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# Design of sparse cosine modulated filter banks based on Hopfield neural network

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**Abstract.** Cosine modulated filter bank (CMFB) is the most basic and important module in multi-rate signal processing system. Sparse FIR linear-phase CMFB can not only reduce the complexity of hardware design, but also ensure good performance. In this paper, a new design method of CMFB is proposed. First, the set of sparse coefficients of the prototype filter is found by using the orthogonal matching pursuit (OMP) algorithm. Then the Hopfield neural network (HNN) is employed to optimize the non-zero coefficients. The analytical and simulation results demonstrate that the new approach not only ensures that the sparse CMFBs satisfy near perfect reconstruction (NPR) , but also improves the efficiency of hardware operation.

## 1. Introduction

Cosine modulated filter banks have many advantages, such as simple design, convenient process of accurately reconstructing signals, parallel processing of multi-rate signals and so on. They are widely used in the fields of speech signal, image signal processing and military applications. There are a class of methods of CMFB [1]-[5], which are simple in design and have the characteristics of low delay, linear phase and perfect reconstruction (PR).

In the past literatures, there are two main methods to design CMFBs. The one is to design them by lattice structure, which guarantees the perfect reconstruction of CMFB and makes them have good robustness, but the stopband attenuation of the designed subband filters is not ideal. The other is to design CMFB by least squares method which has good stopband attenuation, but can't hold the PR property. The Weighted constrained least square (WCLS) algorithm [7] is designed by A. Kumar, which transforms the design problem into linear optimizations of filter coefficients such that their value at $\omega=\pi/2M$ is 0.707. Since the number of order for prototype filter is very large, it causes the bigger delay of the material object. In reference [8], it is proposed that the quantum-behaved particle swarm optimization (QPSO) algorithm. Through fixing stopband cut-off frequency, the passband cut-off frequency is adjusted to minimize the cost function satisfying the condition of PR. As the amount of coefficients of CMFB increases with the number of channels, the quantity of adders and multipliers required is also very large. However, up to now, the sparse design of CMFB has not been proposed.

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In this paper, we propose a novel paradigm developed to design sparse CMFBs based on HNN [6]. The design procedure can be proceeded through two steps: First, the OMP algorithm [9] [10] is applied to obtain the positions of sparse coefficients of prototype filter. Second, the non-zero coefficients are optimized by HNN. As a result, the simulation demonstrates that the analysis filters and synthesis filters obtain good magnitude response, and the overall system satisfies the condition of NPR. The experimental results demonstrate the effectiveness of this new method.

The rest of this paper is organized as follows: In Section 2, the struction of CMFB is formulated briefly. In Section 3, the HNN is applied to the prototype filter design. In Section 4, we present the procedure of designing the sparse linear-phase FIR prototype filter based on the OMP algorithm. The simulation results are showed in Section 5.

## 2. Cosine modulated filter banks

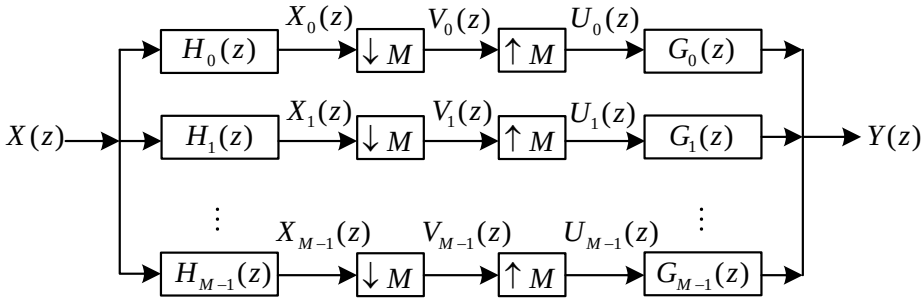



Figure 1. M channel maximally decimated filter bank

A maximally decimated M channel filter bank is demonstrated in Figure 1. $\{H_k(z)\}_{k=0}^{M-1}$ is the analysis filters and $\{G_k(z)\}_{k=0}^{M-1}$ is the synthesis filters. In the CMFBs, if $h(n)$ represents the impulse response coefficients of the prototype filter, the coefficients of the analysis filters $h_k(n)$ and synthesis filters $g_k(n)$ are represented as:

$$h_k(n) = 2h(n)\cos[(k+\frac{1}{2})(n-\frac{N}{2})\frac{\pi}{M} + (-1)^k\frac{\pi}{4}] \tag{1}$$

$$g_k(n) = 2h(n)\cos[(k+0.5)(n-\frac{N}{2})\frac{\pi}{M} - (-1)^k\frac{\pi}{4}] \tag{2}$$

where $n = 0, 1, \cdots, N-1$, $k = 0, 1, \cdots, N-1$, and $N$ represents the order of prototype filter. Through cosine modulation of the prototype filter, the analysis filters and synthesis filters can be obtained. The reconstructed output $Y(z)$ is

$$Y(z) = \left[\frac{1}{M}\sum_{k=0}^{M-1}H_k(z)G_k(z)\right]X(z) + \sum_{l=0}^{M-1}\left[\frac{1}{M}\sum_{k=0}^{M-1}H_k(zW_M^l)G_k(z)\right]X(zW_M^l)\,. \tag{3}$$

To ensure the PR property, the distortion transfer function $T_0(z) = (1/M)\sum_{k=0}^{M-1}H_k(z)G_k(z)$ and the aliasing transfer function $T_l(z) = (1/M)\sum_{k=0}^{M-1}H_k(zW_M^l)G_k(z)$ must satisfy the following conditions:

$$T_l(z) = 0, \quad l = 1, 2, \cdots, M-1. \tag{4}$$

$$T_0(z) = z^{-k} \tag{5}$$

Without loss of generality, we only consider the design of Type-II linear-phase FIR prototype filter (i.e. $h(n)=h(N-1-n)$ for all $0\le n\le N-1$ with $N$ being even). For other types, an argument similar to the one developed in this paper can be followed. Then the frequency response can be expressed as

$$H(e^{j\omega})=e^{-j\frac{N}{2}\omega}H(\omega)=e^{-j\frac{N}{2}\omega}\left(\sum_{i=1}^{N/2}b_i\cos((i-\frac{1}{2})\omega)\right) \tag{6}$$

where $b_i=2h(N/2-i)$, $H(\omega)$ is real-value amplitude response of the prototype filter, the PR conditions of equation (4) and equation (5) for linear-phase CMFB can be reduced to

$$\left|H(\omega)\right|^2+\left|H(\omega-\frac{\pi}{M})\right|^2=1,\quad \omega\in[0,\frac{\pi}{M}] \tag{7}$$

$$\left|H(\omega)\right|^2=0,\quad \omega\in(\frac{\pi}{M},\pi] \tag{8}$$

## 3. Prototype filter design based on HNN

Given the design specifications, the mean square error between the desired response $H_d(\omega_l)$ and the corresponding filter response to be designed can be expressed as:

$$E_2=\sum_{l=0}^{L}\left|E(\omega_l)\right|^2=\sum_{l=0}^{L}\left|H_d(\omega_l)-H(\omega_l)\right|^2 \tag{9}$$

where $\{\omega_l\}_{l=0}^{L}$ are sampling frequencies on $[0,\pi]$. Substituting (6) into (9), we can obtain

$$E_2=\sum_{l=0}^{L}\left[H_d(\omega_l)^2+\sum_{i=1}^{N/2}\sum_{j=1}^{N/2}b_i\cos((i-\frac{1}{2})\omega_l)\cos((j-\frac{1}{2})\omega_l)b_j-2H_d(\omega_l)\sum_{i=1}^{N/2}\cos((i-\frac{1}{2})\omega_l)b_i\right] \tag{10}$$

Since $H_d(\omega_l)$ is independent of the filter coefficients to be designed, the first item in the equation can be ignored. Thus the error function is:

$$E=\sum_{l=1}^{L}\left[\sum_{i=1}^{N/2}\sum_{j=1}^{N/2}b_i\cos((i-\frac{1}{2})\omega_l)\cos((j-\frac{1}{2})\omega_l)b_j-2H_d(\omega_l)\sum_{i=1}^{N/2}\cos((i-\frac{1}{2})\omega_l)b_i\right] \tag{11}$$

