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To cite this article: Jiajun Zou *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **569** 022005

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# Calculation of Tensile Strength of Unidirectional Composites Based on Probability Theory and Monte Carlo Method

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**Abstract.** In this paper, Monte Carlo method combined with the random critical-core probability model is proposed to calculate the longitudinal tensile strength of unidirectional composites. This method considers two-dimensional distribution of fibers on the cross-section, while the theoretical analysis method only takes the linear distribution of fibers into account. Using the weakest link principle, the failure probability and average strength of the unidirectional composites are obtained. The results show that the calculated values of longitudinal tensile strength of T300/5208 composite and unidirectional C/C composites agree well with the experimental results.

## 1. Introduction

The traditional strength theory often encounters great difficulties when solving the problems of strength and fracture of composites which are always a key issue in the study of composites. The discreteness of fiber strength determines that the failure of composites is a random process and a stochastic process of fiber breakage, interfacial debonding and matrix cracking. To deal with those problems, the statistical theory of meso-mechanics should be used.

Daniels[1] proposed the theory of fiber bundle statistical strength. Gucer and Gurland[2] developed Daniels' theory and studied the tensile strength of unidirectional composites via using a chain-of-bundles model firstly. Rosen[3] regarded unidirectional composites as chains of bundles, each of which is called a link or a  $\delta$  layer. If the fibers and the matrix break in the same layer, the link will be destroyed and as long as one link in the chain breaks, it means the failure of the composite. This is the famous Gucer-Guland-Rosen chain model. However, their theories use the average load sharing rule, ignoring the concentration effect of the stress, which is not consistent with the actual situation.

Zweben et al.[4-5] proposed a statistical theory of crack propagation based on experimental observations. In their analysis, the local load sharing rule was adopted. Argon[6] also developed a similar theory, which proposed the conditions of crack instability propagation and the method of calculating the stress concentration factor as well as established the failure criterion of composite materials. Smith et al.[7] and Batdorf[8-9] also proposed a similar theory. The above various crack propagation statistical theories are based on the chain-of-bundles model, which limits the crack propagation to a short layer and is inconsistent with the actual situation.

This paper considers that the ineffective length increases with crack propagation. Monte Carlo method is used to calculate the probability of initial crack cluster containing one broken fiber propagating unsteadily at any stress level. Then combined with the weakest link theory, the average tensile strength of the composite can be obtained.



## 2. Statistical theory of crack propagation

When the composite is loaded in the fiber direction, the weakest fiber will fail firstly, thus causing the failed fiber to lose its load carrying capacity within a certain length near the breaking point. The failed fiber's load will be transferred to the nearby fibers, which leads to the formation of a fiber break cluster, further increasing the stress concentration until a crack cluster becomes sufficiently large. The crack cluster begins to propagate unsteadily and the overall failure of composites happens. According to this view, the fiber strength distribution and the stress concentration around the broken fiber are two key factors determining the longitudinal tensile strength.

Coleman[10] pointed out that the Weibull distribution could be used to describe the strength of carbon fiber. The cumulative probability distribution is as follows

$$F(\sigma_f, L) = 1 - \exp \left[ -\frac{L}{L_0} \left( \frac{\sigma_f}{\sigma_0} \right)^m \right] \quad (1)$$

where  $F(\sigma_f, L)$  is the failure probability when stress  $\sigma_f$  is applied to the fiber of the length  $L$ ;  $L_0$  is the characteristic length;  $\sigma_0$  is the characteristic strength corresponding to the characteristic length;  $m$  is the Weibull modulus.

