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# A Method of Stable Extended Kalman filter

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**Abstract.** The extend kalman filter provides an stability solution to the approximate nonlinear-Gaussian filter problem. Due to the limitation of the word length of the processor, the rounding errors is easy to occur in the calculation, which causes the covariance matrix to lose its positive definite and convergence. In order to solve this problem, The proposed stable extended kalman filter (SEKF) method is applied to the problem of non-positive and divergence in recursive filter. This method introduces the matrix decomposition and constructs a covariance square root adaptive filter which is used to solve the numerical stability problem of the extend kalman filter. Experimental results show that SEKF can effectively guarantee symmetric positive definite in recursive calculation, and perform remarkably better than EKF algorithm.

## 1. Introduction

The emergence of Kalman filter overcomes the limitations of the classical Wiener filter algorithm<sup>[1]</sup>. Kalman filter is an optimized recursive solution that is the smallest of mean square error to realize estimation. In the time domain, the algorithm makes use of the state space method to design the filter, which can estimate the stochastic process of multi-dimensional systems and non-stationary systems. The Kalman filter can optimally estimate linear dynamic systems under random noise because of its own advantages: simple calculation, self-adaptive, recursive operation. The Kalman filter has been used in such diverse areas as pattern recognition and target tracking. More recently, it has been used in some engineering applications such as radar systems, military missile tracking, industrial control, robot control and navigation<sup>[2]</sup>. Kalman filter can provide an optimal filter result for linear Gaussian problem. However, in various engineering applications, most of the filter problems are basically based on nonlinear Gaussian filter, so an approximate linear non-Gaussian filter methodology that is known as extended Kalman filter is finally proposed<sup>[3]</sup>. Extended Kalman filter provides an efficient algorithm for approximate linearized least mean square estimation of discrete linear system states<sup>[4-5]</sup>.

In practical engineering, rounding errors due to the limitation of the word length lead to lose symmetry and positive definiteness of covariance matrix, which results in filter divergence between the filtered result value and the true value. The main reason is that the calculation is truncated due to insufficient word length of the hardware platform used in the filter. The recursive process of the filter methodology gradually accumulates the calculation error, and finally the difference between the filtered value and the true value becomes larger and larger. Finally, the filter divergence is caused by rounding error. In order to avoid filter divergence, we can reduce the truncation error by upgrading the hardware platform for increasing the double word length, but the hardware cost will be increased. The SEKF filter constructs an adaptive method of covariance square root by matrix decomposition, which achieves the symmetric positive definiteness of matrix covariance in the filter process, and ensures the filter stability without adding the hardware cost.



## 2. Extended Kalman Filter

In extended Kalman filter, state prediction and observation prediction are calculated by nonlinear functions, and state transition matrix  $F$  and observation matrix  $H$  can be replaced by the Jacobian matrix of the nonlinear functions  $f$  and  $h$ , which are obtained by using the partial derivatives. The extended Kalman filter algorithm involves two stages: state prediction and measurement update. The standard extended Kalman filter equations for the prediction stage are

$$X_{k,k-1} = F_{k,k-1} X_{k-1} \quad (3)$$

$$P_{k,k-1} = F_{k,k-1} P_{k-1,k-1} F_{k,k-1}^T + Q_k \quad (4)$$

The measurement update equations are given by

$$K_{k,k} = P_{k,k-1} H_{k,k}^T [H_{k,k} P_{k,k-1} H_{k,k}^T + R_k]^{-1} \quad (5)$$

$$X_{k,k} = X_{k,k-1} + K_{k,k} (Z_k - H_{k,k} X_{k,k-1}) \quad (6)$$

$$P_{k,k} = (I - K_{k,k} H_{k,k}) P_{k,k-1} \quad (7)$$

The extended Kalman filter is a process of sequential calculation method, which can compute the current optimal estimation value according to the input data of the current time and store the estimate of the previous time. The advantage of extended Kalman filter is that it does not need to store a large amount of historical information, thus it reduces the dependence on the storage capacity of the computer. The extended Kalman filter meets the requirements of real-time and less-storage in the engineering [6].