In order to optimize (11) and guarantee the real-time conditions, in this design, the HNN is employed to design prototype filter for CMBF. The Lyapunov energy function of HNN [6] is:

$$E_L=-\frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{p}T_{ij}v_iv_j-\sum_{i=1}^{p}I_iv_i \tag{12}$$

where $p$ is the number of neurons, $v_i$ indicates the output state of the $i$-th neuron, $T_{ij}$ and $I_i$ the weight and bias of the $i$-th neuron in the neural network respectively. Comparing equations (11) and (12), the parameters of the HNN are defined as:

$$T_{ij}=-\sum_{l=1}^{L}\cos[(i-\frac{1}{2})\omega_l]\cos[(j-\frac{1}{2})\omega_l] \tag{13}$$

$$I_i=\sum_{l=1}^{L}H_d(\omega_l)\cos[(i-\frac{1}{2})\omega_l] \tag{14}$$

$$b_i=v_i,\quad i=1,2,\cdots p \tag{15}$$

with $p=N/2$.

After the two parameters in the neural network are determined, the dynamic nonlinear differential equation of the HNN can be employed to optimize

$$C_i\frac{du_i}{dt}=-\frac{u_i}{\rho_i}+\sum_{j=1}^{p}T_{ij}v_j+I_i \tag{16}$$

where $u_i$ represents the initial state of the $i$-th neuron, $C_i$ and $\rho_i$ represents the resistance of the input capacitor and parallel connection. $-u_i/\rho_i$ can be ignored owing to $\rho_i=\infty$. In this system, the relationship between input voltage $u_i$ and output voltage $v_i$ is:

$$v_i = g(u_i) = u_i/\lambda \qquad -b\lambda < u_i < b\lambda, 1 \le i \le p \tag{17}$$

where $b$ is the dynamic range and $1/\lambda$ is the slope limiter.

## 4. Sparse linear-phase FIR prototype filter design for NPR CMFBs

In this section, the sparse CMFB is developed by the iterative optimization, which is based on the OMP algorithm and the HNN algorithm. Given the design specifications, the problem of designing sparse linear-phase FIR prototype filter for CMFB can be formulated as

$$\min \quad \|\mathbf{b}\|_0 \tag{18a}$$

$$s.t. \quad |\mathbf{Cb}-\mathbf{d}| \le e \tag{18b}$$

where $e$ represents ripples, $\mathbf{b}=2[h(N/2-1),\cdots,h(N/2-m),\cdots,h(1),h(0)]^T$ is the vector form of the impulse response of the designed prototype filter, $\mathbf{d}=\left[H_d(\omega_1),\cdots,H_d(\omega_l),\cdots,H_d(\omega_L)\right]^T$ is the desired frequency response of prototype filter. $\mathbf{C}$ is a sampling matrix, i.e.,

$$\begin{aligned}\mathbf{C} &= \left[\mathbf{c}_1\ \mathbf{c}_2\ \cdots\ \mathbf{c}_{N/2}\right] \\ &= \begin{bmatrix} \cos(1-\frac{1}{2})\omega_1 & \cos(2-\frac{1}{2})\omega_1 & \cdots & \cos(\frac{N}{2}-\frac{1}{2})\omega_1 \\ \cos(1-\frac{1}{2})\omega_2 & \cos(2-\frac{1}{2})\omega_2 & \cdots & \cos(\frac{N}{2}-\frac{1}{2})\omega_2 \\ \vdots & \vdots & \ddots & \vdots \\ \cos(1-\frac{1}{2})\omega_L & \cos(2-\frac{1}{2})\omega_L & \cdots & \cos(\frac{N}{2}-\frac{1}{2})\omega_L \end{bmatrix}\end{aligned} \tag{19}$$

