IOP Conference Series: Materials Science and Engineering

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To cite this article: Xueyang Sun *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **569** 042008

A method of evaluating performance of missile three-loop autopilot

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Abstract. Three-loop autopilot is widely used in missile acceleration control. The parameters of the autopilot influence the performance of the autopilot in both frequency-domain and time-domain. However, what we know about the influences are quite qualitative, such as that angular speed feedback parameter provides additional damping to the missile. Problems appear if the control parameters cannot satisfy all the requirements of the autopilot. Based on the qualitative conclusions, it is impossible to meet the frequency-domain or time-domain performance requirements by adjusting the control parameters. This paper provides a method to obtain the first order time constant of the autopilot and frequency-domain performance such as amplitude margin and phase margin of the autopilot. The method can be used to make a rapid evaluation of missile three-loop autopilot performance and provides theoretical guidance to optimize control parameters.

1. Introduction

Three-loop autopilot makes missile performance insensitively to the varying aerodynamic parameters and external persistent disturbances. The angular speed loop and attitude loop of the autopilot can provide additional stability and relax static stability of missile. The acceleration loop ensures that the missile keeps tracking the acceleration command rapidly. The three-loop autopilot is shown in figure 1. The control parameters of the autopilot can be obtained by pole assignment approach, LQR approach, loop-shaping design approach, and so on. However, the relationship between the control parameters and the performance of the autopilot is not distinct. From [1] and [2], merely some qualitative conclusions about the relationship can be obtained.

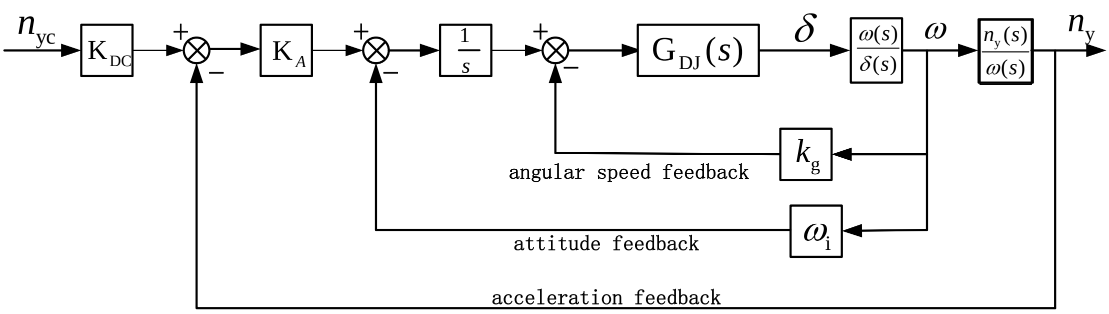

Figure 1. Three-loop autopilot of missile.

The definition of symbols in figure 1 is described in table 1.

IOP Conf. Series: Materials Science and Engineering **569** (2019) 042008 doi:10.1088/1757-899X/569/4/042008

Table 1. The definition of symbols used in figure1.

Symbols	Definition
n_{yc}	Missile acceleration command
n_{y}	Missile acceleration output
δ	Servo deflection
ω	Missile angular speed
$\mathrm{G}_{\mathrm{DJ}}(s)$	Transfer function of servo
$\dfrac{\omega(s)}{\delta(s)}$	Transfer function of missile dynamic (from servo deflection to missile angular speed)
$\dfrac{n_{\mathrm{y}}(s)}{\omega(s)}$	Transfer function of missile dynamic (from missile angular speed to missile acceleration)
K_{A}	Control parameter
ω_{i}	Control parameter
k_{g}	Control parameter
K_{DC}	Normalization coefficient, which can be obtained by control parameters

The expressions of the $\dfrac{\omega(s)}{\delta(s)}$ and $\dfrac{n_{\mathrm{y}}(s)}{\omega(s)}$ are shown in (1) and (2):

$$\frac{\omega(s)}{\delta(s)}=\frac{K_{\mathrm{m}}(T_1 s+1)}{T_{\mathrm{m}}^2 s^2+2\xi_{\mathrm{m}}T_{\mathrm{m}}s+1} \tag{1}$$

$$\frac{n_{\mathrm{y}}(s)}{\omega(s)}=\frac{V}{T_1 s+1} \tag{2}$$

where T_{m} indicates the time constant of the missile, ξ_{m} indicates the missile aerodynamic damping, K_{m} indicates the missile aerodynamic gain, T_1 indicates the time constant of the missile aerodynamic, and V indicates the missile velocity.