## 3. Related Work

As described above in section 1, a large number of matrix calculations need to be completed in the EKF, and it is easily limited by the word length of the processor. In the calculation process, the error is easily lost due to the occurrence of rounding errors. Consequently non-positive definiteness leads to filter divergence. In the process of filter, the covariance matrix  $P$  is a positive definite symmetric matrix, so matrix decomposition can be introduced to construct adaptive filter algorithm of covariance square root to solve the numerical stability problem. According to matrix theory, a positive definite symmetric matrix can be decomposed into the form of LULT, where  $L$  is the unit lower triangular matrix and  $U$  is the diagonal matrix with positive diagonal. Obviously, the positive definiteness of the  $P$  matrix is the same as that of the  $U$  matrix. Thus, in the recursive calculation of covariance, if the diagonal element of  $U$  matrix is greater than zero, the positive definiteness of the  $P$  matrix is also greater than zero.

### 3.1. Matrix decomposition

In the extended Kalman filter, the  $P$  matrix is a symmetric positive definite matrix. [7], When the main order of the matrix is not zero, Matrix  $P$  has a unique decomposition  $P=L \times D$ . In the diagonal elements of matrix  $D$ ,  $d_{ii}$  is not equal to 0, each row of the matrix  $D$  is presented in turn.

$$\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ l_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ l_{n1} & l_{n2} & \cdots & 1 \end{pmatrix} \times \begin{pmatrix} 1 & d_{12} & \cdots & d_{1n} \\ 0 & 1 & \cdots & d_{2n} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \quad (8)$$

Formula (8) can be decomposed into  $UD$ :

$$U = \begin{pmatrix} d_{11} & 0 & \dots & 0 \\ 0 & d_{21} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & d_{nn} \end{pmatrix} \quad \hat{D} = \begin{pmatrix} \frac{1}{d_{11}} & \frac{d_{12}}{d_{11}} & \dots & \frac{d_{1n}}{d_{11}} \\ 0 & \frac{1}{d_{21}} & \dots & \frac{d_{2n}}{d_{21}} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \frac{1}{d_{nn}} \end{pmatrix} \quad (9)$$

where  $P$  is a symmetric matrix and  $P$  is equal to  $P^T$ , i.e.,

$$(LUD)^T = \hat{D}^T \times U^T \times L^T = L \times U \times \hat{D} \quad (10)$$

Accordingly, the diagonal matrix is equal to the transpose of the diagonal matrix, and decomposition has uniqueness, i.e.,  $L^T = \hat{D}$ ,  $P = L \times U \times L^T$

$$L = \begin{pmatrix} 1 & 0 & \dots & 0 \\ l_{21} & 1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ l_{n1} & l_{n2} & \dots & 1 \end{pmatrix} \quad U = \begin{pmatrix} d_{11} & 0 & \dots & 0 \\ 0 & d_{21} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & d_{nn} \end{pmatrix} \quad (11)$$

where  $L^T$  is the transposed matrix of  $L$ .

This matrix decomposition can be used not only to solve linear algebraic equations in computer programs<sup>[10]</sup>, but also to solve the filter divergence problem by appropriate conversion. Therefore, the  $L$  matrix and the  $U$  matrix can be stored by  $N$ -dimension array.

### 3.2. Stable Extended Kalman filter

In engineering project, the application of the extended Kalman filter algorithm is mainly limited by the effective word length of the processor, it is easy to generate rounding errors in the filtering process, resulting in the loss of symmetry and positive definiteness of covariance matrix, and the filter error will be oscillating, so the filtering results will be distorted, and sometimes even the filter may be diverged<sup>[8,9]</sup>. In order to solve this problem, this paper introduces the extended Kalman filter method of matrix decomposition.

