
50	5.85E-5	8.31E-6	1.63E-35
	1.43278	1.15E-5	8.26E-35
	0.003097	3.37E-9	1.29E-33
100	0.000232	4.01E-6	3.57E-35
	0.57353	1.61E-5	9.79E-35
	0.012405	3.84E-9	3.91E-34

In this table assumed that ($\gamma=0.0102$, $\beta=0.0291$, $\alpha=1$) in the equation of generated samples. And suppose that ($\gamma=0.000166$, $\beta=0.0077$, $\alpha=0.00099$) are the initial values which are needing for (Newton–Raphson) in equations (12, 17 and 28).

Table (2): The MSE
(($\gamma=0.0102$, $\beta=0.0291$

n	MLE ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)	RSS ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)	OLS ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)
10	3.52E-5	1.26E-5	1.18E-32
	0.072995	0.00088	2.05E-31
	0.000125	4.15E-9	3.6E-11
30	0.000263	3.4E-5	1.07E-33
	0.655548	9.03E-5	1.2E-32
	0.001165	6.33E-9	3.6E-11
50	0.000882	4.29E-5	1.71E-33
	1.872483	3.85E-5	2.5E-32
	0.003168	1.51E-9	3.6E-11
100	0.003199	2.91E-5	1.21E-33
	7.395463	5.63E-5	1.61E-32
	0.012654	2.67E-9	3.61E-11

values where
, $\alpha=1$)

In this table, having the same initial value that assumed in table (1) to generate the samples. The initial values which are for (Newton–Raphson) in (12, 17, 28) are ($\gamma=0.0005$, $\beta=0.003333$, $\alpha=0.0006$).

Table (3): The MSE values where ($\gamma=1.2$, $\beta=1.1$ and $\alpha=1$)

n	MLE ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)	RSS ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)	OLS ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)
10	1.07E-6	3.65E-6	2.16E-34
	0.001594	7.62844E-6	8.27E-34
	2.73E-8	2.73E-8	2.47E-33
30	3.33E-9	3.65E-6	1.4E-33
	0.01437	3.01E-6	6.77E-33
	0.000588	7.15E-6	2.66E-32
50	3.19E-6	1.25E-8	4.01E-35
	2.69E-6	0.039659	1.41E-35
	1.81E-8	0.001423	9.88E-34
100	1.93E-8	3.33E-6	2.76E-10
	0.158667	1.61E-5	7.1E-35
	0.006542	1.6646E-9	5.68E-34

In this table assumed that ($\gamma=1.2$, $\beta=1.1$ and $\alpha=1$) in the equation of generated samples. And suppose that ($\gamma=0.004$, $\beta=0.01$ and $\alpha=0.00066$) are the initial values which are needing for (Newton –Raphson) in (12,17 and 28) equations.

Table (4): The MSE values where ($\gamma=1.2$, $\beta=1.1$, $\alpha=1$)

n	MLE ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)	RSS ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)	OLS ($\hat{\gamma}, \hat{\beta}, \hat{\alpha}$)
10	7.85E-5	3.4E-5	8.1E-7
	0.073333	0.000291	1.8E-31
	8.48E-5	8.18E-9	3.01E-34
30	0.000782	2.66E-5	2.55E-34
	0.662101	0.000279	4.31E-33
	0.000755	1.07E-8	2.34E-35
50	0.00225	2.4E-5	6.17E-34
	1.841437	0.000223	5.72E-33
	0.002086	1.65E-9	3.27E-35
100	0.008722	3E-5	9.76E-34
	7.356559	4.7E-5	1.06E-32
	0.008382	2.41E-9	4.74E-35

In this table, having the same initial value that assumed in table (3) to generate the samples. The initial values which are needing for (Newton –Raphson) in (12,17 and 28) equations are ($\gamma=0.001$, $\beta=0.023$, $\alpha=0.00077$).

Noting, according to the classical estimation methods and depended to the measure of the mean square error that utilizing for estimating unknown parameters, the best method of estimation parameters of new mixture distribution is the ordinary least square method.

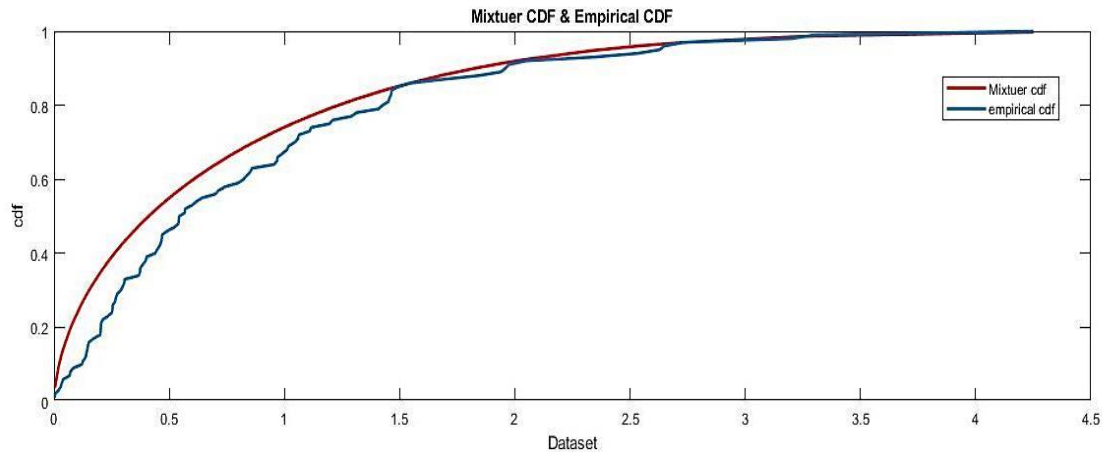


Figure (3): Comparing empirical CDF(blue) with Mixture CDF(red) of n=100 dataset

5. Real application

Murthy et al (2004), has taken the data for life times of 20 electronic components[8]. It was follows: 0.03, 0.12, 0.22, 0.35, 0.73, 0.79, 1.25, 1.41, 1.52, 1.79, 1.8, 1.94, 2.38, 2.4, 2.87, 2.99, 3.14, 3.17, 4.72, 5.09.

Table 5: of MSE of estimation parameters of 20 electronic samples

n	MLE	RSS	OLS
	$(\hat{\gamma}, \hat{\beta}, \hat{\alpha})$	$(\hat{\gamma}, \hat{\beta}, \hat{\alpha})$	$(\hat{\gamma}, \hat{\beta}, \hat{\alpha})$
20	2.25116E-05	1.59E-08	2.55743E-34
	0.000202779	1.1E-06	1.60925E-35
	0.001643384	1.22E-06	1.42235E-34

In this table, assumed that the initial values which are needing for(Newton –Raphson)in (12,17and28) equations are ($\gamma=0.0003$, $\beta=0.0009$, $\alpha=0.00099$). Noted that the Ordinary least square method is the best of parameters estimation methods.

Conclusion. In this research, a New Mixture Distribution was proposed. Statistical and mathematical characteristics were studied. Using classical estimation methods to estimate unknown parameters of New Mixture Distribution. Besides that, simulation study was used to generate different sizes of samples. Utilizing the mean square error to compare between estimation methods referred to above. Noted that the ordinary least square method is the best method because it has a minimum error for all different sizes of samples. Finally, applied these methods using real complete data.

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