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The criterion of non-stationary cyclic fatigue of bodies, taking into account the presence of an incubation period of destruction

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Abstract. A functional of nonstationary cyclic fatigue of bodies was constructed under the assumption that the destruction process occurs in time after the deformation process.
Damage conditions are derived, which determines the number of non-stationary loading cycles, in which damage and cyclic fatigue occurs in the body, which makes it possible to find the non-stationary number of loading cycles before failure.

Introduction

Fatigue or cyclical durability is understood here as a violation of the continuity of the body as a result of the occurrence and accumulation of damage in it during a cyclic change of plastic deformation. The criterion of fatigue failure presented in [1] takes place in the case when the processes of cyclic deformation and cyclic destruction begin simultaneously. In some cases, experimental studies using infrared spectroscopy and acoustic emission methods have shown that damage accumulation begins after a certain incubation time or number of loading cycles after the start of the deformation process. The incubation period significantly depends on the stress amplitude and temperature and is commensurate with the pre-failure period [2,3]. This fact requires the derivation of new ratios of long-term strength and cyclic fatigue. Taking into account the above noted, in [4] theories of deformation and long-term strength of viscoelastic bodies with arbitrary loads in time were developed. These theories also apply to viscoelastoplastic bodies under monotonic loads.

Materials of theoretical research

We will use the concept of accumulation of cyclic damage. Let from the solution of the boundary value problem of plasticity theory at each point $x=(x_1,x_2,x_3)$ of body at any n-cycle in the general case of cyclic loading are known $\sigma_{ij}(x,n)$ $(i,j=1,2,3)$– the greatest differences between the maximum and minimum values of the components of the stress tensor. In addition, let the temperature field be set $T(x,n)$ in n-cycle, which is considered monotonous with respect to n. In further entries of the argument x we omit. Intensity $\sigma_t(n)$ of components $\sigma_{ij}(n)$ will be: $\sigma_t(n)=\left(\frac{3}{2}s_{ij}(n)s_{ij}(n)\right)^{1/2}$, where $s_{ij}(n)=\sigma_{ij}(n)-\sigma(n)\delta_{ij}$, $\sigma(n)=\sigma_{ij}(n)\delta_{ij}/3$, δ_{ij}– Kronecker symbols. We have: $0\le n\le N_*$, where N_* is the desired number of nonstationary loading cycles with an arbitrary change in intensity σ_i from n, for $\sigma_t=\sigma_t(n)\ne const$. Let now the degree of damage in the case of non-stationary cyclic loading is characterized by $\Pi(n)$. Taking into account the normalization we take: $0\le\Pi(n)\le 1$. While maintaining

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the initial natural state of the body $\Pi(n)=0$. In addition, we assume that $\Pi(n)=0$ with incubation numbers of loading cycles $0 \le n \le N'$, где $N' = N'(x_\alpha)$– The desired number of non-stationary loading cycles before damage points (x_α) of body with arbitrary dependence $\sigma_t = \sigma_t(n) \ne const$ and monotonous temperature $T = T(n)$. At the numbers of loading cycles $N' \le k \le N_*$ parameter Π uniquely determined by parameters $\sigma_t(n)$, $T(n)$, moreover, it is a functional, continuous with respect to $\sigma_t(n)$, $T(n)$. In case $0 \le \Pi < 1$, what takes place in the interval $0 \le n < N_*$, body condition is solid. Destruction points (x_α) body occurs at loading numbers $N_*(x_\alpha)$ for $\Pi(N_*)=1$.

Consider the functionality of the form

$$\Pi(n) = H(n-N')\left[\varphi_1(\sigma_t(n),T(n)) + \int_0^n M(n-k)\varphi_2(\sigma_t(k),T(k))dk\right], \tag{1}$$

where $H(n)$– the Heaviside unit function, which is used here to eliminate the negative part of the functional (1). Number N' determined from the condition $\Pi(N')=0$:

$$\varphi_1(\sigma_t(N'),T(N')) + \int_0^{N'} M(N'-k)\varphi_2(\sigma_t(k),T(k))dk = 0. \tag{2}$$

On the condition

$$\lim_{\sigma_t \to 0, T=T^0} \varphi_2(\sigma_t, T) = 0, \tag{3}$$

where T^0– the initial body temperature, at which there are no initial stresses and deformations, the functional (2) characterizes the damage, the properties of which are given above. Condition (3) is dictated by the property Π, where $\Pi = 0$ while maintaining the initial natural state of the body. The ratio (2) determines the number of non-stationary loading cycles N' which begins the process of accumulating damage in the presence of σ_t, T. We call condition (2) a condition of cyclic damage. The number of non-stationary loading cycles N_* s from (1) provided $\Pi(N_*)=1$:

$$\varphi_1(\sigma_t(N_*),T(N_*)) + \int_0^{N*} M(N_*-k)\varphi_k(\sigma_t(k),T(k))dk = 1. \tag{4}$$

The relation (4) is a condition of cyclic fatigue or cyclic durability at given σ_t and T.