The SEKF achieves optimality through an iterative feedback with two update steps, which are the prediction step and the update step. In the prediction step, the time update equations are responsible for projecting forward (in time) current state and error covariance estimates to obtain the a priori estimates for the next time step.

$$X_{k,k-1} = F_{k,k-1} X_{k-1} \quad (12)$$

$$P_{k,k-1} = T_{k,k-1} U_{k-1,k-1} T_{k,k-1}^T + Q_k \quad (13)$$

where  $T_{k,k-1} = F_{k,k-1} L_{k-1,k-1}$

In the update step, the measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

$$K_{k,k} = G_{k,k} S_{k,k}^{-1} \quad (14)$$

$$X_{k,k} = X_{k,k-1} + G_{k,k} S_{k,k}^{-1} (Z_{k,k} - H_{k,k} X_{k,k-1}) \quad (15)$$

$$P_{k,k} = L_{k,k-1} (U_{k,k-1} - J_{k,k} S_{k,k}^{-1} J_{k,k}^T) L_{k,k-1}^T + Q_k \quad (16)$$

where  $J_{k,k} = U_{k,k-1} L_{k,k-1}^T H_{k,k}^T$ ,  $G_{k,k} = L_{k,k-1} J_{k,k}$

$$S_{k,k} = H_{k,k} G_{k,k} + R_k \quad (17)$$

The above equations describe the recursive process of introducing matrix decomposition in the filter process.

#### 4. Experiments

In order to verify the effectiveness of the algorithm, the algorithm was simulated by Monte-Carlo, and the number of simulations was set to 200 times. Considering the target maneuver tracking, this paper compares the EKF and SEKF algorithms by simulating the typical constant acceleration maneuvering motion. The sampling period is  $T=1s$ . The experiment takes a uniform acceleration maneuver as an example, assuming that the starting position of the target is 9500m and the speed is 70m/s, and the heading angle is  $120^\circ$ , the random variables of process noise and measurement noise have approximately the following probability distributions:

$$Q \sim N(10, 2),$$

$$R \sim N(8, 1),$$

The accuracy of target state can be estimated by statistical root mean square error (RMSE), and the result has already been shown in Table. 1.

Table 1. Root mean square comparison of two filter algorithms

Algorithm	EKF	SEKF
Distance (m)	67.673	27.048
Velocity (m/s)	6.807	3.017

By the decomposition of the covariance P matrix, the algorithm reduces the rounding error caused by the inversion of the P matrix. It can be seen from Table 2 that SEKF ascends operation speed by reducing the computational complexity of the algorithm. SEKF is beneficial to the full utilization of resources and meets the requirements of target tracking in complex scenarios.

Table 2. Running time comparison

Algorithm	EKF	SEKF
Time(s)	0.00513	0.00381

Through the root mean square statistical analysis, the filter effect of the two algorithms in the target tracking can be shown in Fig.1 and Fig.2. When the filter process is starting, the SEKF algorithm converges quickly, and the EKF algorithm does not converge, furthermore, the EKF result always oscillates. It can be clearly seen from Fig. 1 and Fig. 2 that the SEKF effect is better than the EKF algorithm. What's more, SEKF has higher precision capability on tracking target