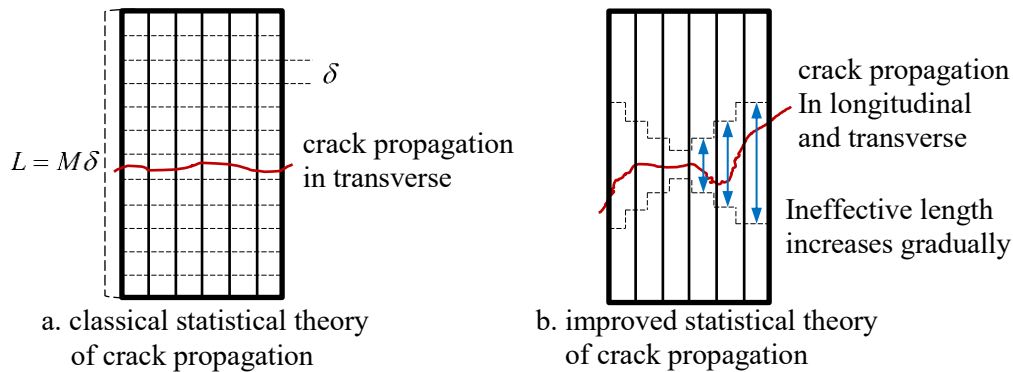


Figure 1 crack propagation mode

The Gucer-Guland-Rosen chains model considers unidirectional composites as a collection of  $M$  chains. Each chain is composed of  $N$  fibers, called a  $\delta$  layer. The model is shown in Figure 1a and  $\delta$  is called the ineffective length. The crack propagation statistical model considers that when a certain fiber breaks in a layer, the fibers adjacent to the fracture might break due to stress concentration. After that, the fracture mode will continue to happen. Zweben assumes that when the number of broken fibers in one layer reaches  $R$ , the crack cluster will be propagated unsteadily. Both the whole layer and the composite material will fail.  $R$ , called the critical crack cluster size, is a positive integer that needs to be determined by the nature of the composite.

The weaker fiber is broken firstly, and the adjacent fiber might undergo successive fracture because of stress concentration. If the adjacent fibers' fracture location is within the range of the ineffective length, it is considered that the original crack cluster has propagated. If it is outside the ineffective length, it is not related to the original crack cluster. In this paper, as the crack cluster propagates, the ineffective length increases gradually. The crack cluster propagates in both the longitudinal and transverse directions, as shown in Figure 1b. Until a critical dimension, the composite fails.

There are  $MN$  elements in the composites represented by the chains model. It's assumed that the initial fracture fibers are far enough apart, ignoring the possibility that they can combine to form a crack cluster. The initial fracture fiber does not affect each other during the propagation of the crack cluster. The expected number of the initial cracks is as follows

$$n = MNF(\sigma_f, \delta_0) = \frac{LN}{\delta_0} F(\sigma_f, \delta_0) \quad (2)$$

where  $LN$  is the total length of the fibers in the composite.

The probability of composites failure can be expressed as

$$P_c(\sigma_f) = 1 - [1 - P_n(\sigma_f)]^n \quad (3)$$

where  $P_n(\sigma_f)$  is the probability of a single initial crack propagation to critical crack cluster. In general, the fiber stress  $\sigma_{fu}$  corresponding to  $P_c(\sigma_f)$  that equals 0.5 [3][6] is taken as the average fiber stress. The tensile strength of the composites can be obtained by using the following formula

$$\sigma_{cu} = V_f \sigma_{fu} \quad (4)$$

$V_f$  is the volume fraction of fibers. When  $P_n(\sigma_f)$  is known, the above equation can be solved. The Monte Carlo method will be used to obtain  $P_n(\sigma_f)$ .

### 3. Monte Carlo simulation of crack propagation

The Monte Carlo method can overcome the difficulty of theoretical analysis and obtain the probability distribution of crack cluster instability propagation. At present, this simulation is limited to the unidirectional composites, which can not only theoretically predict the damage and damage process, but also study the size effect of the specimens. The effects of material properties such as fiber, matrix and interfacial strength on the strength can also be discussed and analyzed.

#### 3.1. Ineffective length and stress concentration factor

The ineffective length is defined as twice as the distance fibers stress recovering to a certain percentage  $\phi$  of far-field stress, as shown in Figure 2. The Kelly-Tyson shear-lag model[11] can offer an approximate solution of the ineffective length. The expression is as follows

$$\delta_0 = 2 \times \frac{\pi (d/2)^2 \sigma_f}{\pi d \tau_i} = \frac{\sigma_f d}{2 \tau_i} \quad (5)$$

where  $d$  is the fiber diameter and  $\tau_i$  is the interfacial shear strength.

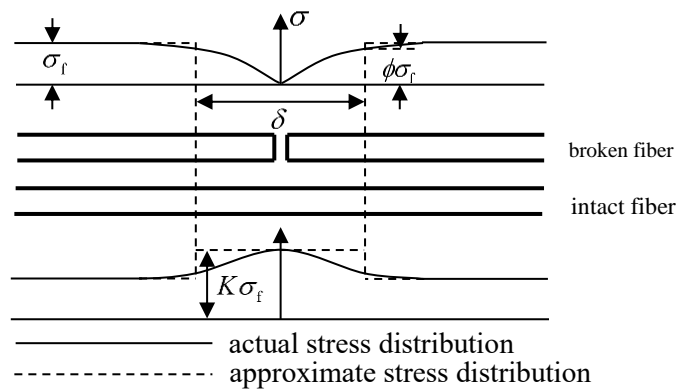


Figure 2 schematic view of ineffective length

The cross-section of the crack cluster is approximately circular. When calculating the ineffective length of the crack cluster, the crack cluster containing  $i$  broken fibers is equivalent to a thick fiber whose section area is equal to the sum of  $i$  broken fibers. Its diameter is  $\sqrt{id}$ . The ineffective length of the equivalent fiber can be approximated as

$$\delta_i \approx 2 \times \frac{\pi (d/2)^2 \sigma_f i}{\pi D_i \tau} = \sqrt{i} \frac{\sigma_f d}{2 \tau} = \sqrt{i} \delta_0 \quad (6)$$

when  $i$  is equal to 1 and  $\delta_1 = \delta_0$ . It is assumed that the broken fibers' load will be evenly distributed in the fiber adjacent to the crack cluster, and the stress concentration factor is

$$K = 1 + \frac{r}{c} \quad (7)$$

where  $r$  is the number of broken fibers in a crack cluster and  $c$  is the number of fibers adjacent to the crack cluster.