Then, normalize each column of $\mathbf{C}$ defined in (19), i.e.,

$$\mathbf{A} = \left[\mathbf{a}_1,\ \mathbf{a}_2,\ \cdots,\ \mathbf{a}_{N/2}\right] = \left[\frac{\mathbf{c}_1}{\|\mathbf{c}_1\|_2},\ \frac{\mathbf{c}_2}{\|\mathbf{c}_2\|_2},\cdots,\ \frac{\mathbf{c}_{N/2}}{\|\mathbf{c}_{N/2}\|_2}\right] \tag{20}$$

Thus, in view of (20), (18) becomes

$$\min \quad \|\mathbf{x}\|_0 \tag{21a}$$

$$s.t. \quad |\mathbf{Ax}-\mathbf{d}| \le e \tag{21b}$$

where $\mathbf{x}=\left[2h(N/2-1)\|\mathbf{c}_1\|_2,\ \cdots,\ 2h(l)\|\mathbf{c}_{N/2-1}\|_2, 2h(0)\|\mathbf{c}_{N/2}\|_2\right]$.

Utilizing the OMP algorithm to solve (21), the procedure of designing prototype filters proceeds as follows:

Step 1: Initialize the residual vector $\mathbf{r}^{(0)}=\mathbf{d}$. Choose the initial index set $S^{(0)}$ and the matrix of atoms $\Phi^{(0)}$ being an empty set. Let $k=1$ and $\Omega^{(0)}=\{1, 2, \cdots, N/2\}$.

Step 2: For $k \ge 1$, find the index $m_k (m_k \in \Omega^{(k)})$ such that the column vector $\mathbf{a}_{m_k}$ maximizes $\left\langle \mathbf{r}_{k-1}, \mathbf{a}_{m_k} \right\rangle$, $\langle \cdot,\cdot \rangle$ represents the inner product operation. Then update the set $S^{(k)}=S^{(k-1)} \cup \{m_k\}$, $\Omega^{(k)}=\Omega^{(k-1)}-\{m_k\}$. $\Phi^{(k)}=[\mathrm{c}_{m_1}^{(k)}, \mathrm{c}_{m_2}^{(k)}, \cdots, \mathrm{c}_{m_k}^{(k)}]$ is the matrix of chosen atoms.

Step 3: Define an $(N/2)\times(N/2)$ matrix $\mathbf{T}$ with its element $T_{ij}$ being computed through (13) and an $(N/2)\times 1$ vector $\mathbf{I}$ with its element $I_i$ being computed through (14). Substituting the set $S^{(k)}$ into (13) and (14) to obtain the weight matrix $\mathbf{T}^{(k)}$ and bias vector $\mathbf{I}^{(k)}$ of HNN, i.e.,

$$\mathbf{T}^{(k)}=\mathbf{T}_{S^{(k)}} \qquad \mathbf{I}^{(k)}=\mathbf{I}_{S^{(k)}} \tag{22}$$

where $\mathbf{T}_{S^{(k)}}$ contains the columns of $\mathbf{T}$ indexed by $S^{(k)}$, $\mathbf{I}_{S^{(k)}}$ contains the elements of $\mathbf{I}$ indexed by $S^{(k)}$. The HNN is employed to optimize the parameters of non-zero position. If it reaches the maximum number of iterations, and the energy function of the system still does not reach the minimum value, step 5 is carried out directly; if the HNN reaches the stability, that is, the error function reaches the minimum value. Then the output voltage is the vector form $\mathbf{x}^{(k)}$ of the impulse response of the prototype filter corresponding to the set $S^{(k)}=\{m_1,m_2,...,m_k\}$.

Step 4: The impulse response $\mathbf{x}^{(k)}$ is substituted into (1) and (2), then calculate the amplitude distortion $e_{am}^{(k)}$ and the total aliasing error $e_a^{(k)}$ by the followings:

$$e_{am}^{(k)}=\max(1-|T_0(e^{j\omega})|) \tag{23}$$

$$e_a^{(k)}=\max(\sqrt{\sum_{l=1}^{M-1}|T_l(e^{j\omega})|^2}) \tag{24}$$

If $e_{am}^{(k)}$ and $e_a^{(k)}$ satisfy the specifications, the iteration is stopped. Then $\mathbf{x}^{(k)}$ is the sparse prototype filter for NPR CMFB, otherwise enter step 5.

