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TRANSIENT BEHAVIOR IMPROVEMENT OF ITERATIVE LEARNING CONTROL SYSTEM BASED ON PSEUDOSPECTRUM METHOD

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Abstract: Robust monotonic convergence (RMC) should be obtained when designing ILC (Iterative learning control) system. But it may cause convergence rate becoming slower. In order to maintain stability of control system, transient growth is often limited within a certain range, which is one of the most properties of transient behaviors. Pseudospectral of the system transfer matrix is introduced to predict the transient growth of ILC system. At the same time, asymptotic performance can be improved if the transient growth is kept small. Under this case, the convergence rate will also be speeded up. The simulation demonstrates the effective of new method for designing ILC system.

1. Introduction

Iterative learning control(ILC) is used to improve the performance of systems that repeat the same operation many times[1]. At the conclusion of each operation, ILC uses the tracking errors from previous iterations of the repeated motion to generate a feedforward control signal for subsequent iterations. Convergence of the learning process results in a feedforward control signal that is customized for the repeated motion, yielding lower tracking error.

When designing a system, lots of designers follow the robustly monotonically convergent (RMC) rule, which can make system stable, but limit the performance of system[2]. According to the research of Longman, R.W and other researchers, some algorithms were used to avoid the large transient growth, by which system can appeal to the rule of RMC[2-6]. While the system satisfying a robust monotonic convergence (RMC) condition, it may limit performance, where the convergence speed of system can speed up if the system has transient growth.

Here, the transient behavior of closed-loop dynamics are introduced to analyze the performance of a system. The transient behavior of system matrix are very hard to describe. Pseudospectra, whose function is adapted as an analyze tool, is used to estimate the transient behavior of a matrix.

This paper has four parts. In section two, the way of 'lifting' the system is introduced where the SISO system can be transformed into MIMO system. In section three, the transient growth of a matrix and the reason for using pseudospectra for analyzing the transient growth is described. In last section the accuracy of the method of a known model is verified, and some remarks are concluded.

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2. Lifted system representation

2.1 norm optimal ILC

The ILC control problem in this paper is studied in the lifted setting[7, 8]. (2.1) is a discrete-time, single-output linear time-invariant system

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + d(k) \end{cases} \tag{2.1}$$

where, $y(k)$ is the output $u(k)$ is input. The system can be'lifted' by noting that over the finite time horizon[9].

$$\underbrace{\begin{bmatrix} y(m) \\ y(m+1) \\ \vdots \\ y(m+N-1) \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} p_m & 0 & 0 & \cdots & 0 \\ p_{m+1} & p_m & 0 & \ddots & \vdots \\ p_{m+2} & p_{m+1} & p_m & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ p_{m+N-1} & \cdots & p_{m+2} & p_{m+1} & p_m \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix}}_{\mathbf{U}} + \underbrace{\begin{bmatrix} d(m) \\ d(m+1) \\ \vdots \\ d(m+N-1) \end{bmatrix}}_{\mathbf{d}} \tag{2.2}$$

where

$$p_i = \begin{cases} 0 & (i < m) \\ CA^{i-1}B & (i \le m) \end{cases},$$

p_i are the system Markov parameters, m is the relative degree, $k = m, m+1, \ldots, m+N-1$

The ILC controller for contour tracking problem results from a quadratic optimization problem[10]. For this problem, an objective function J should be minimizing,

$$J = e_{j+1}^T Q e_{j+1} + u_{j+1}^T S u_{j+1} + (u_{j+1} - u_j)^T R (u_{j+1} - u_j) \tag{2.3}$$

$e_{j+1} = y_r - y_i$, y_r is the reference signal and (Q, R, S) symmetric positive definite matrices (often (Q, R, S) =: (qI, rI, sI)). Note that in some cases (Q, R, S) may be semi-definite matrices, as long as $P^T Q P + R + S$ is positive definite.

where corresponding to the sum of weighted norms of the error $\left\| e_{j+1} \right\|_Q$ defined by $e_{j+1}^T Q e_{j+1}$, the command signal $\left\| u_{j+1} \right\|_S$ defined by $(u_{j+1} - u_j)^T R (u_{j+1} - u_j)$, and the rate of change of the command signal $\left\| u_{j+1} - u_j \right\|_R$ defined by $u_{j+1}^T S u_{j+1}$,

Combining $y_r = e_j + Pu_j$ and $e_{j+1} = y_r - Pu_{j+1}$, we can get

$$e_{j+1} = e_j - P(u_{j+1} - u_j) \tag{2.4}$$

By replacing (2.4) in (2.3) and subsequently differentiating J with respect to u_{j+1} and setting this derivative equal to zero, the norm-optimal ILC controller can be calculated.