### 3.2. Monte Carlo simulation process

The stress  $\sigma_f$  applied to fibers is selected and the corresponding initial ineffective length  $\delta_1$  is calculated. Random numbers are generated by the following formula and assigned to the fiber as the strength of the fibers. R is a random number between 0 and 1. One of the fiber's strength is set to zero as the initial crack source.

$$\sigma_i = \sigma_0 \left( -\frac{\ln(1-R)L_0}{\delta_1} \right)^{\frac{1}{m}} \quad (8)$$

If the fiber fails, the stress concentration factor and the crack cluster's ineffective length are updated. The strength of the fiber adjacent to crack cluster will decrease due to the increase of the ineffective length. The fibers' strength is updated by using following formula.

$$\sigma_i = \sigma_i \left( \frac{\delta_1}{\delta_i} \right)^{\frac{1}{m}} \quad (9)$$

Until the crack cluster stops propagating or all the fibers fail, the simulation ends. When P times are simulated, the times that all the fibers fail is Q. The probability of the crack cluster propagating unsteadily at the stress  $\sigma_f$  is

$$P_n(\sigma_f) = \frac{Q}{P} \quad (10)$$

Some different stress levels are selected to get a complete probability distribution.

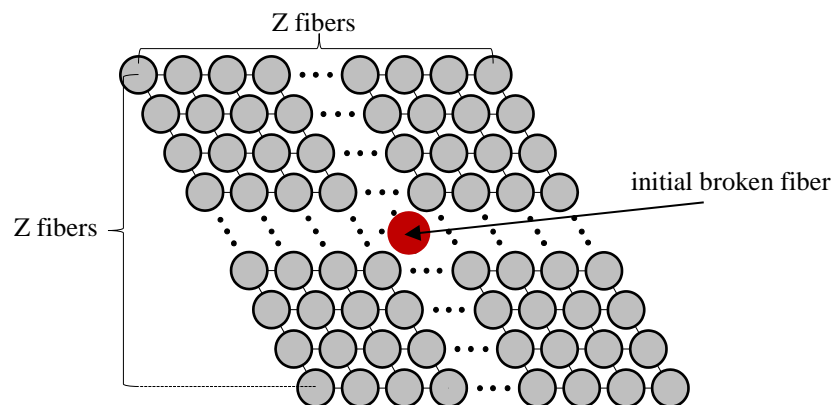


Figure 3 fibers arrangement during Monte Carlo simulation

How to select the number of simulated fibers is also an important issue since the number can not be too much or too little. The critical size of crack cluster is much smaller than the total number of cross-section fibers. In literature[12], the failure probability of any fiber around the crack cluster is greater than or equals  $1 - e^{-1}$  as the criterion of propagating unsteadily. As shown in Figure 3, if Z equals 10 and the simulated fibers is 100, the instability propagation criterion can be satisfied generally, otherwise the number of fibers will be increased.

## 4. Examples

### 4.1. T300/5280 Composite

The material parameters are from the literature[13]. The fiber volume fraction  $V_f$  is 70%, the carbon fiber diameter  $d$  is  $7\mu\text{m}$ , the longitudinal elastic modulus  $E_f$  is 230Gpa. The strength of the filament obeys Weibull distribution,  $L_0$  is 25mm, the corresponding characteristic strength  $\sigma_0$  is 2.98GPa, Weibull modulus  $m$  is 7.68; epoxy matrix elastic modulus  $E_m$  is 3.45Gpa, interfacial shear strength  $\tau_i$

is 25Mpa. The cumulative probability distribution of crack cluster propagating unsteadily is shown in Figure 4, where the dimensionless stress is the stress.

Literature[13] renders the experimental result of longitudinal tensile strength equalling to 1.5GPa, but the size of the specimen is not given. It is estimated from the test standard that the volume of the specimen is about  $8.20 \times 10^{-7} \text{m}^3 \sim 4.10 \times 10^{-6} \text{m}^3$ . If the volume is  $2.0 \times 10^{-6} \text{m}^3$ , the fiber length is 36378m, which is selected to calculate.