Step 5: Compute the new residual vector $\mathbf{r}^{(k)}$ as

$$\mathbf{r}^{(k)}=\mathbf{d}-\Phi^{(k)}\mathbf{x}^{(k)} \tag{25}$$

Then let $k=k+1$ and repeat the procedure from Step2 to Step5.

## 5. Example

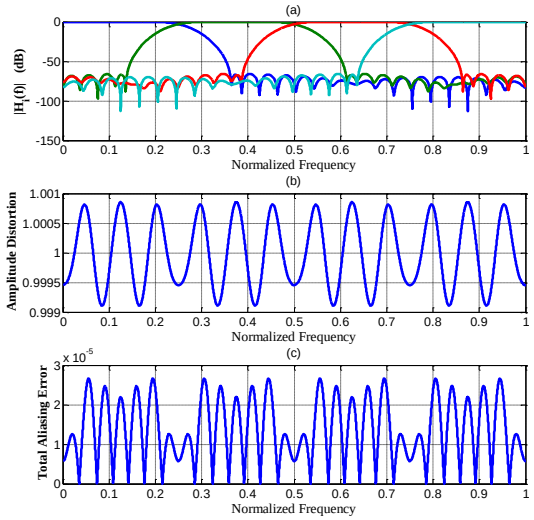



Figure 2. 4-channel CMFB in example 1: (a) Frequency response of the analysis filters; (b) Amplitude distortion; (c) Aliasing distortion.

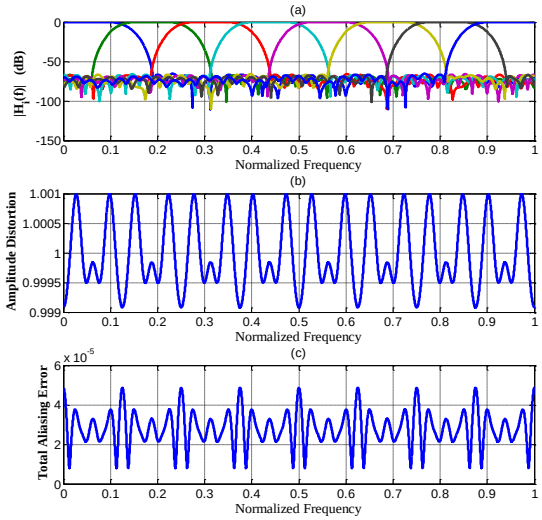



Figure 3. 8-channel CMFB in example 2: (a) Frequency response of the analysis filters; (b) Amplitude distortion; (c) Aliasing distortion.

In this subsection, the performance of the CMFB yielded from the proposed design is compared with the WCLS algorithm [7] and the QPSO algorithm [8]. The obtained results demonstrate that the

proposed scheme not only guarantees the designed CMFBs satisfying the condition of NPR [3], but also reduces the complexity of the CMFB effectively.

Table 1. Comparison of the parameters of filter banks designed by this proposed algorithm and other algorithms

| $M$ | Design method | Filter order | Number of nonzero taps | Amplitude distortion | Total aliasing error |
|---|---|---|---|---|---|
| **4** | WCLS | 80 | 80 | $3.10\times10^{-3}$ | $1.06\times10^{-6}$ |
| | QPSO | 80 | 80 | $7.01\times10^{-4}$ | $8.49\times10^{-8}$ |
| | Proposed | 108 | 52 | $8.92\times10^{-4}$ | $2.66\times10^{-5}$ |
| **8** | WCLS | 144 | 144 | $1.80\times10^{-3}$ | $3.90\times10^{-7}$ |
| | QPSO | 128 | 128 | $3.97\times10^{-4}$ | $2.41\times10^{-6}$ |
| | Proposed | 140 | 78 | $9.20\times10^{-4}$ | $4.87\times10^{-5}$ |
| **16** | WCLS | 224 | 224 | $1.30\times10^{-3}$ | $9.03\times10^{-7}$ |
| | QPSO | 256 | 256 | $4.73\times10^{-4}$ | $2.34\times10^{-6}$ |
| | Proposed | 254 | 156 | $8.91\times10^{-4}$ | $9.76\times10^{-5}$ |