The ILC problem is to select u_j using the historical error from previous trials to reduce the error asymptotically. A common method of the first-order linear ILC updating algorithm is given by

$$u_{j+1} = Q\, u_j + \mathrm{L}\, e_j \tag{2.5}$$

where Q and L are in $\mathbb{R}^{N \times N}$.

$$\begin{aligned} \mathrm{Q} &= (P^TQP + R + S)^{-1}(P^TQP + R) \\ L &= (P^TQP + R + S)^{-1}P^TQ \end{aligned}$$

Combining(2.2) with (2.5), closed-loop control input can be given by

$$u_{j+1} = Tu_j + f_0 \tag{2.6}$$

where, $T = Q\,(I - L\ P)$, and $f_0 = \mathrm{Q\,L}\,e_0$.

To analyze the transient behavior of an exponentially convergent ILC system, defining

$$u_\infty \triangleq \lim_{j\to\infty} u_j \tag{2.7}$$

and

$$\delta_j \triangleq u_\infty - u_j \tag{2.8}$$

According to (2.6), following conclusions can be drawn

$$\delta_{j+1}u = T\delta_j uQ \tag{2.9}$$

or

$$\delta_{j+1}u = T^j\delta_1 u \tag{2.10}$$

The sequence

$$\|T\|, \|T^2\|, \cdots, \|T^j\| \tag{2.11}$$

gives the worst-case bound on the overall decay or transient growth of $\|\delta_j u\|$ during learning. The special case $\|T\| < 1$ are referred to as monotonic convergence because the bounding sequence is necessarily monotonically decreasing,

$$\|T^{j+1}\| \le \|T\|\|T^j\| < \|T^j\| \ .$$

Obviously, transient behavior of the system can be determined by analyzing matrix *T*.

2.2 Lifted system representation of feedback system

A feedback control system configuration is shown in Fig. 1, where **k** is the controller, u_k is its output. **P** is the plant , w is the disturbance is the noise, r is the reference sign, and u is a feedforward control.

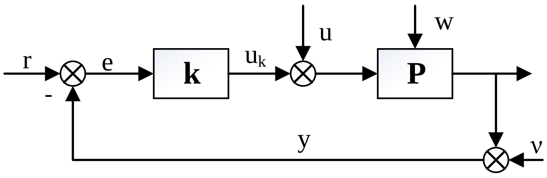


Fig. 1 Configuration of feedback control system

The SISO plant **P** can be described by the state space equation

$$P: \begin{cases} x_{k+1}^P = Ax_k^P + B(uk_k + u_k) + B_d w_k \\ y_k = Cx_k^P + v_k \end{cases} \tag{2.12}$$

where v and w are assumed Gaussian white noise.

The controller is given as following

$$K: \begin{cases} x_{k+1}^c = A_K x_k^c + B_K(r_k - y_k) \\ u_{K,k} = C_K x_k^c + D_K(r_k - y_k) \end{cases} \tag{2.13}$$

The closed loop system is given by

$$T: \begin{cases} x_{k+1} = A_T x_k + B_K \bar{v}_k \\ e_k = C_T x_k + D_k \bar{v}_k \end{cases} \tag{2.14}$$

where

$$\left(\begin{array}{c|c} A_T & B_T \\ \hline C_T & D_T \end{array}\right) = \left(\begin{array}{c|cccc} A_T & B^1 & B^2 & B^3 & B^4 \\ \hline C_T & D^1 & D^2 & D^3 & D^4 \end{array}\right)$$

$$= \left(\begin{array}{cc|cccc} A - BD_K C & BC_K & B & B_d & -BD_K & BD_K \\ -B_K C & A_K & 0 & 0 & -B_K & B_K \\ \hline -C & 0 & 0 & 0 & -1 & 1 \end{array}\right)$$

$$\overline{v}_k = [u_{k,} \omega_k^T, v_k, r]^T \quad ,$$

The task of this system is to follow a desired reference signal r with finite length *N*. The task is that the system repeats over a set of trials, computing u in such a way as to converge to a sufficiently small error *e*.

