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## Trajectory Tracking Method Designing based on Adaptive Fast Convergence Terminal Sliding Mode Control

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# Trajectory Tracking Method Designing based on Adaptive Fast Convergence Terminal Sliding Mode Control

Zhang Wen-guang\*, Yi Wen-jun, Guan Jun, Qu Yue, Liang Zhendong

National Key Laboratory Transient Physics, Nanjing University of Science and Technology, Nanjing, Jiangsu, 210094, China

\*Corresponding author. Email: wgz8911@163.com

**Abstract.** Concerning the plan trajectory tracking problem of gliding missile, the fast convergence sliding mode control is used in the designing of guidance law. To cope with the problem of parameter perturbation, adaptive method is proposed to update the value of parameter timely. Theoretical analysis and numerical simulation show our method can estimate the angular rate of missile precisely, and performs well in trajectory tracking.

## 1. Introduction

Guided munition technology is an important development direction of armament science and technology, and accurate tracking of program trajectory is the precondition of the precise strike of guided munitions. Not only be there all kinds of perturbation in the atmosphere, but also does the measured data contain deviation.

Researchers proposed many control methods concerning the trajectory tracking problem, and achieved good performance [1-5]. Gandolfo et al [6] proposed a tracking control strategy based on algebraic theory and linear systems. Zhang Dayuan et al [7] used feedback linearization method to linear the motion model of guided missile mass point, and then designed a guidance law based on this linear model and Linear quadratic optimal regulator. Cheng Yang et al [8] integrated off-line trajectory optimization with on-line prediction to design the reentry guidance law, and the performance was good.

To cope with the problem of parameters perturbation, an active disturbance rejection controller was proposed [9], which used second order extended state observer to estimate the uncertain terms. Yin Xiaoyun et al [10] utilized the influence of perturbation and deviation of parameters to design the gain factors, which avoided tedious design process, and was easy to realize in engineering project. The common feature of the above mentioned methods is that they all suppose that the upper bound of the perturbation is known. In this paper, we propose an adaptive update strategy for uncertain parameters based on Lyapunov method, which does not need the known upper bound of perturbation.

Timeliness is one of the key dynamic properties of aircraft, and it is an effective way to improve the ability of counter attacks by enhancing the real-time performance of aircraft. For the guidance control system of aircraft, the convergence speed of closed-system is an import index. So, we use fast terminal sliding mode control to design a new tracking guidance law.

## 2. Trajectory Tracking controller design

In this paper, we focus on the trajectory tracking problem in the longitudinal plane. The task of trajectory tracking is controlling the missile through the force produced by duck rudder to gliding



along the plan ballistic. Intuitively, the plan ballistic tracking problem can be decomposed to the tracking problem of a series of track points on the plan ballistic. The geometric graph of homing engagement can be depicted by Fig. 1, where M and T represent the missile and target, respectively,  $\gamma$  represents the velocity azimuth of missile,  $\lambda$  and  $r$  represent light-of-sight angle and the relative distance between missile and target, respectively,  $V$  represents the velocity of missile, and  $a$  denotes the acceleration of missile, and suppose that  $a$  is perpendicular to  $V$ .

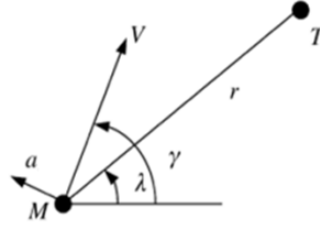


Figure 1. Geometric graph of homing engagement

The relative motion between missile and track point can be modeled as

$$\dot{r} = -V \cos(\theta - \lambda) \quad (1)$$

$$\dot{\lambda} = -\frac{1}{r} V \sin(\theta - \lambda) \quad (2)$$

When the angular rate converges to zero, a successful interception will be achieved [11]. So the object of this paper is to design a guidance law, which makes  $\dot{\lambda} \rightarrow 0$  in finite time.