Example 1: The example is concerned with a 4-channel CMFB, in which the order of the prototype filter is $N=108$ and the number of non-zero coefficients is 52. In Figure 2, the amplitude response of analytic filter banks designed by this algorithm is shown. Through formulas (23) and (24), the amplitude distortion $e_{am}$ and the total aliasing error for the designed CMFB are $8.92\times10^{-4}$ and $2.66\times10^{-5}$, respectively.

In Example 1 of Table 1, under the NPR condition, the number of nonzero coefficients of the prototype filter designed from our scheme is 28 fewer than that of the WCLS algorithm and the QPSO algorithm. The data corresponding to the 4-channel CMFB demonstrates that the number of nonzero tap weights can be reduced significantly by the proposed method.

Example 2: The example is concerned with a 8-channel CMFB, in which the order of the prototype filter is $N=140$ and the number of non-zero coefficients is 78. In Figure 3, the amplitude response of analytic filter banks designed by this algorithm is shown. Through formulas (23) and (24), the amplitude distortion $e_{am}$ and the total aliasing error for the designed CMFB are $9.20\times10^{-4}$ and $4.87\times10^{-5}$, respectively.

In Example 2 of Table 1, under the NPR condition, the number of nonzero coefficients of the prototype filter designed from our scheme is 66 fewer than that of WCLS algorithm and 50 fewer than that of QPSO algorithm respectively. The data corresponding to the 8-channel CMFB demonstrates that the number of nonzero tap weights can be reduced significantly by the proposed method.

Example 3: The example is concerned with a 16-channel CMFB, in which the order of the prototype filter is $N=254$ and the number of non-zero coefficients is 156. In Figure 4, the amplitude response of analytic filter banks designed by this algorithm is shown. Through formulas (23) and (24), the amplitude distortion $e_{am}$ and the total aliasing error for the designed CMFB are $8.91\times10^{-4}$ and $9.76\times10^{-5}$, respectively.

In Example 3 of Table 1, under the NPR condition, the number of nonzero coefficients of the prototype filter designed from our scheme is 68 fewer than that of WCLS algorithm and 100 fewer than that of QPSO algorithm respectively. The data corresponding to the 16-channel CMFB demonstrates that the number of nonzero tap weights can be reduced significantly by the proposed method.

## 6. Conclusion

In this paper, a novel algorithm is developed to design the sparse CMFBs based on the HNN. The positions of sparse coefficients of prototype filter are obtained by OMP algorithm. Then, the HNN is employed to optimize the non-zero coefficients. By the procedure, the implemental complexity of designing the CMFB can be drastically reduced. The simulation results demonstrate the effectiveness of our algorithm.

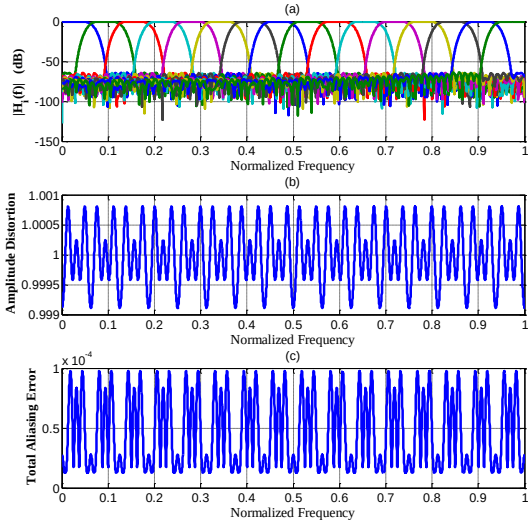



Figure 4. 16-channel CMFB in example 3: (a) Frequency response of the analysis filters; (b) Amplitude distortion; (c) Aliasing distortion.

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