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Granular computing and reasoning based on rough logic

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Abstract: This paper makes an effective combination of rough set and granular computing, and proposes a logic inference form based on rough granular computing. The advantage of rough logic granular computing lies in its demonstration form which is given by array, it can illustrate the formula semantic more directly, and furthermore the reasoning results can be got according to the data information searched momentarily.

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1. Introduction

The expressive method used by granular computing^{[1][2]} which is based on rough set^{[3][4][5][6][7]} is very special. Each granule is expressed in a binary set. The first element of the set is a logic formula, and the second element is a semantic set corresponding to the formula.

Because the two elements of this binary set make up a cohesive whole that shows both syntax and semantic set, so it is called granule. It is both logical and set theory.

This makes it possible to use both logical method and set theory method in deductive or other approximate reasoning^[8]. So this is also the advantage of granular computing.

2. Basic Theory of Rough Logic

Definition 1.1^[9] Suppose that $IS = (U, A)$ is a information system. U is all of the objects discussed. A is the property set. Define (a, v) or a_v is an atomic formula in the rough logic, where $a \in A$ and $v \in V_a$ is the attribute value. The formula which combines the atomic formula with the classical logical conjunction is called a compound formula.

Definition 1.2^[9] Rough logic (RL) is composed of attribute set A , attribute values set $V = \bigcup_{a \in A} V_a$, usual logical connective word, parentheses and the following formula

- (1) All the atomic formula of form a_v is the wff in RL , $a \in A, v \in V_a$;
- (2) Suppose that ϕ and φ are wffs in RL , then $\neg\phi, \phi \vee \varphi, \phi \wedge \varphi, \phi \rightarrow \varphi, \phi \leftrightarrow \varphi$ are wffs in RL ;
- (3) The formula which is got from finite number of repeated using (1) and (2) is wffs in RL .

Definition 1.3^[9] Giving information system $IS = (U, A)$, suppose that $F(\bullet)$ is a Rough logic formula in IS , define formula $F(\bullet): m(F(\bullet)) = \{u \in U \mid u \approx_{IS} F(\bullet)\}$,



Definition 1.4^[9] Suppose that $F_1, F_2 \in RL_{IS}$ is the Rough logic formula on the given information system, the semantic sets are $m(F_1), m(F_2)$, then it gets:

- (1) $m(\neg F_1) = U - T_{lu}(F_1)$;
- (2) $m(F_1 \vee F_2) = m(F_1) \cup m(F_2)$;
- (3) $m(F_1 \wedge F_2) = m(F_1) \cap m(F_2)$;
- (4) $m(F_1 \rightarrow F_2) = m(\neg F_1) \cup m(F_2)$;
- (5) $m(F_1 \leftrightarrow F_2) = (m(\neg F_1) \cup m(F_2)) \cap (m(\neg F_2) \cup m(F_1))$.

With rough logical formula, the truth value of rough logical formula is defined as follows:

Definition 1.5^[9] Suppose that $F_1, F_2 \in RL_{IS}$ is the rough logic formula in IS, then the true value is defined as follows:

- (1) $T_{lu}(F_1) = K(m(F_1)) / K(U)$;
- (2) $T_{lu}(\neg F_1) = 1 - T_{lu}(F_1)$;
- (3) $T_{lu}(F_1 \vee F_2) = T_{lu}(F_1) \vee T_{lu}(F_2)$;
- (4) $T_{lu}(F_1 \wedge F_2) = T_{lu}(F_1) \wedge T_{lu}(F_2)$;
- (5) $T_{lu}((\forall x)F_1(x)) = \min\{T_{lu}(F_1(e_1)), T_{lu}(F_1(e_2)), \dots, T_{lu}(F_1(e_n))\}$.

3. The theory of Granular computing on rough logic

Definition 2.1^[9] Suppose that $IS = (U, A)$ is a information system, ϕ is the rough logic formula in IS, $m(\phi)$ is its semantic set, define binary pair $G = (\phi, m(\phi))$ as the basic Granular about ϕ in IS.

All the Granular in IS is L_{IS} .