## 2.1 The guidance law design based on adaptive fast terminal sliding mode control

### 2.1.1 adaptive fast terminal sliding mode control

Consider the following one order uncertain nonlinear system:

$$\begin{cases} \dot{x} = f(x) + g(x, d_1, d_2)u = f(x) + [g(x, \hat{d}_1, \hat{d}_2) + \xi_1(x)\Delta d_1 + \xi_2(x)\Delta d_2]u \\ y = x \end{cases} \quad (3)$$

where  $f(x)$ ,  $\xi_1(x)$ ,  $\xi_2(x)$  are known functions,  $d_1$ ,  $d_2$  are uncertain terms,  $\hat{d}_1$ ,  $\hat{d}_2$  represent the estimated values,  $\Delta d_1$ ,  $\Delta d_2$  are error of estimation, which satisfy  $d_1 = \hat{d}_1 + \Delta d_1$ ,  $d_2 = \hat{d}_2 + \Delta d_2$ . The object of controller is to find an appropriate control law  $u$ , such that the output will converge to zero in finite time.

Nonsingular fractional integral fast terminal sliding mode surface is denoted as [12]:

$$\begin{cases} S(t) = x(t) + k_1 x_I = x + k_1 \int_0^t x(t) + k_2 x^{q/p}(t) dt \\ \dot{x}_I = x(t) + k_2 x^{q/p}(t), x_I(0) = -\frac{x(0)}{k_1} \end{cases} \quad (4)$$

where  $k_1 > 0$ ,  $k_2 > 1$ ,  $p$ ,  $q$  are odd numbers, and  $p > q > 0$ ,  $S(0)=0$ .

Differentiating  $S(t)$  with respect to time gives

$$\begin{aligned} \dot{S}(t) &= \dot{x}(t) + k_1 x(t) + k_1 k_2 x^{q/p}(t) = \\ &= f(x) + [g(x, \hat{d}_1, \hat{d}_2) + \xi_1(x)\Delta d_1 + \xi_2(x)\Delta d_2]u + k_1 x(t) + k_1 k_2 x^{q/p}(t) \end{aligned} \quad (5)$$

where  $u$  is selected to be

$$u = -(g(x, \hat{d}_1, \hat{d}_2))^{-1} \cdot [k_3 S(t) + k_4 \text{sign}(S(t)) + f(x) + k_1 x(t) + k_1 k_2 x^{q/p}(t)] \quad (6)$$

where  $k_3 > 0$ ,  $k_4 > 0$ , due to  $p > q > 0$ , the control law is nonsingular.

Selecting  $V_1 = \frac{1}{2} S^2 + \frac{1}{2} N_1 \Delta d_1^2 + \frac{1}{2} N_2 \Delta d_2^2$ , where  $N_1, N_2$  are adjustment coefficients, the derivative of  $V_1$  is

$$\dot{V}_1 = -k_3 S^2(t) - k_4 |S(t)| + \Delta d_1 (S \xi_1(x)u + N_1 \Delta \dot{d}_1) + \Delta d_2 (S \xi_2(x)u + N_2 \Delta \dot{d}_2) \quad (7)$$

let

$$-\dot{\hat{d}}_1 = \Delta \dot{d}_1 = -\frac{S \xi_1(x)u}{N_1} \quad (8)$$

$$-\dot{\hat{d}}_2 = \Delta \dot{d}_2 = -\frac{S \xi_2(x)u}{N_2} \quad (9)$$

which are update laws of  $\hat{d}_1$ ,  $\hat{d}_2$ , so

$$\dot{V}_1 = -k_3 S^2(t) - k_4 |S(t)| \leq 0 \quad (10)$$

Because  $\Delta d_1^2$ ,  $\Delta d_2^2$  are small quantities, and selecting appropriate  $N_1$ ,  $N_2$ , taking  $S(0)=0$  into consideration,  $V_1(0) \approx 0$ . What's more, because  $V_1 \geq 0$ ,  $\dot{V}_1 \leq 0$ ,  $V_1(t) \equiv 0$ ,  $S(t) \equiv 0$  are hold when  $t \geq 0$ , which means the system is kept on the sliding surface, and the arrival time is 0. For sliding mode, we have  $S(t)=0$ , which is equal to  $\dot{x}(t) = -k_1 \int_0^t x(t) + k_2 x^{q/p}(t) dt$ , differentiating both sides of it with respect to time gives

$$\dot{x}(t) = k_1 x(t) + k_1 k_2 x^{q/p}(t) \quad (11)$$

by solving this bernoulli equation, we get  $x(t) = e^{-k_1 t} (-k_2 e^{\frac{k_1(p-q)t}{p}} + c)^{p/(p-q)}$ , where  $c$  is constant.

