# To estimate the rigidity of bending reinforced concrete elements based on dependence of length macro trays from load

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# To estimate the rigidity of bending reinforced concrete elements based on dependence of length macro trays from load

**O V Radaikin[1], L A Sharafutdinov[1], L S Sabitov[2], Yu M Strelkov[2], N F Kashapov[2]**
[1] Kazan State University of Architecture and Civil Engineering
[2]Federal State Autonomous Educational Institution of Higher Education "Kazan (Volga Region) Federal University", Kazan, Kremlevskaya street, 18, 420008, Russian Federation

l.sabitov@bk.ru

**Abstract.** The purpose of this work is to make proposals for calculating the stiffness of normal sections of bending reinforced concrete elements, taking into account the work of the stretched concrete above the macrocrack and stress concentration at its top. In the article, such an account is made by the methods of fracture mechanics using the stress intensity factor $K_I$. One of the stages of calculation consists in the finite-element modeling of macrocracks in *ANSYS*.

## Introduction

In the design of reinforced concrete elements they are allowed to work with macrocracks [1]. At the same time, for structures with a span of more than six meters, compliance with the deflection limitation increases the requirements for the reflection of the actual work of the material.

The issues of calculating the deformations of reinforced concrete bent elements, taking into account the length $l$ of the stretched concrete over the macro-crack $M$ [2], were considered by Ya.M. Nemirovsky. The latter circumstance, and in itself, the complexity of such an approach, as noted in [3], did not allow him at that time to find a wide practical application. In addition, later experiments showed a non-linear relationship between $l$ and $M$ [4].

The proposal of Ya.M.Nemirovsky is developed in our work in the following directions:

1) the use of a theoretically sound approach to fracture mechanics made it possible to obtain an analytical dependence of the crack length on the bending moment, which covers a wider range of parameter values in comparison with the data [2];
2) Our approach additionally takes into account the stress concentration at the tip of the macrocrack, that is, the nonlinear relationship between stresses and strains in the area under consideration.

In the present work, the following prerequisites are used:

a) the macrocracks arising in the stretched zone of concrete of the bent elements - through;
b) short-term static single loading is considered;
c) the beginning of the formation of the first macrocrack condition $M=M_{crc}$ [1];

d) concrete is a quasi-brittle material [5];
e) the macrocrack in the element is modeled by the Irwin-Ourov mathematical sharp section, and the stress-strain state at its top is determined by the stress intensity factor KI;
f) we consider concrete and reinforcement to be isotropic and elastically deformable materials - the initial connection between stresses and strains is described by Hooke's law. The non-linearity of the deformation of a reinforced concrete element is caused by the spread of a macrocrack in the stretched concrete;
g) deformation of short-term creep and shrinkage are taken into account through the given modulus of deformation $E_{b,red}$, determined according to [1].

Imagine a formula for determining the stiffness of reinforced concrete section in the following form:

$$B_f = E_{b,red} I_{red} \tag{1}$$

where $E_{b,red} = R_{bn} / \varepsilon_{b1,red}$ – reduced modulus of concrete deformation [1], $I_{red}$ – the reduced moment of inertia of a reinforced concrete section relative to its center of gravity, determined taking into account the cross-sectional area of the concrete of a compressed and stretched zone and the cross-sectional area of the tensioned reinforcement with a reduction factor $\alpha_{s2} = E_s / E_{b,red}$:

$$I_{red} = I_b + \alpha_{s2} I_s \tag{2}$$

$$I_s = A_s (y-a)^2 \tag{3}$$

$$I_b = \frac{b(h-l)^3}{12} + b(h-l)\left[x_{ct} - \frac{(h-l)}{2}\right]^2 \tag{4}$$

$$x_{ct} = \frac{\frac{b}{2}(h-l)^2 + \alpha_{s2} A_s (h-a)}{b(h-l) + \alpha_{s2} A_s} \tag{5}$$

where $l$ – crack length; $x_{ct}$ – total concrete height of compressed and stretched zone ($x_{ct} = x_c + x_t$); $I_b$ – moment of inertia of concrete; the remaining designations are taken as in the Manual [1].

Figure 1 shows a diagram of the distribution of stresses and forces in reinforced concrete elements after the appearance of cracks in them.

