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# Speed estimation of PMSM using SRUKF algorithm

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**Abstract.** Speed estimation is a key technology to realize sensorless control for the PMSM. Based on the unscented Kalman filter (UKF) method without linearization of nonlinear system equations, this letter provides square root unscented Kalman filter (SRUKF) algorithm that operates through iterating the square roots of the covariance matrixes obtained by QR decomposition and Cholesky decomposition. The presented method can further improve speed estimation performance through decreasing the effect of truncation error and enhancing the convergence and stability of algorithm. Simulation results of sensorless control system demonstrate the feasibility and effectiveness of the proposed algorithm.

## 1. Introduction

Permanent magnet synchronous motor (PMSM) is a new type electrical machine, which has been widely used in the fields of robotics, numerical control machine tools, electric vehicles and so on[1]. Traditional closed-loop vector control system of PMSM usually applies mechanical photoelectric sensors to obtain rotor information, eg speed or position. However, these sensors in the control system can not only yield additional cost, but also decrease adaptability and reliability[2]. To solve these shortcomings, sensorless control method is proposed. This method works by using estimation algorithm to identify rotor status after obtaining its voltage, current and flux linkage. In this way, mechanical sensors can be replaced by estimation algorithm[3]. So far, many different types of PMSM rotor state estimation methods have been proposed, including model reference adaptive system (MRAS) method[4], sliding mode observer (SMO) method[5], and extended Kalman filter (EKF) method[6]

The key technology of the EKF method is to use first-order linearization to deal with non-linear systems, and then use the standard Kalman filter method to estimate status of the motor rotor. However, linearization of motor models can produce big truncation errors that have negative impact on statistical property of system status, which decrease estimation accuracy. The UKF method is based on UT transform, adopting Kalman linear filtering framework and reserving high-order term of nonlinear function[7]. UT transform is used in nonlinear transfer of the mean and covariance of one-step prediction equation. Compare to EKF, UKF observer can not only reduce the inherent linearization errors, but also achieve high estimation accuracy and save computation time. However, the UKF may yield round-off errors and truncation errors during computation. It may result in a loss of positive definiteness of the error equation matrix, which decreases convergence stability, estimation accuracy and robustness of observation[8].

In order to solve the shortcomings of the above two methods, sensorless control method of PMSM based on square root unscented Kalman filter (SRUKF) algorithm[9] is proposed in this letter. This method uses the square root matrix of the error covariance matrix to replace the covariance matrix of

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the UKF algorithm for iteration, thus ensuring the positive definite of the covariance matrix and avoiding filtering algorithm divergence that caused by the propagation of accumulative error that exist in the calculation. Moreover, the algorithm uses the system state covariance matrix for Cholesky decomposition and QR decomposition, which enhances the numerical accuracy of the PMSM system state covariance matrix update process, and has better rotor state estimation accuracy and robustness of nonlinear PMSM.

## 2. SRUKFE estimation Algorithm of Nonlinear Systems

Suppose that, a discrete-time nonlinear system with the following mathematical model:

$$\begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k) + \boldsymbol{w}_k \\ \boldsymbol{y}_{k+1} = \boldsymbol{h}(\boldsymbol{x}_k, \boldsymbol{u}_k) + \boldsymbol{v}_k \end{cases} \tag{1}$$

where $\boldsymbol{x}_k \in \mathbb{R}^n$、$\boldsymbol{y}_k \in \mathbb{R}^m$ and $\boldsymbol{u}_k \in \mathbb{R}^r$ are the system state vector, measurement vector and control vector at time k respectively; $\boldsymbol{f}(\cdot)$and $\boldsymbol{h}(\cdot)$ represent deterministic nonlinear functions; Process noise $\boldsymbol{w}_k \in \mathbb{R}^n$ and measurement noise $\boldsymbol{v}_k \in \mathbb{R}^m$ are both white Gaussian noise, it are independent of each other and the mean value of zero. Their covariance matrices are Q and R respectively.