T_{m}, ξ_{m}, K_{m} and T_1 are represented as shown in (3):

$$\begin{cases} T_{\mathrm{m}}=(-a_{24}-a_{22}\cdot a_{34})^{-2} \\ \xi_{\mathrm{m}}=\dfrac{1}{2}\cdot(-a_{22}+a_{34})\cdot(-a_{24}-a_{22}\cdot a_{34})^{-2} \\ K_{\mathrm{m}}=-\dfrac{a_{25}a_{34}}{a_{24}+a_{22}\cdot a_{34}} \\ T_1=\dfrac{1}{a_{34}} \end{cases} \tag{3}$$

where a_{24}, a_{25}, a_{22}, a_{34} indicate the missile aerodynamic coefficients(referring to [3] for details).

2. The relationship between autopilot performance and control parameters

In this section, simplified derivations are used to obtain the relationship of control parameters and the autopilot performance (both in time-domain and frequency-domain), and the results of the derivations reveal the relationship succinctly and quantitatively.

2.1. First order time constant evaluation

The bandwidth of the servo is much higher than the cross-over frequency which is obtained by breaking the autopilot loop at the acceleration feedback. Omitting the influence of the servo, we obtain the open-loop transfer function which is broken at the acceleration feedback as shown in (4).

$$G(s)=\frac{KA\cdot K_{\mathrm{m}}\cdot V}{T_{\mathrm{m}}^{2}s^{3}+(2T_{\mathrm{m}}\xi_{\mathrm{m}}+k_{\mathrm{g}}K_{\mathrm{m}}T_{1})s^{2}+(1+w_{\mathrm{i}}K_{\mathrm{m}}T_{1}+k_{\mathrm{g}}K_{\mathrm{m}})s+w_{\mathrm{i}}\cdot K_{\mathrm{m}}} \tag{4}$$

Let $s = jw$, then (4) can be rewritten as:

$$G(jw)=\frac{KA\cdot K_{\mathrm{m}}\cdot V}{w_{\mathrm{i}}K_{\mathrm{m}}-(2T_{\mathrm{m}}\xi_{\mathrm{m}}+k_{\mathrm{g}}K_{\mathrm{m}}T_{1})w^{2}+jw\cdot(1+w_{\mathrm{i}}K_{\mathrm{m}}T_{1}+k_{\mathrm{g}}K_{\mathrm{m}}-T_{\mathrm{m}}^{2}w^{2})} \tag{5}$$

supposing the cross-over frequency of the (5) is w_c, then we can obtain:

$$KA\cdot K_{\mathrm{m}}\cdot V=\left[(w_iK_m-(2T_m\xi_m+k_gK_mT_1)w_c^2)^2+w_c^2\cdot(1+w_iK_mT_1+k_gK_m-T_m^2w_c^2)^2\right]^{1/2} \tag{6}$$

considering $w_{\mathrm{c}}T_{\mathrm{m}}<<1,w_{\mathrm{c}}^{2}T_{\mathrm{m}}\xi_{\mathrm{m}}\approx 0$,we can simplify (6) as:

$$(w_{\mathrm{i}}K_{\mathrm{m}}-k_{\mathrm{g}}K_{\mathrm{m}}T_{1}w_{\mathrm{c}}^{2})^{2}+w_{\mathrm{c}}^{2}\cdot(1+w_{\mathrm{i}}K_{\mathrm{m}}T_{1}+k_{\mathrm{g}}K_{\mathrm{m}})^{2}\approx KA^{2}\cdot K_{\mathrm{m}}^{2}\cdot V^{2} \tag{7}$$

the first term of the left equation is much smaller than the second term, then we can get:

$$w_{\mathrm{c}}\approx\frac{KA\cdot K_{\mathrm{m}}\cdot V}{w_{\mathrm{i}}K_{\mathrm{m}}T_{1}+1}\approx KA\frac{a_{25}\cdot a_{34}\cdot V}{w_{\mathrm{i}}\cdot a_{25}-a_{24}} \tag{8}$$

if the autopilot is equivalent to the first order inertia system, then the autopilot transfer function can be approximated as:

$$G(s)\approx\frac{1}{w_{\mathrm{c}}^{-1}s+1} \tag{9}$$

where w_{c}^{-1} is the first order time constant of the autopilot. From the above results of derivations, the autopilot time-domain performance is obtained:

$$\begin{cases}T_{\mathrm{d}}\approx 0.69\cdot\omega_{\mathrm{c}}^{-1}\\ T_{\mathrm{r}}\approx 2.2\cdot\omega_{\mathrm{c}}^{-1}\\ T_{\mathrm{s}}\approx 3.0\cdot\omega_{\mathrm{c}}^{-1}\end{cases} \tag{10}$$

where T_{d} indicates the delay time, T_{r} indicates the settling time and T_{s} indicates the rise time.