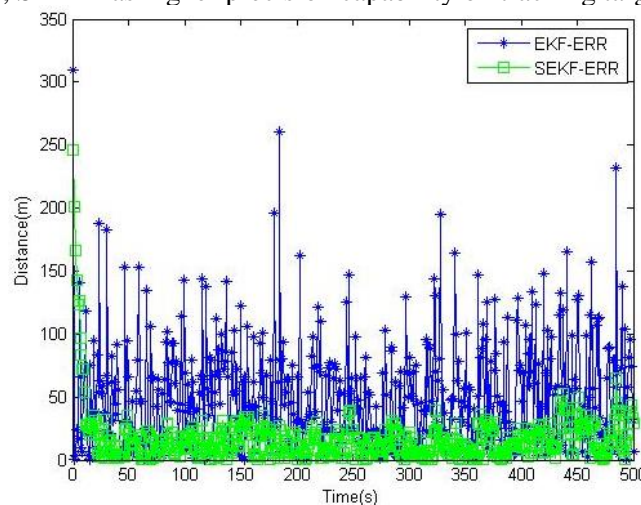


Fig.1 Comparison of tracking result of position

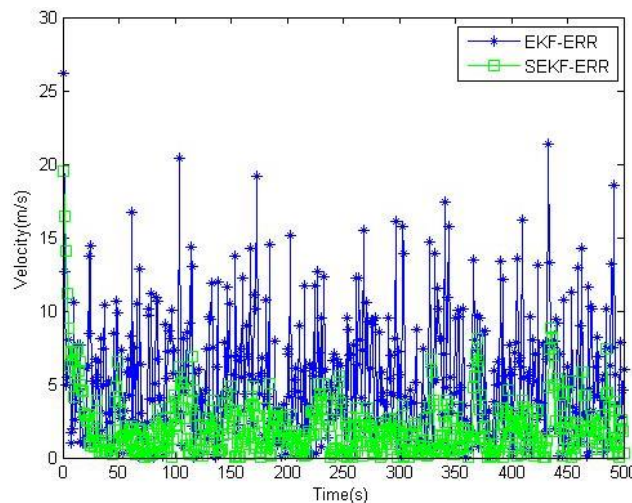


Fig.2 Comparison of tracking result of velocity

In addition, the SEKF algorithm is simple, so it is easy to implement. SEKF has the characteristic of faster speed, stable and accuracy, so it can suppress divergence very well. The main reason is that SEKF can better adapt to change in complex environments, so that the algorithm always keeps track of the target.

## 5. Conclusions

The extended Kalman filter algorithm is characterized by simplicity and easy implementation. However, in the nonlinear gaussian environment, the tracking performance may be degraded due to computer word length truncation. Therefore, a SEKF filter algorithm is proposed for solving divergence problem. From the simulation results, the proposed algorithm has better performance than the extended Kalman filter in both filter stability and computation time. The algorithm improves the tracking accuracy and has certain application value in engineering.

## References

- [1] Cui,J.T.,Chen,G.R. (2013)Kalman filter and its real-time application[M].Tsinghua University Press,the Beijing.
- [2] Fu,M.Y.,Deng,Z.H.,Zhang,J.W. (2010)Kalman Filter Theory and Its Application in Navigation System[M].Science Press, the Beijing.
- [3] Yang,Z.,Shen,Z.J. Attitude Estimation of Fixed-wing UAVs with Extended Kalman Filter[J].Aeronautical Science and Technology,2017,28(11):16-21
- [4] Ning,Q.H.,Zhang,Y.B.,Liu L. Optimization Algorithm for Target Tracking Based on Extended Kalman Filtering[J],Journal of Detection & Control,2016,38(1):91-94.
- [5] Bucy,R.S.,Renne,K.D. Digital Synthesis of Nonlinear Filter[J]. Auto matica,1971,7(3):287-289.
- [6] Mao,X.H.,Wu,J. Research into Kalman Filtering Algorithm[J].Hipboard Electronic Countermeasure,2017,40(3):65-68.
- [7] Ning,D.B.,Zhang,B. Adaptive Kalman target tracking filter based on information matrix[J].Journal of Southwest University,2016,38(7) :173-178.
- [8] Yan,Y.,Li,Y. A Tracking Control Method for Servo System Based on Improved Kalman Filter Algorithm[J]. Ship Electronic Engineering,2017,37(6) :25-28.
- [9] Willigenburg,L.G.V.UDU factored discrete-time Lyapunov recursions solve optimal reduced-order LQG problems[J], European Journal of Control,2004,10:588-601.
- [10] Li,Q.K.,Ge,Z.j.,Li,C.G.Robust Kalman Filtering Algorithm Based on Fuzzy Control [J]. Journal of Geomatics Science and Technology,2008,25(3):209-212.