We now define the functions φ_1, φ_2 and M. They are assumed to be characteristic functions of the material. Therefore, when determining them, you can use the results of experiments that are represented by the ratios [3]: $N_1 = N_1(\sigma_0, T_0)$, $N_0 = N_0(\sigma_0, T_0)$, где $T_0 = const$, $\sigma_0 = const$. Parameter σ_0 is a double amplitude of stresses in the case of cyclic loading according to the law of symmetric cycles of stresses, N_1 and N_0– experimentally determined characteristic functions of the material: N_1– the number of stationary loading cycles preceding the damage process, N_0– the number of stationary cycles to failure at various constants σ_0 and T_0. From relations (2) and (4) with constant σ_0 and T_0 (in this case $N' \to N_1$, $N_* \to N_0$) define functions φ_1 and φ_2:

$$\varphi_1(\sigma_0, T_0) = -\frac{\int_0^{N_1} M(\xi) d\xi}{\int_{N_1}^{N_0} M(\xi) d\xi}, \qquad \varphi_2(\sigma_0, T_0) = \frac{1}{\int_{N_1}^{N_0} M(\xi) d\xi}. \tag{5}$$

As we see, φ_1 and φ_2 are connected with functions N_0, N_1 and M. The properties of functions N_i $(i = 0,1)$ are: $N_i(\sigma, T) > 0$, $\partial N_i / \partial \sigma < 0$, $\partial N_i / \partial T < 0$. n this case, the choice of function $M(n)$ ictated by the fulfillment of condition (3). We apply the average theorem to the integral in the second formula (5). We will get

$$\varphi_2(\sigma_0, T_0) = \{N_0(\sigma_0, T)[1 - A(\sigma_0.T_0)]M(\theta N_0(\sigma_0, T_0))\}^{-1},$$

where $0 \le \theta \le 1$, $A(\sigma_0, T_0) = N_1(\sigma_0, T_0) / N_0(\sigma_0, T_0)$, $0 \le A(\sigma_0, T_0) < 1$.

rom the obtained ratio, taking into account it follows that condition (3) will be satisfied if $M(n)$ will have n^m, where $m > -1$. Based on this, we take: $M(n) = M_0 n^m$, where $M_0 = const > 0$, $m = const > -1$. Given this formula for $M(n)$ from relations (5) we define φ_1 and φ_2. After replacement σ_0 on $\sigma_t(n)$, T_0 on $T(n)$ in the resulting expressions for φ_1 and φ_2, convert functional (1) to mind:

$$\Pi(n) = H(n - N')\left[-\frac{N_1^{1+m}(\sigma_t(n), T(n))}{N_0^{1+m}(\sigma_t(n), T(n)) - N_1^{1+m}(\sigma_t(n), T(n))} + \right.$$

$$\left. + (1+m)\int_0^n \frac{(n-k)^m dk}{N_0^{1+m}(\sigma_t(k), T(k)) - N_1^{1+m}(\sigma_t(k), T(k))}\right]. \tag{6}$$

Conditions (2) and (4) are:

$$\frac{N_1^{1+m}(\sigma_t(N'), T(N'))}{N_0^{1+m}(\sigma_t(N'), T(N')) - N_1^{1+m}(\sigma_t(N'), T(N'))} = (1+m)\int_0^{N'} \frac{(N'-k)^m dk}{N_1^{1+m}(\sigma_t(k), T(k)) - N_1^{1+m}(\sigma_t(k), T(k))}, \tag{7}$$

$$\frac{N_0^{1+m}(\sigma_t(N_*), T(N_*))}{N_0^{1+m}(\sigma_t(N_*), T(N_*)) - N_1^{1+m}(\sigma_t(N_*), T(N_*))} = (1+m)\int_0^{N_*} \frac{(N_*-k)^m dk}{N_1^{1+m}(\sigma_t(k), T(k)) - N_1^{1+m}(\sigma_t(k), T(k))}, \tag{8}$$

Define unknown constant m. With $\sigma_t = \sigma_0 = const$, $T = T_0 = const$ from (6) follows $(N_* \to N_0, N' \to N_1, \Pi \to \Pi_0)$:

$$\Pi_0(\sigma_0, T_0, n) = H(n - N_1(\sigma_0, T_0))\frac{n^{1+m} - N_1^{1+m}(\sigma_0, T_0)}{N_0^{1+m}(\sigma_0, T_0) - N_1^{1+m}(\sigma_0, T_0)}. \tag{9}$$