2.2. Frequency-domain performance evaluation

Ignoring the nonlinearity of the servo, the servo can be equivalent to a second order transfer function as:

$$G_{\mathrm{DJ}}(s)=\frac{1}{T_{0}^{2}\mathrm{s}^{2}+2T_{0}\xi_{0}s+1} \tag{11}$$

breaking the autopilot at the output of the servo, the open-loop transfer function of the autopilot is:

$$GH(s)=G_{\mathrm{DJ}}(s)\cdot\frac{k_{\mathrm{g}}K_{\mathrm{m}}T_{1}s^{2}+(w_{\mathrm{i}}K_{\mathrm{m}}T_{1}+k_{\mathrm{g}}K_{\mathrm{m}})s+w_{\mathrm{i}}K_{\mathrm{m}}+KA\cdot K_{\mathrm{m}}\cdot V}{T_{\mathrm{m}}^{2}s^{3}+2T_{\mathrm{m}}\xi_{\mathrm{m}}s^{2}+s} \tag{12}$$

supposing the cross-over frequency of (12) is w_{cR}, the servo bandwidth is usually more than three times higher than w_{cR}, then we can obtain:

$$\left|GH(jw_{cR})\right|=1 \tag{13}$$

let $s = jw_{\mathrm{cR}}$ and substituting (12) into (13):

$$T_m^4 w_{cR}^6 + (4T_m^2\xi_m^2 - 2T_m^2 - k_g^2 K_m^2 T_1^2) w_{cR}^4 + [1-(w_i K_m T_1 + k_g K_m)^2 + 2k_g K_m T_1 \cdot (w_i K_m + K_m^2 V)] w_{cR}^2 = (w_i K_m + K_m^2 V)^2 \tag{14}$$

because $w_{cR} \cdot T_m \approx 1$,then $\left|w_{cR} T_m^2\right| << 1$ .Furthermore $4T_m^2\xi_m^2$ is much smaller than $2T_m^2$ and $k_g^2 K_m^2 T_1^2$,equation(14) can be rewritten as:

$$T_m^4 w_{cR}^6 + (-2T_m^2 - k_g^2 K_m^2 T_1^2) w_{cR}^4 \approx 0 \tag{15}$$

from (15), we can get:

$$w_{cR} \approx (2T_m^2 - k_g^2 K_m^2 T_1^2)^{1/2} \cdot T_m^{-2} \tag{16}$$

substituting (3) into (16), we can get the assessment of w_{cR} as:

$$w_{cR} \approx \sqrt{-2a_{24} + k_g^2 a_{25}^2} \tag{17}$$

from (14) and (15), an inequation can be obtained:

$$k_g^2 K_m^2 T_1^2 w_{cR}^4 >> (w_i K_m + K_m^2 V)^2 \tag{18}$$

according to (11), (12) and (16), the phase margin of the $GH(s)$ is:

$$\phi(w_{cR}) \approx 90^\circ + \arctan\left(\frac{(w_i K_m T_1 + k_g K_m) w_{cR}}{w_i K_m + KA \cdot k_a - k_g K_m T_1 w_{cR}^2}\right) - \arctan\left(\frac{2T_m \xi_m w_{cR}}{1 - T_m^2 w_{cR}^2}\right) - \arctan\left(\frac{2T_0 \xi_0 w_{cR}}{1 - T_0^2 w_{cR}^2}\right) \tag{19}$$

inequation (18) can be used to simplify equation (19), then equation (19) can be rewritten as:

$$\phi(w_{cR}) \approx 90^\circ - \arctan\left(\frac{(w_i T_1 + k_g)}{k_g T_1 w_{cR}}\right) - \arctan\left(\frac{2T_m \xi_m w_{cR}}{1 - T_m^2 w_{cR}^2}\right) - \arctan\left(\frac{2T_0 \xi_0 w_{cR}}{1 - T_0^2 w_{cR}^2}\right) \tag{20}$$

substituting (3) into (20) and defining $n = (T_0 w_{cR})^{-1}$, then the assessment of phase margin is shown in (21).