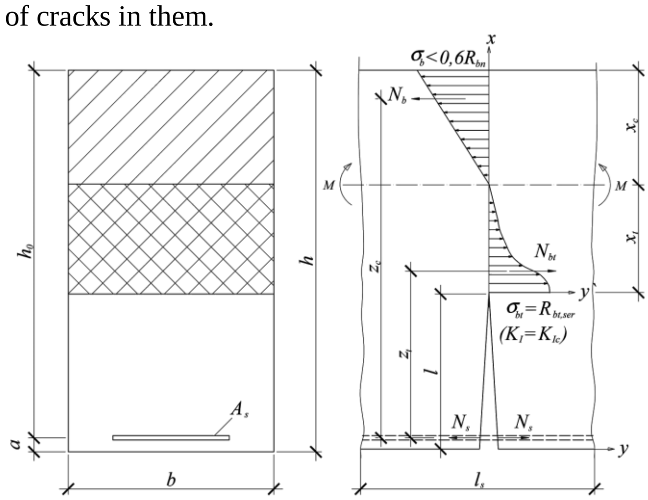


Figure 1. Scheme of the distribution of stresses and forces in reinforced concrete elements after the appearance of cracks in them

We write the equilibrium equations for the considered elements:

$$\sum M_{N_s} = 0 \quad => \quad M+N_b z_c-N_{bt} z_t=0 \tag{6}$$

$$\sum Y = 0 \quad => \quad N_b-N_{bt}+N_s=0 \tag{7}$$

where the resultant efforts $N_b = \frac{1}{2}\sigma_b x_c b$, $N_{bt} = b\int_l^{l+x_t}\sigma_{bt}dx$, $N_s = \sigma_s A_s$ and the corresponding distance between them $z_c = h_0 - \frac{1}{3}x_c$, $z_t = l - a + \frac{\int_l^{l+x_t}\sigma_{bt}xdx}{\int_l^{l+x_t}\sigma_{bt}dx}$. Here $x$ – the current coordinate of the point along the vertical axis with the origin located on the lower stretched face of the concrete.

According to clause “e”, the calculated assumptions for stresses in tensile reinforcement should not exceed its yield strength.:

$$\sigma_s = \frac{M - N_{bt}(z_c - z_t)}{A_s z_c} \le R_s \tag{8}$$

The stress pattern of the concrete of the compressed zone is assumed to be triangular, which is true when $\sigma_b < 0{,}6R_{bn}$:

$$\sigma_b = \frac{M}{I_{red,l}}(x - l - x_t), \quad l + x_t \le x \le h \tag{9}$$

where $I_{red,l}$ – the reduced moment of inertia of the reinforced concrete section relative to the crack tip, taking into account the cross-sectional area of the concrete of the compressed and stretched zone and the cross-sectional area of the tensioned reinforcement.

The stresses in the concrete of the stretched zone consist of two solutions — singular (first term) and regular (second term):

$$\sigma_{bt} = \frac{K_I}{\sqrt{2\pi(x-l)}} + \frac{M}{I_{red,l}}(x - l), \quad l \le x \le l + x_t \tag{10}$$

where $K_I$ – stress intensity factor ($\sigma_{bt}$ and $K_I$ are determined numerically in the PC *ANSYS*).

The start condition of the macrocrack is written in the form:

$$K_I = K_{Ic},\ \sigma_{bt}\big|_{x=l} = R_{bt,ser} \tag{11}$$

where $K_{Ic}$ - critical stress intensity factor (determined from an experiment on standard samples [6]).

The joint solution of equations (6) - (11) allows determining the stress-deformed state of a bent element taking into account the work of the stretched concrete over the macrocrack and concentration at its top, including the dependence of the length of the macrocrack on the load.