If $m(\phi) = \emptyset$, then can call ϕ is false in IS, it can be written as $\not\models_{IS} G$;

If $m(\phi) = U$, then can call ϕ is true in IS, it can be written as $\models_{IS} G$;

If $\emptyset \subseteq m(\phi) \subseteq U$, then can call ϕ is satisfied in IS, it can be written as $\approx_{IS} G$;

If $m_*(\phi) = \{x \in U : x \approx_{IS^*} \phi\} = B_*(m(\phi))$, Where m_* is the lower semantic function of ϕ , then ϕ can be called rough lower true in IS, it can be written as $\models_{R_*} G$;

If $m_*(\phi) = \{x \in U : x \approx_{IS^*} \phi\} = \emptyset$, then ϕ can be called rough lower false in IS, it can be written as $\not\models_{R_*} G$;

If $m^*(\phi) = \{x \in U : x \approx_{IS^*} \phi\} = B^*(m(\phi))$, Where m_* is the lower semantic function of ϕ , then ϕ can be called rough upper true in IS, it can be written as $\models_{R^*} G$;

If $m^*(\phi) = \{x \in U : x \approx_{IS^*} \phi\} = \emptyset$, then ϕ can be called rough upper false in IS, it can be written as $\not\models_{R^*} G$;

If $\emptyset \subseteq m_*(\phi) \subseteq B_*(m(\phi)) \subseteq B^*(m(\phi)) \subseteq U$, then ϕ can be called rough satisfied in IS, it can be written as $\approx_R G$.

If $m^*(\phi) = \{x \in U : x \approx_{IS^*} \phi\} = U$, then ϕ can be called rough true in IS, it can be written as $\models_R G$.

If $m^*(\phi) = \{x \in U : x \approx_{IS^*} \phi\} = U$ and $m_*(\phi) = \{x \in U : x \approx_{IS^*} \phi\} = \emptyset$, then ϕ can be

called rough incompatible in IS .

For decision rules $\phi \rightarrow \varphi$ in a decision system, it can be written as (ϕ, φ) , Its semantic set is written as $(m(\phi), m(\varphi))$, The basic Grain of the rule $\phi \rightarrow \varphi$ is defined as $((\phi, m(\phi)), (\varphi, m(\varphi)))$.

Definition 2.2^[9] Suppose that $G = (\phi, m(\phi)), G' = (\varphi, m(\varphi))$ are two Grain of statements. Their operation definitions of connectives $\neg$ (negation), $\oplus$ (or), $\otimes$ (and), $\Rightarrow$ (implication) $\Leftrightarrow$ (equivalence) are as follows:

- (1) $\neg(\phi, m(\phi)) = (\neg\phi, U - m(\phi))$;
- (2) $(\phi, m(\phi)) \oplus (\varphi, m(\varphi)) = (\phi \vee \varphi, m(\phi) \cup m(\varphi))$;
- (3) $(\phi, m(\phi)) \otimes (\varphi, m(\varphi)) = (\phi \wedge \varphi, m(\phi) \cap m(\varphi))$;
- (4) $(\varphi, m(\varphi)) \Rightarrow (\phi, m(\phi)) = (\varphi \rightarrow \phi, m(\varphi) \subseteq m(\phi))$
 $\vee (m_*(\varphi) \subseteq m_*(\phi) \wedge m^*(\varphi) \subseteq m^*(\phi))$
- (5) $(\varphi, m(\varphi)) \Leftrightarrow (\phi, m(\phi)) = (\varphi \leftrightarrow \phi, (m(\varphi) \subseteq m(\phi) \wedge m(\phi) \subseteq m(\varphi)) \vee ((m_*(\varphi) \subseteq m_*(\phi) \wedge m_*(\phi) \subseteq m_*(\varphi)) \wedge (m^*(\varphi) \subseteq m^*(\phi) \wedge m^*(\phi) \subseteq m^*(\varphi))))$

4. Semantic Reasoning Based On Rough Granular

Based on the grammar and semantic of granular computing, the semantic reasoning of granular is given.

Theorem 3.1 Suppose that $G = (\phi, m(\phi)), G' = (\varphi, m(\varphi)) \in L_{IS}$, then: if $\models_R (G \Rightarrow G') \otimes G$, then $\models_R G'$.