$$\phi(w_{cR}) \approx 90^\circ - \arctan\left(\frac{(w_i + a_{34} k_g)}{a_{34} \cdot k_g w_{cR}}\right) - \arctan\left(\frac{(a_{22} - a_{34}) w_{cR}}{a_{24} + w_{cR}^2}\right) - \arctan\left(\frac{2\xi_0 n}{n^2 - 1}\right) \tag{21}$$

Another frequency-domain performance we focus on is the amplitude margin. Defining $w = w_p$ which leads to $\phi(w_p) = 0^\circ$. Employing the same method which is used in the above derivations, the expression of w_p can be approximated as:

$$w_p \approx \left[2\xi_0 (a_{22} - a_{34}) T_0^{-1} - T_0^{-2} + a_{24}\right]^{1/2} \tag{22}$$

and the expression of autopilot amplitude margin can be approximated as:

$$G_m \approx 20\lg\left\{k_g a_{25} w_p^{-1} \left[(1 - \frac{1}{N^2})^2 + \frac{4\xi_0}{N^2}\right]^{-2}\right\} \tag{23}$$

where $N = (T_0 w_p)^{-1}$.

The frequency-domain performance of the autopilot can be obtained by equation (17), (21) and (23).

3. Verification of the evaluation method

An air-to-surface missile aerodynamic coefficients in the range of Ma from 0.4Ma~1.05Ma are shown in table 2.

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Table 2. Aerodynamic coefficients.

Ma	0.4	0.6	0.8	0.9	0.95	1.0	1.05
a_{22}	-1.72	-2.57	-3.92	-4.66	-5.11	-5.62	-7.26
a_{34}	0.77	0.90	1.25	1.60	1.76	1.99	2.35
a_{25}	210	220	230	265	275	300	315
a_{24}	-174	-186	-198	-223	-235	-247	-259

The parameters of the missile pitch autopilot are shown in table 3.

Table 3. Control parameters.

Ma	0.4	0.6	0.8	0.9	0.95	1.0	1.05
k_{g}	0.054	0.05	0.048	0.039	0.037	0.034	0.035
w_{i}	0.77	0.795	0.864	0.83	0.845	0.826	0.893
KA	0.067	0.036	0.022	0.013	0.012	0.01	0.009

The servo transfer function has the same expression as (11), and $T_0 = 0.0133$, $\xi_0 = 0.65$ .The comparisons of the autopilot performance and the evaluation results are shown in table 4~table 7.

Table 4. Comparison result of w_{c} .

Ma	0.4	0.6	0.8	0.9	0.95	1.0	1.05
evaluation	4.84	4.87	4.90	4.79	4.91	4.79	4.94
calculation	5.04	5.03	5.01	4.81	4.93	4.76	4.86

Table 5. Comparison result of w_{cR} .

Ma	0.4	0.6	0.8	0.9	0.95	1.0	1.05
evaluation	18.36	19.06	20.04	21.22	21.83	22.49	23.61
calculation	18.62	19.37	20.43	21.72	22.33	23.03	24.24

Table 6. Comparison result of $\phi(w_{\mathrm{cR}})$.

Ma	0.4	0.6	0.8	0.9	0.95	1.0	1.05
evaluation	85.63	87.82	90.52	95.38	96.16	97.94	99.70
calculation	82.51	84.33	86.43	91.76	92.35	94.37	95.80

Table 7. Comparison result of G_{m} .

Ma	0.4	0.6	0.8	0.9	0.95	1.0	1.05
evaluation	11.58	11.85	12.23	12.93	12.95	13.43	13.59
calculation	9.11	9.44	10.35	10.68	10.67	11.25	12.31

Analyzing the data in table 4~table 7, some conclusions can be drawn as:

- In time-domain, the evaluations of the w_{c} is highly consistent with the calculation result, and the relative error is no more than 4%.

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- In frequency-domain, the evaluations of w_{cR} and $\phi(w_{cR})$ are highly identical with the calculation results, and the relative error is no more than 5%.
- Though the relative error of evaluation G_m is lightly large, which reaches about 20%, the precision of the assessment is acceptable.

4. Conclusion

This article provides a new and succinct method to obtain the missile performance by simplifying the formula derivations. From the results of the derivations, we obtain the relationship between the missile autopilot performance and the control parameters. Employing the same method of formula derivations, we can acquire that simplified performance expressions of two-loop acceleration autopilot are identical with those of the three-loop autopilot. Approximated expressions of other types autopilot performance, which are composed of control parameters and missile aerodynamic coefficients, can be obtained as well. The approximated expressions provide theoretical guidance to optimize autopilot parameters and evaluate autopilot performance rapidly.

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