Defined the state estimator of the system is $\hat{\boldsymbol{x}}_k$、the state predictor is $\hat{\boldsymbol{x}}_{k|k-1}$ and the measured predictor is $\hat{\boldsymbol{y}}_k$. The basic steps of the SRUKF-based nonlinear system state estimation algorithm are as follows:

**Step 1**, initialization of the algorithm:

$$\hat{\boldsymbol{x}}_0 = E[\boldsymbol{x}_0] \tag{2}$$

$$\boldsymbol{P}_0 = E\left[ \boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0 \quad \boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0 {}^{\mathrm{T}} \right] \tag{3}$$

$$\boldsymbol{S}_{x,0} = \mathrm{chol}(\boldsymbol{P}_0)^{\mathrm{T}} \tag{4}$$

Where the initial estimated state $\hat{\boldsymbol{x}}_0$ is the mean value of $\boldsymbol{x}_0$; $\boldsymbol{P}_0$ is the estimated error covariance matrix; Chol is a MATLAB instruction, it represents the lower triangular Cholesky decomposition of the matrix; $\boldsymbol{S}_{x,0}$ represents the square root of the estimated error covariance of the initial state.

**Step 2**, computing the 2n+1 Sigma sampling point set $\boldsymbol{\xi}_{i,k-1}$ of the estimated state of the system:

$$\begin{cases} \boldsymbol{\xi}_{0,k-1} = \hat{\boldsymbol{x}}_{k-1} \\ \boldsymbol{\xi}_{i,k-1} = \hat{\boldsymbol{x}}_{k-1} + (\sqrt{(n+\lambda)}\boldsymbol{S}_{x,k-1})_i, i = 1, 2, \cdots, n \\ \boldsymbol{\xi}_{i,k-1} = \hat{\boldsymbol{x}}_{k-1} - (\sqrt{(n+\lambda)}\boldsymbol{S}_{x,k-1})_i, i = n+1, \cdots, 2n \end{cases} \tag{5}$$

Equation (5): $n$ is the dimension of the system state vector; $\lambda = \alpha^2(n+\kappa) - n$ represents a scaling parameter used to reduce the prediction error, $\kappa$is a proportional scale constant, $\alpha$is a parameter that determines the degree of dispersion or distribution of the sampling point in the vicinity, and its value range is usually between 0.0001 and 1. $(\sqrt{(n+\lambda)}\boldsymbol{S}_{x,k-1})_i$ indicates the i column of $\sqrt{(n+\lambda)}\boldsymbol{S}_{x,k-1}$.

**Step 3**, one-step prediction of system state quantity using Sigma sample point set:

$$\hat{\boldsymbol{\xi}}^x_{i,k/k-1} = \boldsymbol{f}(\boldsymbol{\xi}_{i,k-1}, \boldsymbol{u}_{k-1}) \tag{6}$$

$$\begin{cases} \omega_{m,0} = \lambda / (n+\lambda) \\ \omega_{m,i} = 0.5\lambda / (n+\lambda), \quad i = 1 \sim 2n \end{cases} \tag{7}$$

$$\hat{\boldsymbol{x}}_{k|k-1} = \sum_{i=0}^{2n} \omega_{m,i} \hat{\boldsymbol{\xi}}^x_{i,k|k-1} \tag{8}$$

Equation (8): $\omega_{m,i}$ represents the weighting coefficient of the mean value. This step first uses the state equation to find the state prediction value $\hat{\boldsymbol{\xi}}^x_{i,k|k-1}$ of 2n+1 sampling points. Then use the equation

(8) to weight-average these predicted values to obtain a one-step predicted value $\hat{\boldsymbol{x}}_{k|k-1}$ of the state.

**Step 4**, covariance square root matrix $\boldsymbol{S}_{x,k|k-1}$ for calculating state prediction error:

$$\begin{cases}\omega_{c,0}=\lambda/(n+\lambda)+1-\alpha^2+\beta \\ \omega_{c,i}=0.5\lambda/(n+\lambda),\quad i=1\sim 2n\end{cases} \tag{9}$$

$$\boldsymbol{S}^{-}_{x,k|k-1}=\text{qr}\left[\sqrt{\omega_{c,i}}\ \ \hat{\boldsymbol{\xi}}^{x}_{1:2n,k|k-1}-\hat{\boldsymbol{x}}_{k|k-1}\ \ ,\sqrt{\boldsymbol{Q}}\right] \tag{10}$$

$$\boldsymbol{S}_{x,k|k-1}=\text{cholupdate}\ \ \boldsymbol{S}^{-}_{x,k|k-1},\hat{\boldsymbol{\xi}}^{x}_{0,k|k-1}-\hat{\boldsymbol{x}}_{k|k-1},\omega_{c,0} \tag{11}$$

Where $\omega_{c,i}$ represents the weighting coefficient of the covariance; $\beta$ is a non-negative coefficient that reflects the motion information of higher order error terms; The symbols qr and chol update are MATLAB instructions, which respectively represent QR decomposition operation and Cholesky factor first-order update operation. We can see from equations (10) and (11) that the square root of the covariance of the state prediction error is mainly obtained by QR decomposition and Cholesky factor update two-step calculation.