The lifted system of (2.14) is [11]

$$\mathrm{P} : e^l = P_1 u^l + P_2 \omega^l + P_3 v^l + P_4 r + P_x x_0{}^l \tag{2.15}$$

where the lifted input u^l represents the input to the system in the l^{th} trial:

$$u^l \in \mathbb{R}^N := [u_0^l, u_1^l, \cdots, u_{N-1}^l]^T \quad .$$

the lifted representation of the error in trial *l* is given by $e^l \in \mathbb{R}^N := [e_0^l, e_1^l, \cdots e_{N-1}^l]^T$

and

$$p_i = \begin{pmatrix} G_0^i & 0 & \cdots & 0 \\ G_1^i & G_0^i & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ G_N^i & G_{N-1}^i & G_1^i & G_0^i \end{pmatrix}$$

where

$$G_0^i = D_i, G_k^i = C_T A_T^{i-1} B^i \quad k = 1,2\cdots \quad . \quad P_x : (C_T \; (C_T A_T)^T \cdots (C_T A_T^{N-1})^T)$$

The map P_1 will be considered only, since it's role is essential to the design of learning controllers.

3. Transient behavior of a matrix

Let A_n be a complex $n \times n$ matrix. The critical behavior of the norms $\left\| A_n^k \right\|$ is of considerable interest in connection with several problems. Critical behavior means that the norms of the powers grow exponentially to infinity or that they run through a critical transient phase before decaying exponentially to zero. The norm of $\left\| A_n^k \right\|$ converges to zero when $k \to \infty$, where $\| . \|$ is spectral norm if and only if the spectral radius of A_n ($\rho(A_n)$) is less than one[12]. However, sole knowledge of the spectral radius or even of all eigenvalues of A_n, does not tell us whether the norms $\left\| A_n^k \right\|$ run through a critical transient phase, that is, whether there is *k*, for which $\left\| A_n^k \right\|$, becomes very large before eventually decaying exponentially to zero.

3.1 Pseudospectra

The spectrum of a matrix is the set of all its eigenvalues, the *ε*-pseudospectrum of matrix *T* is defined as

$$\Lambda_\varepsilon = \quad z \in C : z \in \Lambda(X), and \left\| X - T \right\| \le \varepsilon \tag{3.1}$$

where *ε* is a fixed value, $\Lambda(T)$ is the matrix *T* spectrum When *ε* takes a different value, it represents

the pseudospectral of the different ranges of the matrix. Pseudospectral is shown in Fig. 2 diagram of a Demmel matrix. The black dots in the Fig. 2 are the eigenvalues of the matrix. The left side shows different values of ε to obtain different pseudospectral boundaries. The border color corresponds to the right-side color bar. The bars indicate that ε takes an index of different logarithms.
For the first-order discrete system

$$x_{k+1} = Ax_k \tag{3.2}$$

whose pseudospectral radius is defined as:

$$\rho_{\varepsilon}(A) = \max\{|z| : z \in \Lambda_{\varepsilon}(A)\} \tag{3.3}$$

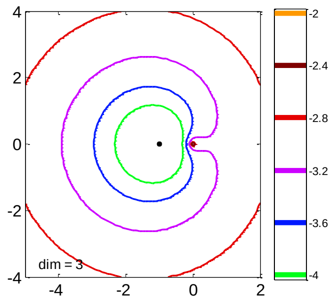


Fig. 2 pseudospectra of Demmel matrix

3.2 Analyze of transient growth

The role of pseudospectra in connection with the norms of powers of matrices and operators is as follows: norms of powers can be related to the resolvent norm, and pseudospectra decode information about the resolvent norm in a visual manner. A relation between resolvent and power norms is established by the Kreiss matrix theorem[13].

Kreiss Matrix Theorem

For any $n \times n$ matrix T, it has the following conclusion

$$\sup_{\varepsilon>0} \frac{\rho_{\varepsilon}(T)-1}{\varepsilon} \leq \sup_{k} \left\| T^k \right\| \leq en \sup_{\varepsilon>0} \frac{\rho_{\varepsilon}(T)-1}{\varepsilon} \tag{3.4}$$

where *e* is the exponential constant, $\rho_{\varepsilon}(T)$ is the $\varepsilon-$ pseudospectra radius of matrix.

