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Mathematical modeling of heat treatment of three-layered material

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Abstract. The paper presents mathematical models of the thermal behavior of the three-layered material in the presence of a thin air gap between the two of them. Both convective and radiative-convective heat transfers with the environment are considered. As a result of an acceptance of a set of assumptions, the relationships to estimate the change of the material temperature in the layers over time were found for each of these cases.

1. Introduction
The material under consideration is a three-layer plane wall (strip) infinitely extended in the longitudinal direction. The first layer has a thickness $h_т$, its material properties are characterized by density $\rho_т$, thermal conductivity $\lambda_т$, specific heat capacity $c_т$. The second layer is a relatively thin fabric with a thickness h_m freely lying on the first layer. In the general case, its material is porous, and you should distinguish between macroporosity characterizing the cavities between the threads of the fabric and the microporosity of the threads material.

2. Body text
Both the total porosity ε_m and the average (equivalent) thermophysical parameters – density ρ_m, thermal conductivity coefficient λ_m, and specific heat capacity c_m – will possibly be anisotropic. The last layer is a layer of polymeric particles discretely located on the surface of the fabric. It is of great difficulty to take into account the effect of these discrete particles on the strip's thermal behavior. To avoid this, we replace the particles with an equivalent continuous layer with thickness h_p. Let us suppose that the parameters of this layer are close to the parameters of the material of the particles: density ρ_p, thermal conductivity λ_p, and specific heat capacity c_p.

The following important circumstance characterizing the structure of the strip under consideration is the conditions thermal contact of its layers. It is believed that the particles are contiguous to the fabric surface rather tightly there is ideal thermal contact between the materials of the introduced continuous layer and the reinforcing fabric. Since the fabric lies freely on the first layer its surface has a certain relief, an interlayer of gas (air) is formed in the interfacial region of the fabric layer and the first layer. If the radiative (radiant) heat transfer between the surfaces separated by a gas interlayer is not taken into account then the thermal conductivity of the contact is $1/R_k = 1/R_f + 1/R_g$, where $1/R_f$ is the thermal conductivity of the actual contact, $1/R_g$ is the conductivity of the gas interlayer.

Let us suppose that the material of this interlayer (conditional transition layer) with thickness h_c, is a mixture of the first layer, fabric, and air their volume concentration is $\beta_т$, β_m and β_a ($\beta_т + \beta_m + \beta_a = 1$). The averaged thermophysical parameters of this material are $\rho_c = \beta_т\rho_т + \beta_m\rho_m + \beta_a\rho_a$, $\lambda_c = 1/\left(\beta_т/\lambda_т + \beta_m/\lambda_m + \beta_a/\lambda_a\right)$, $c_c = \beta_т c_т + \beta_m c_m + \beta_a c_a$, the index "a" is used to denote the characteristics of air. Hence the thermal resistance of the considered layer is $R_c = h_c/\lambda_c$ its value is close to $R_к$.

Next, we single out the following heat transfer variants of the strip surfaces with the environment: double-sided convective and radiative-convective from the side of polymeric particles, convective on the outer surface of the first layer.

Convective heat transfer of the strip. Provided that the surface temperature of the strip that is in contact with the gaseous medium is equal to some average value T_w the thermophysical parameters of the gas weakly depend on the temperature at the surface of the strip's test section of length l, we accept [1, 2] that the average value of the convective heat transfer coefficient is $\bar{\alpha}_k = 0.66 k_л \left(\lambda_g/l\right) \mathrm{Re}^{0.5}\,\mathrm{Pr}^{1/3}$ for laminar gas flow and $\bar{\alpha}_k = 0.04 k_т \left(\lambda_g/l\right) \mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.43} \left(\mathrm{Pr}/\mathrm{Pr}_w\right)^{0.25}$ for turbulent one. Here, k_l, $k_т$ – correction coefficients, λ_g – thermal conductivity coefficient of the surrounding gas environment, $\mathrm{Re} = wl/\nu_g$, $\mathrm{Pr} = \nu_g/a_g$, $a_g = \lambda_g/\left(\rho_g c_g\right)$; ρ_g, c_g, ν_g – density, specific heat capacity and kinematic viscosity of the gas; $w = \left|u_т \pm u_g\right|$, $u_т$, u_g – motion speed of the strip and gaseous environment.