Considering that  $x(0) \neq 0$  and  $x(t)=0$ , we can obtain

$$t = \frac{p}{k_1(p-q)} \ln \frac{x(0)^{(p-q)/p + k_2}}{k_2} \quad (12)$$

which is the convergence time of system Eq. (3).

### 2.1.2 Guidance Law Design

The kinetic equation of gliding missile in longitudinal plane can be modeled as [13]

$$\frac{dV}{dt} = \frac{F_x \cos \alpha - F_y \sin \alpha}{m} \quad (13)$$

$$\frac{d\theta}{dt} = \frac{F_x \sin \alpha + F_y \cos \alpha}{mv} \quad (14)$$

$$\frac{d\vartheta}{dt} = \omega_z \quad (15)$$

$$\frac{d\omega_z}{dt} = \frac{k_m}{J_z} (C_{n0} + k_{ne} \delta_z) \quad (16)$$

$$\frac{dx}{dt} = V \cos \theta \quad (17)$$

$$\frac{dy}{dt} = V \sin \theta \quad (18)$$

$$\alpha = \vartheta - \theta \quad (19)$$

where  $V$  represents the velocity of gliding missile,  $\theta$  denotes the velocity azimuth,  $\vartheta$  is body pitch angle,  $\omega_z$  represents pitch angle velocity,  $k_m$  represents missile shaping coefficient,  $J_z$  denotes rotational inertia,  $C_{n0}$  denotes dynamic moment coefficient,  $k_{ne}$  represents efficiency coefficient of rudder blade,  $\delta_z$  denotes angle of elevator deflection,  $x$ ,  $y$  denote the coordinate of missile,  $\alpha$  represents attack angle,  $m$  denotes the mass of missile, and  $F_x, F_y$  are defined as

$$F_x = k_F \rho V^2 C_x, F_y = k_F \rho V^2 C_y \quad (20)$$

$$C_x = C_{x0} + k_{xz} \delta_z, C_y = C_{y0} + k_{yz} \delta_z \quad (21)$$

where  $K_F$  denotes missile shaping coefficient,  $C_x$  and  $C_y$  denote the total drag coefficient and the total lift coefficient, respectively.  $C_{x0}$  represents the drag coefficient of the body of missile, and  $C_{y0}$  represents the lift coefficient of the body of missile, and  $k_{xz}$ ,  $k_{yz}$  are efficiency coefficients, suppose

$$k_{xz} = \hat{k}_{xz} + \Delta k_{xz}, k_{yz} = \hat{k}_{yz} + \Delta k_{yz} \quad (22)$$

Let  $\zeta(t) = \dot{\lambda}(t)$ , according to Eq. (2), differentiating it gives

$$\dot{\zeta}(t) = \frac{r[V \sin(\theta - \lambda) + (\dot{\theta} - \dot{\lambda}) V \cos(\theta - \lambda)] - \dot{r} V \sin(\theta - \lambda)}{r^2} \quad (23)$$

Substituting Eqs. (13), (14), (20), (21) into Eq. (22) gives

$$\begin{aligned} \dot{\varsigma}(t) = & \frac{-1}{r^2} \{ rk_F \rho V^2 \frac{1}{m} [\sin(\theta - \lambda) \cdot C_{x0} \cos(\alpha) + \cos(\theta - \lambda) \cdot C_{x0} \sin(\alpha)] + \\ & rk_F \rho V^2 \frac{1}{m} [-\sin(\theta - \lambda) \cdot C_{y0} \sin(\alpha) - \\ & \cos(\theta - \lambda) \cdot C_{y0} \cos(\alpha)] \} - rV \cos(\theta - \lambda) \dot{\lambda} - \dot{r}V \sin(\theta - \lambda) - \\ & \frac{k_F \rho V^2}{mr} \{ [\cos(\alpha) \sin(\theta - \lambda) + \sin(\alpha) \cos(\theta - \lambda)] (k_{xz} + \Delta k_{xz}) + \\ & [\sin(\alpha) \sin(\theta - \lambda) - \cos(\theta) \cos(\theta - \lambda)] (k_{yz} + \Delta k_{yz}) \} \end{aligned} \quad (24)$$