$$\rho_{\varepsilon}(T) = \max\{|z| : z \in \Lambda_{\varepsilon}(T)\}$$

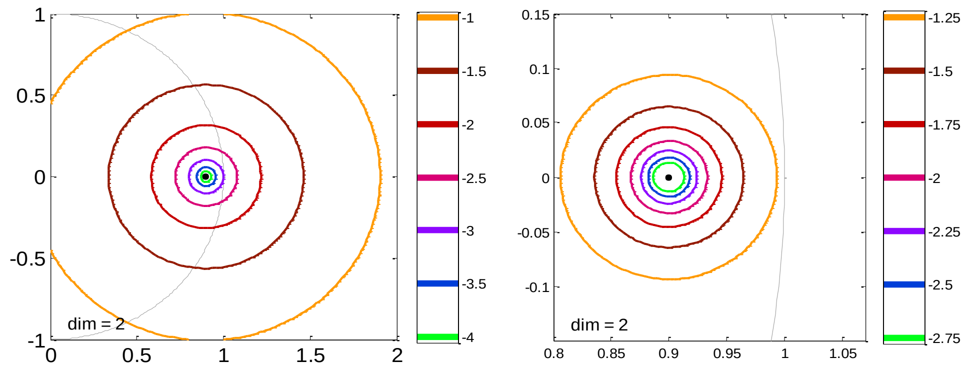
$$\Lambda_{\varepsilon}(T) = \quad z \in C : z \in \Lambda(X), and \left\| X - T \right\| \leq \varepsilon$$

where $\Lambda(X)$ is all eigenvalue of the matrix *X*. If the matrix *T* is normal, then (2.11) is trivially given by $\left\| T^k \right\| = \rho(T)^k$ for all *k*.

$\sup_{\varepsilon>0} \frac{\rho_{\varepsilon}(T)-1}{\varepsilon}$ is called Kreiss constant, which gives upper and lower bounds on the maximum transient growth. However it is difficult to compute the constant. Therefore, plots of pseudospectra are often used to estimate the magnitude of transient growth.

To show the visualisation aspect of the pseudospectra, the following two matrices are considered

$$T_1 = \begin{bmatrix} 0.9 & 0 \\ 10 & 0.9 \end{bmatrix}, \mathrm{T}_2 = \begin{bmatrix} 0.9 & 0 \\ 0.1 & 0.9 \end{bmatrix}$$



(a)the pseudospectra of T_1 (b) the pseudospectra of T_2

Fig. 3 the pseudospectra of T_1 and T_2

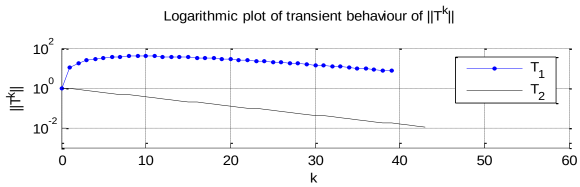


Fig. 4 transient behavior of $T_1$and T_2.

The boundaries (or level sets) of pseudospectra for these Matrices for various values of ε computed using Eigtool[14] is shown in Fig. 3. The eigenvalues are located at 0.9 for each matrix of system, but the pseudospectra of each matrix are rather different. The pseudospectra of T_2 are clustered closely around its eigenvalues, T_1's are much larger, its pseudospectra extend well outside of the unit circle, even for very small values of ε. This phenomenon can be used to check whether transient growth exists. Thus, Fig. 3 indicates that transient growth is expected for T_1. This is verified by direct computations of the transients $T_1$and T_2, in Fig. 4

Accoding to the research of transient behavior in ILC[2-6], the transient growth are limited to the acceptable range that making system stable, and the convergence rate of the system can be speed up simultaneously.

3.3 Advantageous of Pseudospectral

It is true that for a purely linear, constant-coefficient, homogeneous problem, eigenvalues govern the asymptotic behavior as $t \to \infty$. If the problem is normal, this statement is robust; the eigenvalue also has relevance to short-time or transient behavior, and moreover, their influence tends to persist if the problem is altered in small ways. If the problem is far from normal, however, conclusions based on eigenvalues are in general not robust. Firstly, there may be a long transient that looks quite different from the asymptote and has no connection to the eigenvalues. Secondly, the asymptote may change beyond recognition if the problem is modified slightly. Eigenvalues do not always govern the transient behavior of a nonnormal system, nor the asymptotic behavior in the presence of nonlinear terms, variable coefficients, lower order terms, inhomogeneous forcing data, or other complications. Few applied problems are free of all these effects, where it is rare that one is interested so purely in the limit $t \to \infty$ as one may at first imagine. These issues are at the heart of convergence and stability investigations in numerical analyze.

4. Example

In this section, a servo control problem is considered for a typical fourth order mechanical system[9].

The plant and the feedback controller are given in equation (4.1) and (4.2) respectively. The servo task is considered with a duration of 200 samples.

$$\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) = \left(\begin{array}{cccc|c} 3.9050 & -1.451 & 0.9732 & -0.4970 & 0.0019 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ \hline 10^{-3}\,(0.4780 & 1.3605 & 1.3049 & 0.2381) & 0 \end{array}\right) \tag{4.1}$$

$$\left(\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right) = \left(\begin{array}{c|c} 0.9048 & 1 \\ \hline -0.8564 & 10 \end{array}\right) \tag{4.2}$$

As described in section 2, transition matrix T is used to analyze the transient behavior of the system. Three norm-optimal ILCs are designed with different R weightings, as listed in Table 1.