As a result of our theoretical and experimental numerical studies, the following formula was proposed for it:

$$M(l) = M_0 \frac{\Phi_I(l)}{\Phi_I(l_{ult})},\ 0 \le l < l_{ult} \tag{12}$$

where $l$, $l_{ult}$, – respectively, the current and maximum length of the crack in the section; $\Phi_I(l)$- calibration function, which depends on several factors - it can be determined numerically by the results of modeling the elastic work of an element with a crack under the action of a unit load ($M_1$=1MH×м) - the values of this function depending on the length of a macrocrack, the class of concrete strength, percent reinforcement and modulus of elasticity of reinforcement were tabulated and tabulated (to reduce the volume of the article we do not give here); $M_0$ is the maximum experimental value of the bending moment at which the growth of a transverse crack stops, as it rests against the compressed zone of concrete, and the stretched concrete is almost completely shut off from work. The value of this moment $M_0$ can be obtained by the experimental formula of A.S. Zalesov [7]:

$$M_0 = M_{crc} + \psi R_{bt,m} bh^2$$
$$\psi = 15 \frac{A_s}{bh_0} \frac{E_s}{E_b} \le 0,6 \qquad (13)$$

Expressing from formula (12) $l$, you can build the desired dependence:

$$l=f(M, E_b, \mu, M_0, l_{ult}) \qquad (14)$$

To do this, first calculate the $l_{ult}$ by the formula, which we present here without output:

$$l_{ult} = h - h_0\left(1,5 - \sqrt{10\alpha_0 - 2,25}\right) \qquad (15)$$

where $h_0$ – working section height, $\alpha_0 = \frac{M_0}{R_b bh_0^2}$.

Then, using the above tables, I determine the value $\Phi_I(l_{ult})$ and by formula (12) calculate the value $\Phi(l) = \frac{M}{M_0}\Phi(l_{ult})$, according to which the value of the crack length is found from the same tables $l$.

After that, using the expressions (1) - (5), calculate the stiffness and then determine the deflection of the structure by the already known formulas [1].

To find the calibration function $\Phi_I$, we carried out numerical studies of flexural reinforced concrete elements with a transverse crack. In the course of the experiment, 5 of the most significant factors varied - the plan of the experiment is shown in Figure 2. The simulation was carried out in ANSYS using the methodology [8].

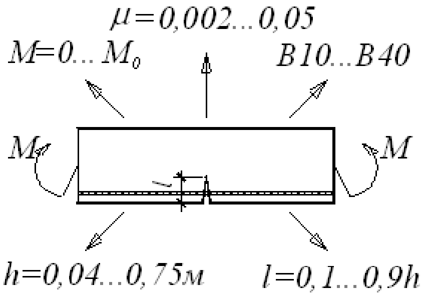


Figure 2. Diagram of numerical studies of reinforced concrete elements

According to the results of the studies carried out in Figure 3, the dependence (14) in dimensionless coordinates is constructed by a solid line for one item. On the same graph, the dots show the experimental data of K A Piradov [4] for a reinforced concrete beam of size $b \times h \times L$ 75×100×840mm from heavy concrete B20. Reinforcement percentage $\mu = 1,8\%$. The value of the protective layer of reinforcement $a$=25mm.

Other beams considered in [4] were also to be compared. As a result, the following discrepancies were found in the results obtained by the author, with the experiments of K A Piradov: no more 8% for μ=0,0089, 10% for μ=0,0180 and 13% for μ=0,0279.

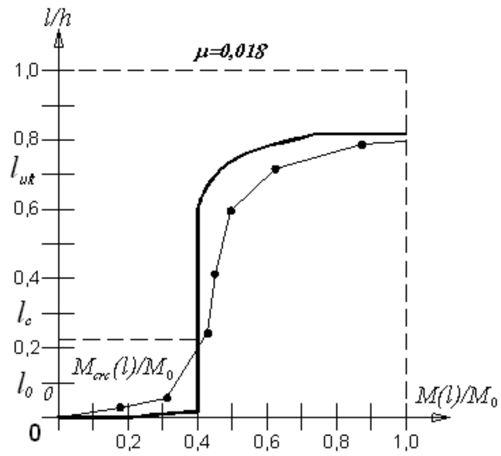


Figure 3. The crack length dependence on the load for a reinforced concrete beam with the percentage of reinforcement μ=0,018: mugs - according to the experiments of K A Piradov [4], the solid line - the results of the author

The volume of the article did not allow to reflect the influence of the initial discontinuities of the material, the prefracture zone at the top of the macrocrack and the coefficient of elasticity of concrete of different strength.

Our subsequent publications will be devoted to this. Further development of the work should be the integration of the results obtained here and the diagram method for calculating reinforced concrete structures [9, 10].

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