therefore

$$\left\{ \begin{aligned} f(\varsigma) = & \frac{-1}{r^2} \{ rk_F \rho V^2 \frac{1}{m} [\sin(\theta - \lambda) \cdot C_{x0} \cos(\alpha) + \cos(\theta - \lambda) \cdot C_{x0} \sin(\alpha)] + \\ & rk_F \rho V^2 \frac{1}{m} [-\sin(\theta - \lambda) \cdot C_{y0} \sin(\alpha) - \cos(\theta - \lambda) \cdot C_{y0} \cos(\alpha)] \} - rV \cos(\theta - \lambda) \dot{\lambda} - \dot{r}V \sin(\theta - \lambda) \\ g(\varsigma, \hat{k}_{xz}, \hat{k}_{yz}) = & \frac{k_F \rho V^2}{mr} \{ [\cos(\alpha) \sin(\theta - \lambda) + \sin(\alpha) \cos(\theta - \lambda)] k_{xz} + \\ & [\sin(\alpha) \sin(\theta - \lambda) - \cos(\theta) \cos(\theta - \lambda)] k_{yz} \} \\ \xi_1(\varsigma) = & \frac{k_F \rho V^2}{mr} \{ [\cos(\alpha) \sin(\theta - \lambda) + \sin(\alpha) \cos(\theta - \lambda)] \} \\ \xi_2(\varsigma) = & \frac{k_F \rho V^2}{mr} \{ [\sin(\alpha) \sin(\theta - \lambda) - \cos(\theta) \cos(\theta - \lambda)] \} \\ & u = \delta_z \\ & Z = \varsigma \end{aligned} \right. \quad (25)$$

where Z represents observed value, select

$$\left\{ \begin{aligned} S(t) = & \varsigma(t) + k_1 \varsigma_I = \varsigma + k_1 \int_0^t \varsigma(t) + k_2 \varsigma^{q/p}(t) dt \\ \dot{\varsigma}_I = & \varsigma(t) + k_2 \varsigma^{q/p}(t), \varsigma_I(0) = -\frac{\varsigma(0)}{k_1} \end{aligned} \right. \quad (26)$$

Differentiating  $S(t)$  with respect to time gives

$$\begin{aligned} \dot{S}(t) = & \dot{\varsigma}(t) + k_1 \dot{\varsigma}(t) + k_1 k_2 \varsigma^{q/p}(t) = f(\varsigma) + [g(\varsigma, \hat{k}_{xz}, \hat{k}_{yz}) + \xi_1(\varsigma) \Delta k_{xz} + \xi_2(\varsigma) \Delta k_{yz}] u + k_1 \varsigma(t) + \\ & k_1 k_2 \varsigma^{q/p}(t) \end{aligned} \quad (27)$$

and the control law is designed as

$$u = -(g(\varsigma, \hat{k}_{xz}, \hat{k}_{yz}))^{-1} \cdot [k_3 S(t) + k_4 \text{sign}(S(t)) + f(\varsigma) + k_1 \varsigma(t) + k_1 k_2 \varsigma^{q/p}(t)] \quad (28)$$

where  $K_1 > 0$ ,  $K_2 > 1$ ,  $p, q$  are odd numbers, and  $p > q > 0$ .

The update law of  $\hat{k}_{xz}$ ,  $\hat{k}_{yz}$  are

$$-\dot{\hat{k}}_{xz} = \Delta \dot{k}_{xz} = -\frac{S \xi_1(\varsigma) u}{N_1} \quad (29)$$

$$-\dot{\hat{k}}_{yz} = \Delta \dot{k}_{yz} = -\frac{S \xi_2(\varsigma) u}{N_2} \quad (30)$$

Stability proof: Choosing  $V_c = \frac{1}{2} S^2 + \frac{1}{2} N_1 \Delta k_{xz}^2 + \frac{1}{2} N_2 \Delta k_{yz}^2$ , differentiating it with respect to time gives

$$\dot{V}_c = S \dot{S} + N_1 k_{xz} \dot{k}_{xz} + N_1 k_{yz} \dot{k}_{yz} \quad (31)$$