Because the strip is very long in the longitudinal direction thermal resistance in this direction is substantially higher than the resistance across the strip, therefore, the heat transfer in the transverse direction will be predominant in the strip. Considering the above circumstance, assuming that internal sources (sinks) of heat are absent the thermophysical parameters of the material of the layers change only slightly with temperature the corresponding thermo-adjoint problem is written. The main difficulties in solving this problem are connected with the mathematical description of heat distribution at the initial stage of heating (cooling) of the strip also called the inertial period. This period is characterized by the gradual heating up or cooling of the material of the layers with the boundary conditions playing a large role In our opinion, since the strip thickness is, in general, comparatively small the thermal conductivity of the material of the first layer, fabric and polymeric particles (excluding the intermediate layer) is quite large this period is not of great interest. The regular period follows the inertia period. During this period the entire strip is involved in the hearting (cooling) process the heating (cooling) rate in different layers is assumed to be the same and equal to a certain average speed.

Focusing on the regular period, we consider two mechanisms of the thermal behavior of the material of the layers. *In the first case,* we assume that in all strip layers the average temperature of the material is almost the same, characterized by the value $\bar{T}$. Accordingly, the average density of the strip material is

$$\bar{\rho} = \gamma_т\rho_т + \gamma_c\rho_c + \gamma_m\rho_m + \gamma_p\rho_p,$$

the average specific heat capacity $\bar{c} = \gamma_т c_т + \gamma_c c_c + \gamma_m c_m + \gamma_p c_p$, where $\gamma_т = h_т/h_0$, $\gamma_c = h_c/h_0$, $\gamma_m = h_m/h_0$, $\gamma_p = h_p/h_0$, $h_0 = h_т + h_c + h_m + h_p$.

From the condition of the heat balance of a strip section with the length l_0 over time $d\tau$ it follows that the thermal energy change in the volume of the strip $V_0 = l_0 h_0 \cdot 1$ of a unit width due to the heat transfer with the surrounding gaseous medium will be:

$$\bar{\rho} \cdot \bar{c} V_0 d\bar{T} = \left[\bar{\alpha}_{k1} \left(T_{g1} - \bar{T} \right) - \bar{\alpha}_{k2} \left(T_{g2} - \bar{T} \right) \right] S_0 d\tau \,, \tag{1}$$

where $S_0 = l_0 \cdot 1$ – a unit width strip surface area; T_{g1}, T_{g2} – the average temperature of the gas medium in contact with the first and the third layers of the strip, respectively; $\bar{\alpha}_{k1}$, $\bar{\alpha}_{k2}$ - the average convective heat transfer coefficients on opposite sides of the strip.

Solving equation (1) we find

$$\bar{T} = \bar{T}\left(\tilde{\tau}\right) = T_0 \exp\left(-\tilde{\tau}\right) + \left(\frac{\bar{\alpha}_{k1} T_{g1} + \bar{\alpha}_{k2} T_{g2}}{\bar{\alpha}_k} \right) \left(1 - \exp\left(-\tilde{\tau}\right)\right). \tag{2}$$

Here $\tilde{\tau} = \tilde{\alpha}_k / \left(\bar{\rho} \cdot \bar{c} \cdot h_o \right)$, $\tilde{\alpha}_k = \bar{\alpha}_{k1} + \bar{\alpha}_{k2}$; T_0 - the average initial temperature of the strip layers.

In particular, for small values of the argument $\tilde{\tau}$ ($\tilde{\tau} \ll 1$) the formula (2) is simplified as follows:

$$\bar{T} = \bar{T}\left(\tilde{\tau}\right) \approx T_0 + \left(\bar{\alpha}_{k1} \Delta T_{g1} + \bar{\alpha}_{k2} \Delta T_{g2} \right) \tilde{\tau} / \tilde{\alpha}_k \,, \tag{3}$$

where $\Delta T_{g1} = T_{g1} - T_0$, $\Delta T_{g2} = T_{g2} - T_0$.

On the contrary, when the value $\tilde{\tau}$ is high ($\tilde{\tau} \gg 1$) (long time *τ*) the strip thickness *h₀*, average density, and specific heat capacity of its material are small; the influence of the multiplier $\exp\left(-\tilde{\tau}\right)$ on the temperature $\bar{T}$ is insignificant.

As a consequence, approximately

$$\bar{T} \approx \left(\bar{\alpha}_{k1} T_{g1} + \bar{\alpha}_{k2} T_{g2} \right) / \tilde{\alpha}_k \,, \tag{4}$$

the average temperature of the strip is determined by mainly the ambient temperature and the intensity of the convective heat transfer on its surfaces.