Proof: Because $\models_R (G \Rightarrow G') \otimes G$, so it gets

$$\begin{aligned} B^*(m(\phi \rightarrow \varphi) \wedge \phi) &= B^*(m(\phi \rightarrow \varphi) \cap m(\phi)) \subseteq B^*(m(\phi \rightarrow \varphi)) \cap B^*(m(\phi)) \\ &= B^*(m(\neg\phi \vee \varphi)) \cap B^*(m(\phi)) = B^*(m(\neg\phi)) \cup B^*(m(\varphi)) \cap B^*(m(\phi)) \\ &= B^*(m(\varphi)) = U \end{aligned}$$

then $\models_R G'$.

Theorem 3.2 Suppose that $G = (\phi, m(\phi)), G' = (\varphi, m(\varphi)) \in L_{IS}$, if $\models_R G \Rightarrow G'$ and $\models_{IS} G$, then $\models_R G'$.

Proof: Because $\models_R G \Rightarrow G'$, so it gets

$$B^*(m(\phi \rightarrow \varphi)) = B^*(m(\neg\phi \vee \varphi)) = B^*(m(\neg\phi)) \cup B^*(m(\varphi)) = U$$

and because $\models_{IS} G$, so $m(\phi) = U$, $B^*(m(\neg\phi)) = U - B_*(m(\phi)) = \emptyset$,

$$B^*(m(\varphi)) = U.$$

Theorem 3.3 Suppose that $G = (\phi, m(\phi)), G' = (\varphi, m(\varphi))$ are two granular statements on rough granular. Then there are two conclusions:

- (1) If $\models_R G \Rightarrow G'$, and $m(\phi) \cup m(\varphi) = U$, then $\models_R G'$;
- (2) If $\models_R G$, and $m(\phi) \cap m(\varphi) = \emptyset$, then $\not\models_R G \Rightarrow G'$.

Proof: (1) Because $\models_R G \Rightarrow G'$, so

$$B^*(m(\phi \rightarrow \varphi)) = B^*(m(\neg\phi \vee \varphi)) = B^*(m(\neg\phi)) \cup B^*(m(\varphi)) = U$$

And $m(\phi) \cup m(\varphi) = U$, then $U - m(\phi) \subseteq m(\varphi)$

$$B^*(m(\neg\phi)) \cup B^*(m(\varphi)) = B^*(m(\varphi)) = U$$

It gets the conclusion.

(2) Because $\models_R G$, so $B^*(m(\phi)) = U$

And because $m(\phi) \cap m(\varphi) = U$, so $m(\varphi) \subseteq U - m(\phi)$, then

$$B_*(m(\varphi)) \subseteq B_*(U - m(\phi)) = U - B^*(m(\phi))$$

So gets the conclusion

$$\begin{aligned} B_*(m(\phi \rightarrow \varphi)) &= B_*(m(\neg\phi \vee \varphi)) = B_*(m(\neg\phi) \cup m(\varphi)) \\ &= B_*(U - m(\phi)) = U - B^*(m(\phi)) = \emptyset \end{aligned}$$

Theorem 3.4 There are Granular $G = (\phi, m(\phi)) \in L_{IS}$ in approximate space, which makes G and $\neg G$ are true in IS .

Proof: If G is rough and not compatible in IS , then $B^*(m(\phi)) = U$, and $B_*(m(\phi)) = \emptyset$. So, $B^*(m(\neg\phi)) = B^*(U - m(\phi)) = U - B_*(m(\phi)) = U$.

Theorem 3.5 For Granular $G = (\phi, m(\phi)) \in L_{IS}$ in approximate space $IS = (U, A)$, If G is rough and not compatible in IS , then $\neg G$ is rough and not compatible in IS .

Proof: Because G is rough and not compatible in IS , then $B^*(m(\phi)) = U$, and $B_*(m(\phi)) = \emptyset$ so,

$$\begin{aligned} B^*(m(\neg\phi)) &= B^*(U - m(\phi)) = U - B_*(m(\phi)) = U \\ B_*(m(\neg\phi)) &= B_*(U - m(\phi)) = U - B^*(m(\phi)) = \emptyset \end{aligned}$$

Then can get the conclusion that G is rough and not compatible in IS .