Substituting Eq. (31) into Eqs. (27), (28), (29), (30) gives

$$\dot{V}_c = -k_3 S^2(t) - k_4 |S(t)| \leq 0 \quad (32)$$

Due to  $V_c \geq 0$ , the system is stable by referring to Lyapunov stability criterion. According to Eq. (12), the convergence time is

$$t_c = \frac{p}{k_1(p-q)} \ln \frac{\varsigma(0)^{(p-q)/p + k_2}}{k_2} \quad (33)$$

Besides, to weaken the chattering problem of sliding mode, boundary layer method[14] can be used: replace  $\text{sign}(s)$  with

$$\text{sat}(s) = \begin{cases} s_{bl}^{-1} \cdot s, & |s| \leq s_{bl} \\ \text{sign}(s), & |s| > s_{bl} \end{cases} \quad (34)$$

where  $s_{bl}$  represents the thickness of boundary.

### 3. Simulation and Results

In this section, we take a certain type of gliding missile as object of study to validate the efficient of the proposed method. The initial conditions of simulation are set as  $v = 273\text{m/s}$ ,  $\theta = 0^\circ$ ,  $\vartheta = 0^\circ$ ,  $x = 0\text{m}$ ,  $y = 16000\text{m}$ , parameter perturbation during simulation is set as

Table 1. Parameter perturbation in simulation

Time of gliding(second)	$\Delta k_{xz}$	$\Delta k_{yz}$
$10 < t \leq 20$	40	-70
$20 < t \leq 40$	-50	30
$t > 40$	-20	-40

The parameters of controller are set as  $k_1 = 0.7, k_2 = 1.4, k_3 = 0.6, k_4 = 1.1$ ,  $N_1 = 10$ ,  $N_2 = 7$ , and the simulation results are shown as follows