**Step 5**, calculate the measured prediction value using the Sigma sample point set:

$$\hat{\boldsymbol{\xi}}^{y}_{i,\,k|k-1}=\boldsymbol{h}(\hat{\boldsymbol{\xi}}^{x}_{i,k|k-1},\boldsymbol{u}_k) \tag{12}$$

$$\hat{\boldsymbol{y}}_k=\sum_{i=0}^{2n}\omega_{m,i}\hat{\boldsymbol{\xi}}^{y}_{i,\,k|k-1} \tag{13}$$

In this step, the measurement prediction values of 2n+1 Sigma point sets are obtained by using equation (12), and then the weighted average of these measurement output prediction values is obtained by using equation (13) to obtain the system measurement prediction value $\hat{\boldsymbol{y}}_k$ .

**Step 6**, calculate the residual covariance square root matrix $\boldsymbol{S}_{y,k}$ and cross-covariance matrix $\boldsymbol{P}^{xy}_k$ between state and measurement:

$$\boldsymbol{S}^{-}_{y,k}=\text{qr}\left[\sqrt{\omega_{c,i}}\ \ \hat{\boldsymbol{\xi}}^{y}_{1:2n,k|k-1}-\hat{\boldsymbol{y}}_k\ \ ,\sqrt{\boldsymbol{R}}\right] \tag{14}$$

$$\boldsymbol{S}_{y,k}=\text{cholupdate}\ \ \boldsymbol{S}^{-}_{y,k},\hat{\boldsymbol{\xi}}^{y}_{0,k|k-1}-\hat{\boldsymbol{y}}_k,\omega_{c,0} \tag{15}$$

$$\boldsymbol{P}^{xy}_k=\sum_{i=0}^{2n}\omega_{c,i}\left[\hat{\boldsymbol{\xi}}^{x}_{i,k|k-1}-\hat{\boldsymbol{x}}_{k|k-1}\right]\left[\hat{\boldsymbol{\xi}}^{y}_{i,k|k-1}-\hat{\boldsymbol{y}}_k\right]^{\mathrm{T}} \tag{16}$$

**Step 7**, calculate the filter gain and update the state and its covariance square root matrix:

$$\boldsymbol{K}_k=(\boldsymbol{P}^{xy}_k/\boldsymbol{S}^{\mathrm{T}}_{y,k})/\boldsymbol{S}^{\mathrm{T}}_{y,k} \tag{17}$$

$$\hat{\boldsymbol{x}}_k=\hat{\boldsymbol{x}}_{k|k-1}+\boldsymbol{K}_k(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_k) \tag{18}$$

$$\boldsymbol{S}_{x,k}=\text{cholupdate}\ \ \boldsymbol{S}_{x,k|k-1},\boldsymbol{K}_k\boldsymbol{S}_{y,k},\text{-}1 \tag{19}$$

The algorithm is based on the nonlinear dynamic system of PMSM, and the UKF algorithm is improved and proposed the SRUKF algorithm to reduce the possibility of algorithm divergence. In general, the algorithm contributes better filtering accuracy and stability. Figure 1 illustrates the above process in sequence flow diagram.

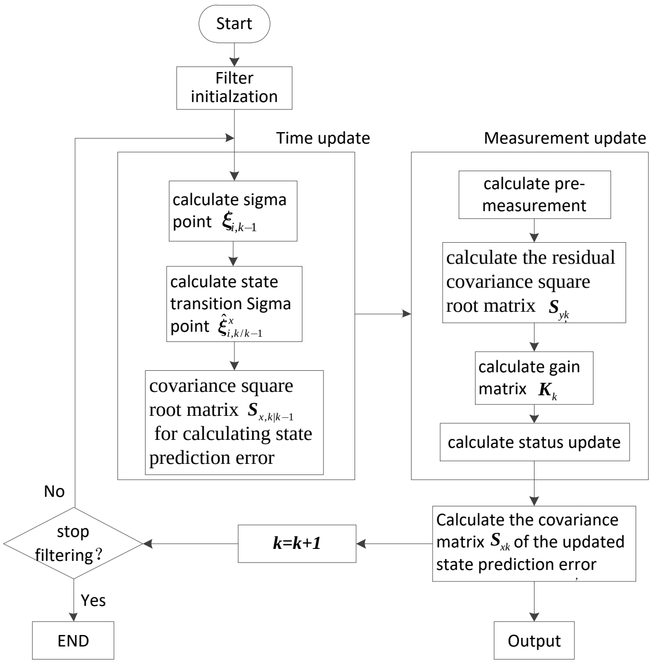


Fig.1 Flow diagram of SRUKF algorithm

## 3. Discrete mathematical model of PMSM

Assume that the two-phase stator winding inductance of the motor is the same, in stationary (α, β) reference frame, the nonlinear mathematical model of PMSM is as follows:

$$\begin{cases} \dfrac{di_\alpha}{dt} = -\dfrac{R}{L_s} i_\alpha + n_p \omega_r \dfrac{\psi_f}{L_s} \sin(n_p\theta) + \dfrac{u_\alpha}{L_s} \\ \dfrac{di_\beta}{dt} = -\dfrac{R}{L_s} i_\beta - n_p \omega_r \dfrac{\psi_f}{L_s} \cos(n_p\theta) + \dfrac{u_\beta}{L_s} \end{cases} \tag{20}$$

where $\dfrac{d}{dt}$ is the derivative operator; $u_\alpha$, $u_\beta$ are the α-axis and β-axis stator voltages respectively; $i_\alpha$, $i_\beta$ are the α-axis and β-axis stator currents respectively; $L_s$ and $R$ are the inductance and resistance of stator winding; $\psi_f$ is rotor flux linkage; $\theta$ and $\omega_r$ are the rotor position angle and angular velocity respectively, $n_p$ is number of the pole pairs. The motor speed changes slowly, implying that $\dot{w}_r$=0, that is the following relationship:

$$\begin{cases} \dfrac{d\omega_r}{dt} = 0 \\ \dfrac{d\theta}{dt} = \omega_r \end{cases} \tag{21}$$

Combined equation(20) with (21), consider the effects of system process noise and measurement noise, introducing state vector $\boldsymbol{x}_k = [i_{\alpha,k}, i_{\beta,k}, \omega_{r,k}, \theta_k]^{\mathrm{T}}$, control vector $\boldsymbol{u}_k = [u_{\alpha,k}, u_{\beta,k}]^{\mathrm{T}}$ and measurement output vector $\boldsymbol{y}_k = [i_{\alpha,k}, i_{\beta,k}]^{\mathrm{T}}$.Therefore, by deriving, the system state dimension n=4, the input dimension r=2, the output dimension m=2, and the PMSM discretization mathematical model of the equation (1) can be obtained. The nonlinear function vectors in the model are $\boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k)$ and $\boldsymbol{h}(\boldsymbol{x}_k, \boldsymbol{u}_k)$.

$$\boldsymbol{f}(\boldsymbol{x}_k,\boldsymbol{u}_k)=\begin{bmatrix}(1-RT_s/L_s)i_{\alpha,k}+T_s\psi_f n_p\omega_{r,k}\sin(n_p\theta_k)/L_s+T_s u_{\alpha,k}/L_s\\(1-RT_s/L_s)i_{\beta,k}+T_s\psi_f n_p\omega_{r,k}\cos(n_p\theta_k)/L_s+T_s u_{\beta,k}/L_s\\\omega_{r,k}\\\theta_k+T_s\omega_{r,k}\end{bmatrix} \tag{22}$$

$$\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)=[i_{\alpha,k},i_{\beta,k}]^{\mathrm{T}} \tag{23}$$

## 4. Simulation results and analysis

Parameters of simulation PMSM:

| Parameter | Value |
|---|---|
| number of the pole pairs | $n_p$ =2 |
| stator resistance | R=1.6Ω |
| stator inductance | $L_S$=0.006365H |
| flux linkage$\Psi_f$ | $\Psi_f$=0.1852Wb |
| rotor friction coefficient | $5.396\times10^{-5}$ |
| maximum load torque | $T_e$=2N·m |
| supply voltage | $U$=300V |

Select the initial state of the motor $\boldsymbol{x}_0$= $[0,0,0,0]^{\mathrm{T}}$, initial estimated error covariance matrix $\boldsymbol{P}_0$= diag($10^{-5}$, $10^{-5}$, 200, 10), noise covariance matrix $\boldsymbol{Q}$= diag($10^{-8}$, $10^{-8}$, 0.1, $10^{-7}$), $\boldsymbol{R}$= diag($10^{-5}$, $10^{-5}$); The sampling period $T_s$=$10^{-6}$s, simulation duration is 1s; weight calculation correlation coefficient $\kappa$=0,$\alpha$=1,$\beta$=2.The load jump and motor shift control scenarios are also considered in the simulation, and the sensorless control simulation results based on EKF and UKF algorithms are compared.