Relation (9) is the equation of experimental damage accumulation curves depending on the number of stationary loading cycles n for $\sigma_0 = const$, $T_0 = const$. Let such curves be known from experience. In this case, (9) is used to determine the constant m. Parameter m an be determined using the results of other experiments. For example, let the cyclic loading of an experimental sample occur according to the following program: $T = T_0 = const$, $\sigma_t(n) = n\sigma_0$, where $\sigma_0 = const$, specified doubled voltage amplitude at the first symmetric cycle. At the time of destruction we have: $\sigma_t(N_*) = N_* \sigma_0 = \sigma_b$, where σ_b is a double voltage amplitude in the cycle N_*, which can be found experimentally. Wherein $N_* = \sigma_b / \sigma_0$. We take into account marked in the ratio (8):

$$\frac{N_0^{1+m}(\sigma_b, T_0)}{N_0^{1+m}(\sigma_b, T_0) - N_1^{1+m}(\sigma_b, T_0)} = (1+m) \int_0^{\sigma_b/\sigma_0} \frac{\left(\frac{\sigma_b}{\sigma_0} - k\right)^m dk}{N_0^{1+m}(k\sigma_0, T_0) - N_1^{1+m}(k\sigma_0, T_0)}.$$

The resulting ratio also allows you to define constant *m* with known functions N_0 и N_1.

Relation (7) is a condition of cyclic damage and determines the desired number of non-stationary cycles N' which begins the process of damage accumulation, relation (8) is a condition of cyclic durability and determines the desired number of non-stationary cycles N_* to failure with arbitrary dependence $\sigma_t \sim n$ and monotonous $T = T(n)$. In the case $N_1(\sigma_t, T)/N_0(\sigma_t, T) \equiv A = const$, and also in the case of $N_1 \equiv 0$ condition (8) gives the same result, which coincides with the fatigue condition presented in [1].

$$(1+m)\int_0^{N_*} \frac{(N_* - k)^m}{N_0^{1+m}(\sigma_t(k), T(k))} = 1.$$

Under the condition $N_1 / N_0 = A = const$ cyclic damage condition (7) goes to the ratio

$$(1+m)\int_0^{N'} \frac{(N' - k)^m dk}{N_0^{1+m}(\sigma_t(k), T(k))} = A^{1+m},$$

kinetic equation (6) - to the relation

$$\Pi(n) = \frac{H(n - N')}{1 - A^{1+m}} \left\{ -A^{1+m} + (1+m)\int_0^n \frac{(n-k)^m}{N_0^{1+m}(\sigma_t(k), T(k))} \right\}$$

Now using the results of fatigue experiments, which are presented in [3] for steel grade EI 437 at a temperature of 1073K, we define the functions N_0, N_1 and constant m. One of the variants of the approximation of functions N_0 and N_1 maybe the following:

$$N_i = N_{is} \exp\left[\beta_i \left(1 - \frac{\sigma_t}{\sigma_s}\right) + b_i \left(1 - \frac{T}{T_s}\right)\right], \ (i = 0,1). \tag{10}$$

At the same time we consider $\sigma_t \neq 0$. Here $\beta_0, b_0, \beta_1, b_1$ are constants; $\sigma_s = const$ - stresses, $T_s = const$ - temperature, which are dimensionless parameters, and are selected from the range of variation σ_t and T as their greatest values, N_{0s} and N_{1s} - the number of loading cycles to failure and to the onset of damage in an experimental sample with $\sigma_t = \sigma_s$, $T = T_s$. For quantities σ_s and T_s will take: $\sigma_s = 600$ MPa, $T_s = 1073$K. It turned out: $N_{0s} \approx 1{,}4 \cdot 10^9$ cycle, $N_{1s} \approx 4{,}1 \cdot 10^8$ cycle, $\beta_0 \approx 1{,}8$, $\beta_1 \approx 1{,}6$, $m \approx 0{,}64$. Note that according to the formulas for N_0 and N_1, when taking into account the found constants, in this case can be taken: $N_1 / N_0 \approx 0{,}35$. The reverse recalculation of the cyclic strength curves showed a stable deviation of the calculated data from the experimental ones, which was about 6%.

If the duration of k-cycle is $t_k - t_{k-1}$ $(t_0 = 0)$, then time t_* and number of cycles N_* before destructionas well as time t' and number of cycles N' before the formation of injuries in the body are related according to the ratios:

$$t_* = \sum_{k=1}^{N*}(t_k - t_{k-1}),\ t' = \sum_{k=1}^{N'}(t_k - t_{k-1}).$$

Finally, we note that the fatigue ratios presented take into account a physically important phenomenon — the influence of the history of cyclic loading on the cyclic strength of bodies.

Conclusions

The criteria of non-stationary cyclic damage and durability are presented, which, in the general case of variable loads, allow to determine the number of non-stationary loading cycles, respectively, before damage and before the destruction of bodies.

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