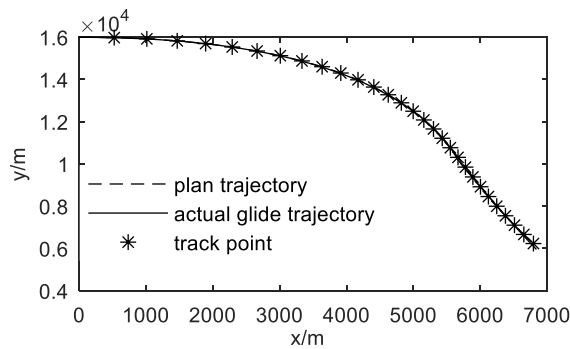


Figure 3.1. Trajectory of missile gliding

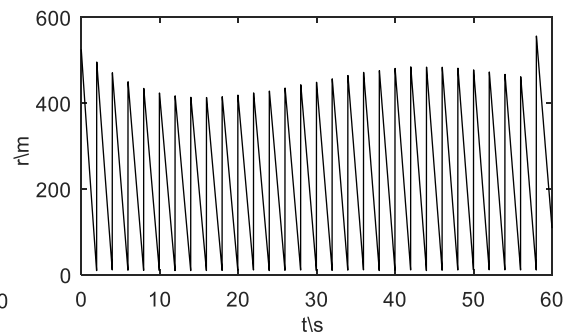


Figure 3.2. Tracking error of tracking points

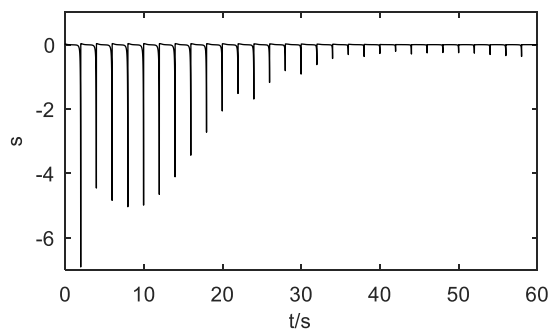


Figure 3.3. Sliding mode variable

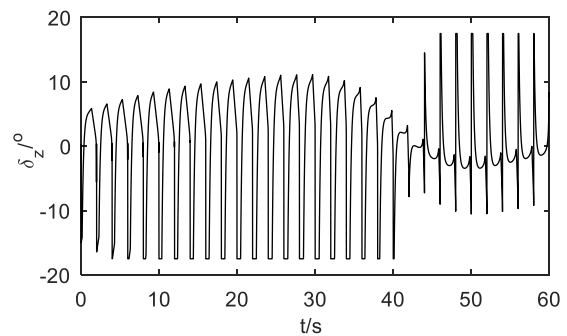


Figure 3.4. Rudder deflection

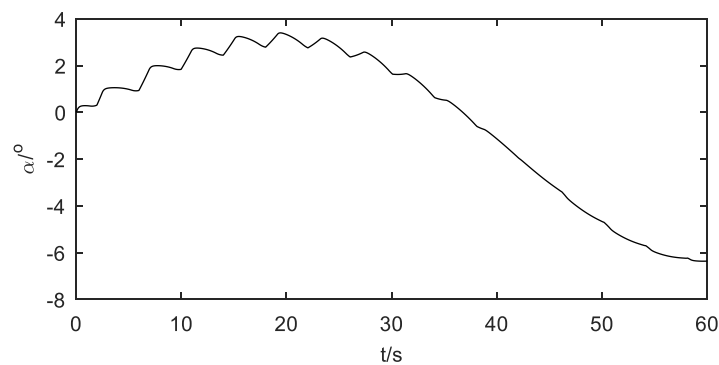


Figure 3.5. Angle of attack

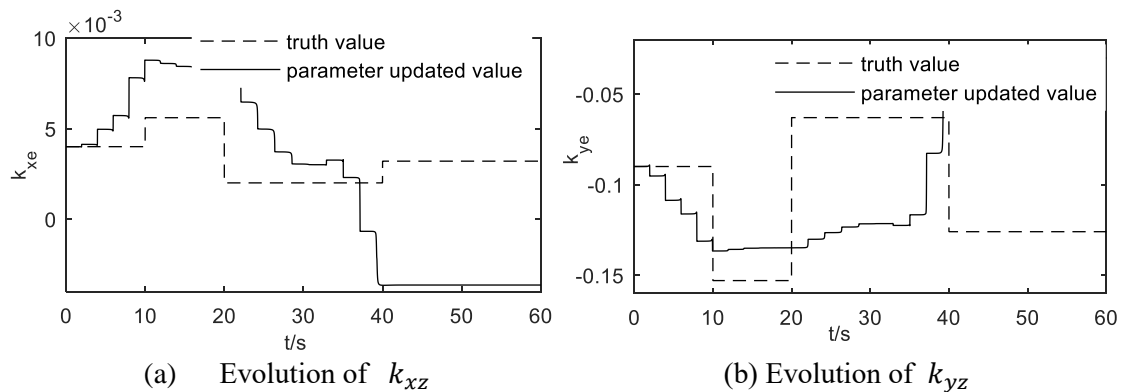


Figure 3.6. Evolutions of parameters

Figures (3.1 – 3.6) show the simulation results. As Figure 3.1 shows, the gliding missile is able to following the plan trajectory, and the tracking error is as Figure 3.2 shows. As presented in Figure 3.3, the sliding mode variable always obtains zero value in the beginning of new point tracking, and holds around that. It should be noted that the sliding mode variable suddenly deviates zero at the end of point tracking. This is because the value of  $r$  becomes very small during that time. Besides, the available deflection angle ranges from  $-18^\circ$  to  $18^\circ$ . Because boundary layer method is adopted to handle  $\text{sign}(s)$  in the control law, there is no chattering phenomenon as Figures (3.4) and (3.5) show. From Figure 3.6 (a, b) we can conclude that the controller does not estimate the parameter perturbation, what it aims to do is making the whole system be stable through the adaptive adjustment of parameters.

#### 4. Conclusion

This paper proposes new guidance law based on adaptive fast terminal sliding mode control for trajectory tracking problem. To deal with the problem of parameter perturbation, we propose an adaptive parameter update strategy, which does not need the known upper bound of perturbation. What's more, the stability of the whole system is proved by employing Lyapunov theory. Simulation results show that the proposed method can cope with parameter perturbation problem, and satisfies the requirement of trajectory tracking.

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