Define the estimated speed as $\hat{\omega}_r$ , actual speed as $\omega_r$ and estimation error of speed as $(\hat{\omega}_r-\omega_r)$ .Set the starting load of the motor to 1N•m and the initial desired speed is 600 r/min. Figure 2 to Figure 4 are the graphs of the actual speed, estimated speed and speed estimation error for expected speed jumping (600r/min to 1000r/min in 0.5s moment). Figure 5 to Figure 7 are the graphs of the actual, speed estimated speed and speed estimation error for load hopping (1N•m to 2N•m in 0.5s moment).

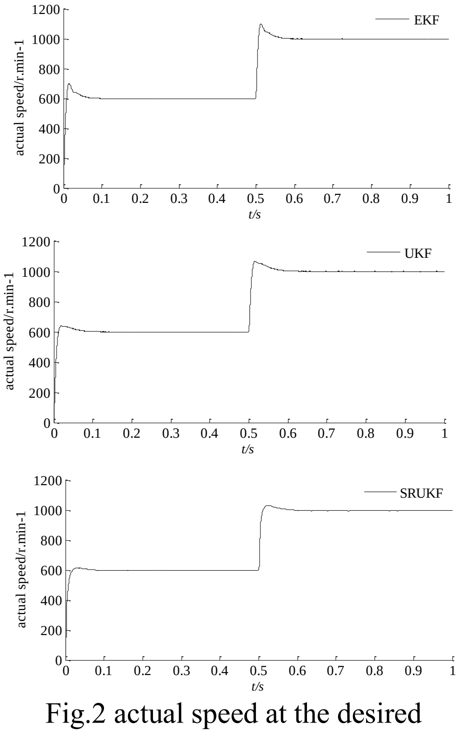


Fig.2 actual speed at the desired speed jump

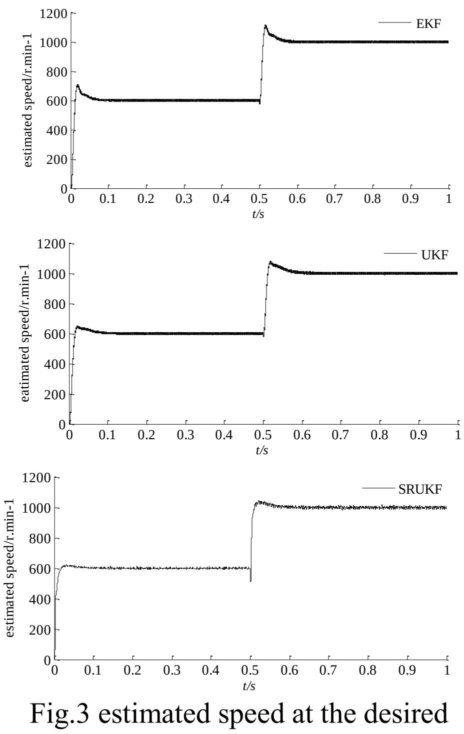


Fig.3 estimated speed at the desired speed jump

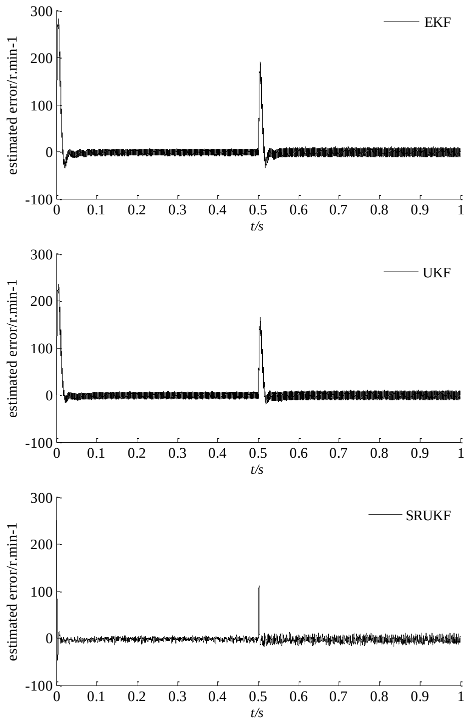


Fig.4 estimated error at the desired speed jump

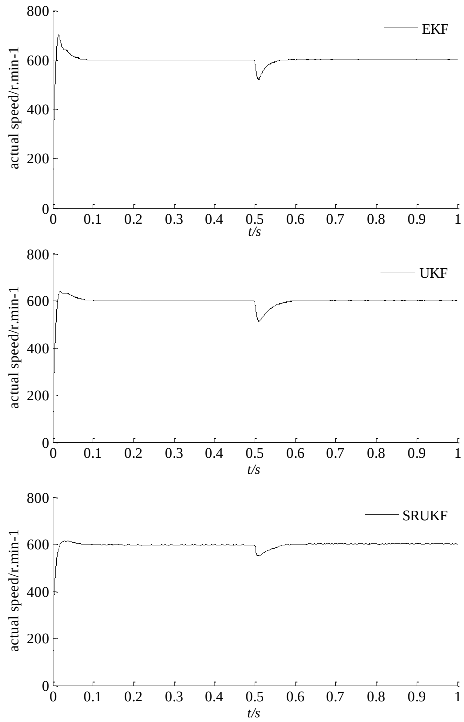


Fig.5 actual speed at the desired load jump

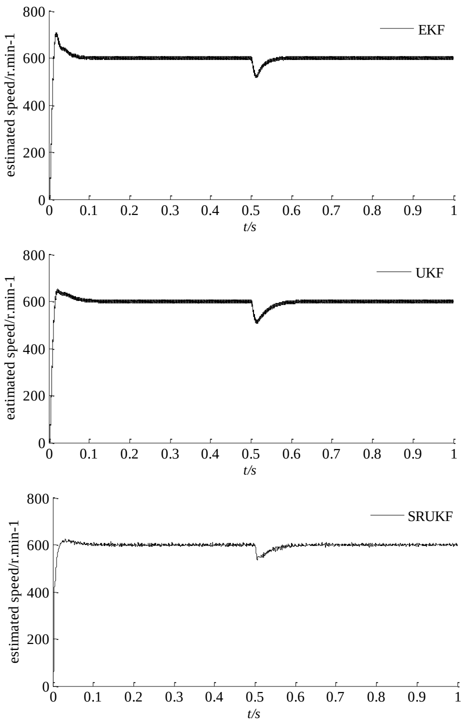


Fig.6 estimated speed at the desired load jump

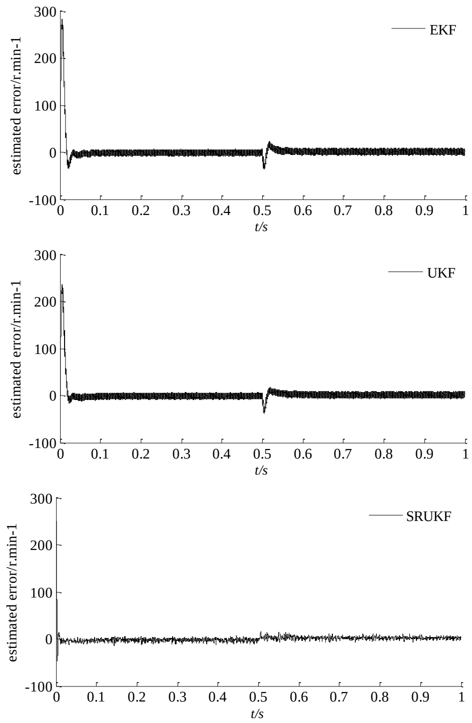


Fig.7 estimated error at the desired load jump

From Fig.2 to Fig.7, we can see that three different speed estimation methods can effectively estimate the motor speed and realize the tracking control of the desired speed by PMSM. However, compared to the other two methods, the SRUKF-based sensorless control method has faster speed estimation dynamics and better desired speed control accuracy;

In particular, according to Fig. 4 and Fig. 7, we can see that in the case of motor speed jump and load jump, the EKF and UKF methods have large error estimation errors, and the SRUKF method's speed estimation error varies around zero. The value is the smallest. This indicates that the sensorless control of PMSM by SRUKF method not only has better speed estimation response speed and accuracy, but also good observation robustness to load parameters and expected speed changes. Moreover, compared with the traditional EKF method and UKF method, the sensorless dynamic and static performance of the control is better.

## 5. Conclusion

Based on the standard UKF algorithm, the SRUKF-based rotor status estimation method performs computation through QR decomposition and Cholesky decomposition, and iterate directly with the square root of the covariance matrix specific to the status. The conclusion of the simulation analysis is that the SRUKF algorithm not only makes the system have better response speed and precision, but also has good ant-disturbance ability. Therefore, the algorithm proposed by this paper can deliver good sensorless rotor-speed-control performance for PMSM.

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