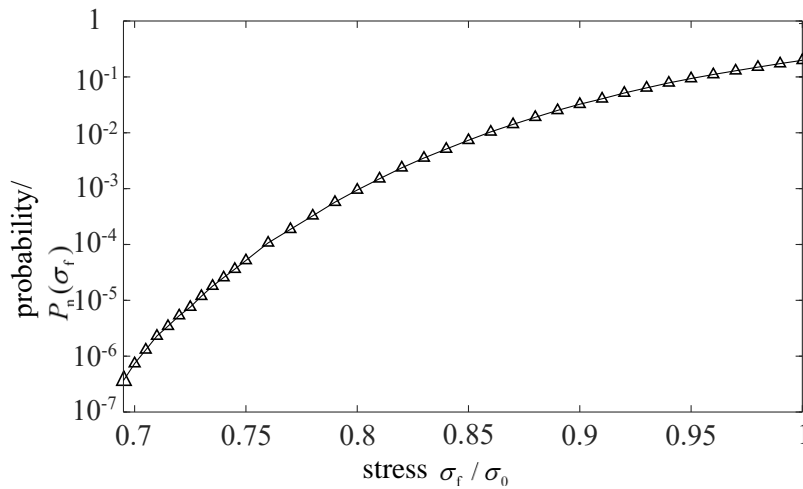


Figure 4 probability distribution of failure crack cluster of T300/5208 composite

Solving the formula(6), the result that  $\sigma_f$  equals to 2.15GPa could be obtained. Using formula(6), the tensile strength equal to 1.51GPa can be gained. The experimental value is 1.5GPa and the error is 0.67%.

#### 4.2. Unidirectional C/C Composites

The material parameters are from the literature[14]: fiber volume fraction  $V_f$  is 33.1%, carbon fiber diameter  $d$  is 7μm, longitudinal elastic modulus  $E_f$  is 235GPa. The strength of fiber filament approximately obeys double Weibull distribution, characteristic length is 20mm, corresponding characteristic strength  $\sigma_{01}$  is 1.79GPa,  $\sigma_{02}$  is 2.67GPa, Weibull modulus  $m_1$  is 6.7,  $m_2$  is 3.1, the interfacial shear strength is 8.1MPa. The size of the specimen is 25mm×4mm×3mm, the total length of fibers is 2580m. The probability cumulative function of fibers' strength is expressed by formula(11). The cumulative probability distribution of crack cluster propagating unsteadily is shown in Figure 5.

$$F(\sigma_f, L) = 1 - \exp \left[ -\frac{L}{L_0} \left( \left( \frac{\sigma_f}{\sigma_{01}} \right)^{m_1} + \left( \frac{\sigma_f}{\sigma_{02}} \right)^{m_2} \right) \right] \quad (11)$$

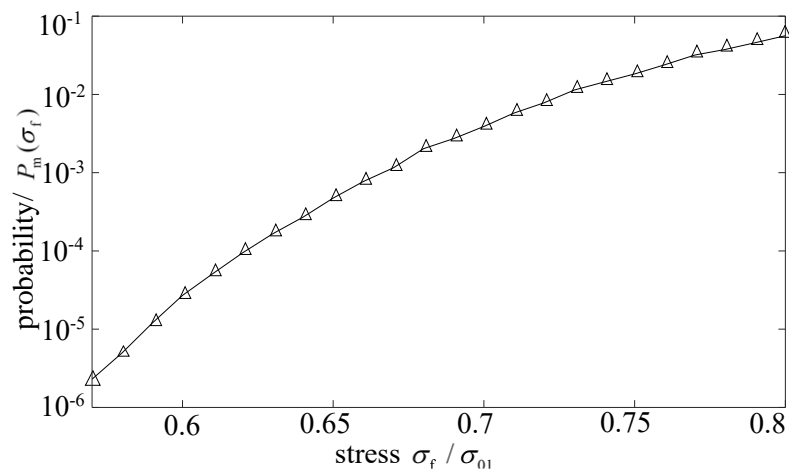


Figure 5 probability distribution of failure crack cluster of C/C composite

Using the method proposed in this paper, we obtained that the fiber stress  $\sigma_{fu}$  is 1093.5MPa. Then by formula(6), the results that the strength of the composite is 362 MPa, experimental value is 352.4MPa and the errors is 2.7% are acquired.

## 5. Conclusions

The fracture process of composites is affected by various factors, which leads to different physical processes. In addition to the properties of fibers and matrix, the micro-fracture mechanism is mainly influenced by the interfacial bonding strength. The proposed model can describe the random failure process of unidirectional composites well, and the cracks can propagate randomly in both longitudinal and transverse directions in this model. Therefore, the model can overcome the shortcoming that the chain model only considers the transverse propagation of cracks. The calculation results are in good agreement with the experimental data, which confirms the rationality and correctness of the statistical damage theory established in this paper. The method of calculating the tensile strength proposed in this paper can be further developed, for example, the fusion and influence between different crack cluster can be considered.

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