In the second case, we assume that the heat transfer in the layers is carried out as a transfer in the “mixture” consisting of materials of the first layer, intermediate layer, fabric and polymeric particles in the absence of sources, sinks of heat. The thermal behavior of this “mixture” is described by the equation:

$$\partial T / \partial \tau = \bar{a}\, \partial^2 T / \partial y^2 \,, \tag{5}$$

where thermal diffusivity $\bar{a} = \bar{\lambda} / \left(\bar{\rho} \bar{c} \right)$, $\bar{\lambda} = 1 / \left(\gamma_{т} / \lambda_{т} + \gamma_c / \lambda_c + \gamma_m / \lambda_m + \gamma_p / \lambda_p \right)$ – the average coefficient of molecular thermal conductivity of the material of the mixture under consideration, $T = T\left(y, \tau\right)$ ($0 \le y \le h_0$) – the strip temperature.

At the strip borders when

$$y = 0: \; q_1 = -\bar{\lambda} \left(\frac{\partial T}{\partial y} \right) = \bar{\alpha}_{k1} \left(T_{g1} - T \right), \tag{6}$$

$$y = h_0: \; q_2 = -\bar{\lambda} \left(\frac{\partial T}{\partial y} \right) = \bar{\alpha}_{k2} \left(T - T_{g2} \right); \tag{7}$$

at the initial time $\tau = 0$

$$T = T_0 \,. \tag{8}$$

The problem specified in this way (5) – (8) is solved by the variable separation method. Omitting the details of this solution, we present only the final dependencies for estimating the strip temperature for small values of the parameter $F_0 = \bar{a} \tau / h_0^2$:

$$T \approx A_1 + A_2 \bar{y} + \left(B_1 \sin\left(\beta \bar{y}\right) + B_2 \cos\left(\beta \bar{y}\right) \right) \left(1 - \frac{\tilde{\alpha}_k \tau}{h_0 \bar{\rho} \cdot \bar{c}} \right), \tag{9}$$

and for high F_0:

$$T = A_1 + A_2 \bar{y} \approx \left(\bar{\alpha}_{k1} T_{g1} + \bar{\alpha}_{k2} T_{g2} \right) / \tilde{\alpha}_k \,. \tag{10}$$

Here $\bar{y} = y/h_0$, coefficients

$$A_1 \approx \left(\alpha_{k1}^* T_{g1} + \alpha_{k2}^* T_{g2}\right)\Big/\alpha_k^*,\ A_2 \approx \alpha_{k1}^* \alpha_{k2}^* \left(T_{g2} - T_{g1}\right)\Big/\alpha_k^*,$$

$$B_1 = B_1(\beta) = \alpha_{k1}^* \left(T_0 - A_1\right)\Big/\left(\beta \alpha_{k0}^*\right),\ B_2 = \left(T_0 - A_1\right)\Big/\alpha_{k0}^*; \tag{11}$$

parameters $\beta \approx \sqrt{\alpha_k^*}$, $\alpha_k^* = \alpha_{k1}^* + \alpha_{k2}^*$, $\alpha_{k0}^* = 1 + 0{,}5\alpha_{k1}^*$, $\alpha_{k1}^* = \bar{\alpha}_{k1} h_0 / \bar{\lambda}$, $\alpha_{k2}^* = \bar{\alpha}_{k2} h_0 / \bar{\lambda}$.

We note that dependence (10) coincides with (4).

Radiative-convective heat transfer is realized from the side of polymeric particles located on the fabric. It is assumed that the working surface of the heating element is flat, located parallel to the flat surface of the fabric its longitudinal (relative to the fabric) size is $a_и$, and transverse is $b_и$, respectively, the area is $a_и b_и$. This element is fixed the fabric moves with the speed $u_т$ relative to it. The gaseous medium between the element and the fabric is considered optically transparent (does not absorb radiation) it moves at a speed $w = u_т \pm u_g$ with respect to the fabric, its average temperature is T_g. The surfaces of the heating element and the fabric are grey the absorption coefficient does not depend on the wavelength of the incident radiation, accordingly, on temperature and other characteristics of the radiating body. The emissivity factor (coefficient of radiation) of the heating element surface is denoted by ε_1, the surfaces of the fabric with polymeric particles – ε_2.

Next, we single out two variants of the thermal contact of the heating element with the surface temperature T_1 and area $S_1 = S_и$, and the fabric section with the temperature T_2 and area S_2. In the first case, we assume that the areas are commensurable ($S_1 \approx S_2$) and reasonably large. In the second case, the heating element area is smaller than the area of the fabric section ($S_1 < S_2$).