Table 1 Weighting matrices for norm-optimal ILC example

q	s	r	design
100	0.1	120000	T_1
100	0.1	110000	T_2
100	0.1	1900000	T_3

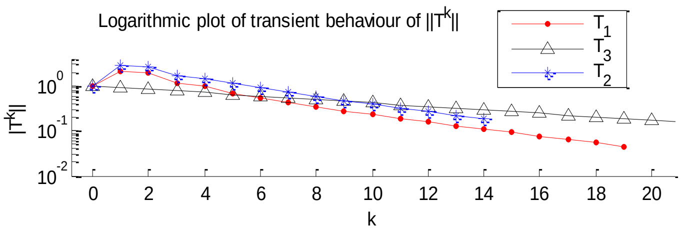
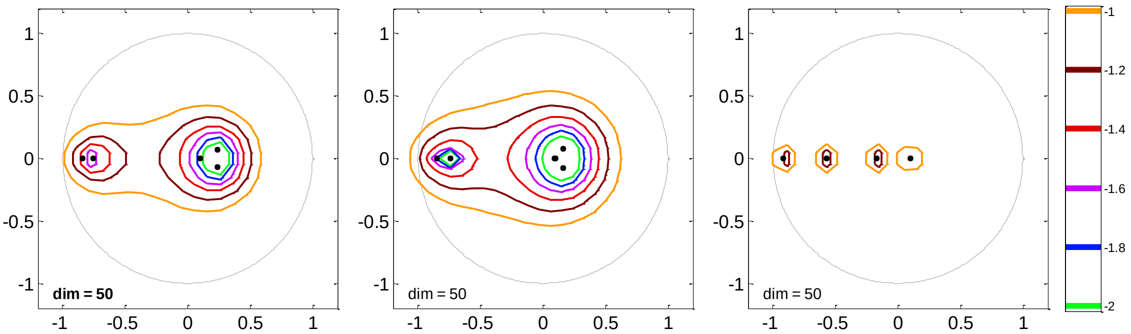


Fig. 5 Learning transient bound for norm-optimal ILC designs



(a)The pseudospectra of T_1 (b) The pseudospectra of T_2 (c) The pseudospectra of T_3

Fig. 6 The pseudospectra of T_1, T_2 and T_3

Three norm-optimal ILC's are designed with different **R** weightings (Table 1). A check of the RMC condition will show that two (T_1 and T_2) of three designs are not RMC[15] . From Fig. 5 it can be found that T_3 follows the condition of RMC, whose convergence rate is slower than T_1 and T_2. With the increase in r , convergence rate will speed up, if transient growth is allowed. It can also find in Fig. 5 that the T_1 and T_2 are the conditions which do not follow the rule of RMC, whereas convergence rate , of T_1 and T_2 is faster than T_3, as designed in Fig. 5. In the case of non-monotonic convergence, increasing the r follows the condition that decreases the transient growth[15, 16], but it also speeds up convergence rate, which is different from some conclusion of paper[17]. It may be affected by the characteristics of the model itself.

Increasing r will pull the levels sets in closer to the eigenvalues, while also moving some of the eigenvalues farther from the origin. Tighter grouping of ε-pseudospectrum of T , the levels reduce the magnitude of the transient growth, while shifting eigenvalues away from the origin accounts for the slower convergence rate observed at large iterations. In Fig. 6, from the pseudospectra of T_1 and T_2, it can be found that fix the $\varepsilon=10^{-1.2}$, then substitute various values of R, let the bound of pseudospectra makes R's border within the unit circle near the unit circle, so it can faster convergence rate. Calculations using several other system perturbations by the authors have yielded the same trend demonstrated here. The pseudospectra of T_3 shows that the difference choice of ε is very close to the eigenvalue of T_3 , although it follows the rule of RMC, whose convergence rate is slower than T_1 and T_2.

5. Conclusion

Transient growth is an undesirable property in ILC, while robust monotonic convergence is a performance-limiting constraint. Thus, it is reasonable to consider some transient growth as a trade-off for improved performance. Pseudospectra is introduced to calculate and bound transient growth. Significantly, the pseudospectra also affords a mathematical basis that illuminates the often misunderstood topic of transient growth in ILC. Norm optimization and pseudospectra are combined to predict the transient peaking. It’s convergence rate weighting factor R plays an important role in reducing transients. These results cannot be gained using eigenvalue or singular value analyze, but in its place are only evident using pseudospectral analyze.

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