Theorem 3.6 For Granular $G = (\phi, m(\phi)) \in L_{IS}$ in approximate space $IS = (U, A)$.

(1) If G is rough and not compatible in IS , then for all $G' = (\varphi, m(\varphi)) \in L_{IS}$, there are $\models_R G \oplus G'$ and $\not\models_R G \otimes G'$, namely $G \oplus G'$ is rough and true in IS , $G \otimes G'$ is rough and false in IS ;

(2) There is $G' = (\varphi, m(\varphi)) \in L_{IS}$, which makes $\models_R G \oplus G'$ and $\not\models_R G \otimes G'$, namely $G \oplus G'$ is rough and true in IS , $G \otimes G'$ is rough and false in IS .

Proof: (1) Because G is rough and not compatible in IS , then $B^*(m(\phi)) = U$, and $B_*(m(\phi)) = \emptyset$. For all the rough Granular $G' = (\varphi, m(\varphi))$,

$$\begin{aligned} B^*(m(\phi \vee \varphi)) &= B^*(m(\phi) \cup m(\varphi)) = B^*(m(\phi)) \cup B^*(m(\varphi)) = U \\ B_*(m(\phi \wedge \varphi)) &= B_*(m(\phi) \cap m(\varphi)) \subseteq B_*(m(\phi)) \cap B_*(m(\varphi)) = \emptyset \end{aligned}$$

then, $B_*(m(\phi \wedge \varphi)) = \emptyset$

(2) For rough Granular $G = (\phi, m(\phi)) \in L_{IS}$, if $G' = (\varphi, m(\varphi)) \in L_{IS}$, and satisfies

$$m(\phi) \cup m(\varphi) = U \text{ and } m(\phi) \cap m(\varphi) = \emptyset$$

Then

$$\begin{aligned} B^*(m(\phi \vee \varphi)) &= B^*(m(\phi) \cup m(\varphi)) = B^*(m(\phi)) \cup B^*(m(\varphi)) = U \\ B_*(m(\phi \wedge \varphi)) &= B_*(m(\phi) \cap m(\varphi)) \subseteq B_*(m(\phi)) \cap B_*(m(\varphi)) = \emptyset \end{aligned}$$

5. Conclusion

This paper mainly discusses rough Granular logic, defines several kinds of Granular operations similar to the classical propositional logic operations, and realizes the deductive reasoning using the method of Granular calculation. The advantages of this reasoning are both logical and collective, so that the formula in each step and its evolution process of semantic set can be intuitive to see in the process of reasoning, then enhance the transparency and generalized of the reasoning.

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Reference

- [1] Miao Duoqian,Wang Guoyin,Granular computing:past,present and future.Beijing:Science press,2007.
- [2] Yanlin. Mathematical logic basis and particle calculation. Beijing:Science press, 2007: 190-196.
- [3] Miao Duoqian,Li Daoduo. Rough set theory, algorithm and application. Beijing: tsinghua university press, 2008.
- [4] Yan Lin,Wang Quanrui,Liuyan. Semantic analysis of rough logic formulas and research on reasoning based on semantic analysis. Artificial intelligence and pattern recognition,2006,19(4): 433-438.
- [5] Wang Yanping,Xu Yi,Dou Jinpei. Fuzzy rough logic semantics and its reasoning..Journal of liaoning university of technology, 2010, 30(4): 259-264.
- [6] Yan Lin. Semantic research and application of rough logic under a special formula of approximate space.Computer engineering and applications, 2004,40(25):84-87.
- [7] Zhang Wenxiu,Wu Weizhi,Liang Jiye,Li Deyu.Rough set theory and methods. Beijing: Science press, 2001.
- [8] Liu Qing. Neighborhood logic and its data reasoning on the neighborhood value information table.Journal of computer science, 2001, 24(4): 405-410.
- [9] Liu Qing.Rough logic and Rough reasoning.Beijing: Science press, 2001, 146-165.