According to the solutions provided in the literature in both cases the intensity of the resulting radiative heat flux is described by the equation:

$$q_r = \sigma_0 \varepsilon_{\text{reduced}} \left(T_1^4 - T_2^4\right), \tag{12}$$

where $\sigma_0 = 5.67 \cdot 10^{-8}\ \text{W}/\left(\text{m}^2 \cdot \text{K}^4\right)$ – Stefan-Boltzmann constant, in the first case, the reduced emissivity factor of the system under consideration $\varepsilon_{\text{reduced}} = 1\big/\left(1/\varepsilon_1 + 1/\varepsilon_2 - 1\right)$, in the second case, the reduced emissivity factor is approximately equal to

$$\varepsilon_{\text{reduced}} = 1\Bigg/\left(1/\varepsilon_1 + \frac{S_1}{S_2}\left(1/\varepsilon_2 - 1\right)\right).$$

It should be noted that for estimating ε_2 you can use the relationship:

$$\varepsilon_2 = \varepsilon_{2p}\zeta_p + \varepsilon_{2m}\left(1 - \zeta_p\right).$$

Here $\zeta_p = \pi d_p^2 \big/ \left(4 S_{p0}^2\right)$, ε_{2p}, ε_{2m} – the emissivity factor of the particles and fabric surfaces, respectively.

At radiative-convective heat transfer which is often observed in engineering the total heat flow density is

$$q = q_r + q_k,$$

where q_k – specific heat flux which characterizes the convective heat transfer of the fabric surface with particles and gaseous medium ($T_{g2} = T_g$),

$$q_k = \bar{\alpha}_{k2}\left(T_2 - T_{g2}\right).$$

In engineering practice it is convenient to determine the radiation heat transfer coefficient in the same way as the convection coefficient is determined, assuming that [3, 4]

$$q_r = \alpha_r \left(T_1 - T_2\right). \tag{13}$$

From a comparison of relationships (12) and (13) it follows that

$$\alpha_r = \sigma_0 \varepsilon_{\text{reduece d}} \left(T_1^4 - T_2^4\right) / \left(T_1 - T_2\right) = \sigma_0 \varepsilon_{\text{reduece d}} \left(T_1^3 + T_1^2 T_2 + T_1 T_2^2 + T_2^3\right).$$

Since the temperature $T_1 > T_2$, approximately

$$\alpha_r \approx \sigma_0 \varepsilon_{\text{reduced}} T_1^3 \left(1 + T_2 / T_1\right). \quad (14)$$

Replacing in (14) the current temperature T_2 with the average temperature over the heating time of particles $\tilde{T}_2 = 0.5\left(T_{21} + T_{22}\right)$ (T_{21}, T_{22} – temperature T_2 before and after the heating end (given by technology)) we obtain the following estimate:

$$\alpha_r \approx \sigma_0 \varepsilon_{\text{reduced}} \tilde{T}_{12},$$

where multiplier is $\tilde{T}_{12} = \left(1 + \tilde{T}_2 / T_1\right) T_1^3$.

Taking into account that the radiative heat flux enters the strip material from the side of the particles layer, we write down:

$$q = \alpha_{rk} \left(T_2 - T_{g2}\right) - \alpha_r \Delta \tilde{T}. \quad (15)$$

Here $\alpha_{rk} = \alpha_r + \bar{\alpha}_{k2}$, $\Delta \tilde{T} = T_1 - T_{g2}$ – temperature difference which in this problem is considered to be a known constant value.

The representation of the total specific heat flux acting on the strip under consideration in the form (15) significantly simplifies the integration of the heat problem [5-8] since you can use the already available solutions for the average temperature $\bar{T}$ (2) and the temperature $T = T\left(\bar{y}, F_0\right)$ (9), (10) where the integration constants and parameter β are determined by the relationships (11).

The value $\tilde{\alpha}_k$ in (2) should be substituted by

$$\tilde{\tilde{\alpha}}_k = \bar{\alpha}_{k1} + \alpha_{rk},$$

the integration constant A_1,and parameter β (11) written in the form:

$$A_1 = \left(\alpha_{k1}^* \left(1 + \alpha_{rk}^*\right) T_{g1} + \alpha_{rk}^* T_{g2} + \alpha_r^* \Delta \tilde{T}\right) / \left(\alpha_{k1}^* + \alpha_{k1}^* \alpha_{rk}^* + \alpha_{rk}^*\right), \; \beta = \sqrt{\alpha_{k1}^* + \alpha_{rk}^*},$$

where $\alpha_{rk}^* = \alpha_{rk} h_0 / \bar{\lambda}$, $\alpha_r^* = \alpha_r h_0 / \bar{\lambda}$.

Accordingly, when determining A_2, B_1 these values of A_1 and β are used.

3. Conclusions

Under certain assumptions, quite simple relationships that allow estimating the thermal behavior dynamics of the three-layered strip taking into account the thermal resistance of the air interlayer and other parameters at its convective and radiative-convective heat transfer with the environment were obtained. These relationships can be useful when determining the rational technological regimes of heat treatment of the material under consideration.

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