IOP Conference Series: Materials Science and Engineering

PAPER • OPEN ACCESS

Expansion of the investment portfolio performance assessment model based on value-at-risk using a time series approach

To cite this article: Sukono *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **567** 012015

IOP Conf. Series: Materials Science and Engineering **567** (2019) 012015 doi:10.1088/1757-899X/567/1/012015

Expansion of the investment portfolio performance assessment model based on value-at-risk using a time series approach

Sukono[1], D Susanti[1], E S Hasbullah[1], Y Hidayat[2], Subiyanto[3]

[1]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, INDONESIA
[2]Department of Statistics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, INDONESIA
[3]Department of Marine Sciences, Faculty of Fishery and Marine Sciences, Universitas Padjadjaran, INDONESIA

Corresponding author: sukono@uanpad.ac.id

Abstract. Portfolio performance assessment needs to be carried out before or after the investment decision is taken, in order to minimize the possibility of risk loss. This paper discusses the expansion of the investment portfolio performance appraisal model based on Value-at-Risk, where the analyzed stock returns on mean and volatility is non-constant.The aim is to increase the likelihood of achieving investment objectives by investors. In this paper the mean is estimated using autoregressive moving average models, while the non-constant volatility is estimated using generally autoregressive conditional heteroscedastic models. The estimator's ofmean and non-constant volatility are then used for the analysis of investment portfolio optimization. Portfolio optimization issues are followed based on the basic framework of the Mean-Value-at-Risk model. The solution to the investment portfolio optimization problem is done by using the Lagrange multiplier technique and the Kuhn-Tucker method.Assessment of investment portfolio performance is based on Reward to Value-at-Risk, which is then used to compare the two investment portfolios A and B are analyzed. The results of the analysis show that portfolio A has better performance than portfolio B. So it is recommended to investors to choose an investment portfolio A, to achieve a better level of success.

1. Introduction

In dealing with risky investments, investors must make a decision to choose an efficient portfolio that has better performance. To make a decision, an assessment of an efficient portfolio needs to be done. The performance assessment of an efficient portfolio can be carried out before or after the investment decision is taken [1][2][3]. Efficient portfolio performance evaluation is to increase the likelihood of achieving investor goals. In conditions of uncertainty, investors cannot choose investment opportunities only by considering the level of profit offered. Investors need to consider the element of risk [4][5]. Therefore, the assessment of investment performance will be based on the level of profit and risk [6][7].

Portfolio risk is the possibility of a level of profit deviating from what is expected. Therefore, certain dispersal measures are often used as a measure of risk [8][9]. Standard deviation or variance is often used as a measure of investment portfolio risk [10]. However, many loss risk events exceed the standard deviation or variance.Therefore, the idea arises to measure risk using a quantile, or more popularly called Value-at-Risk (*VaR*) [10][11]. The amount of *VaR* depends on the average parameter value and volatility of a stock return, as well as the probability of possible risk of loss. Stock returns often have a non-constant mean and volatility, and even have long memory effects [12][13].

This paper aims to analyze portfolio performance measurement based on *VaR* risk measure. Average and volatility as constituents in *VaR* will be analyzed using a time series approach. The average is estimated using autoregressive fractionally integrated moving average (ARFIMA) models. Non-constant volatility is estimated using the generally autoregressive conditional heteroscedastic (GARCH) models.For efficient portfolio selection is based on the Mean-VaR portfolio optimization model. While to measure portfolio performance is done using the Reward to Value-at-Risk (*RVaR*) approach. The application of this method is used to analyze ten stocks traded in the capital market in Indonesia. The aim is to compare and choose a portfolio that has better performance than other portfolios. This study is useful for investors to find out the performance of two investment portfolios, where risk is measured by Value-at-Risk, where data follows a time series pattern.

2. Methodology

Suppose P_{it} and r_{it} successively stated prices and stock returns i ($i = 1,...,N$ and N the number of stocks analyzed), at the time t ($t = 1,...,T$, T data observation period). Stock returns r_{it} calculated using formula $r_{it} = \ln(P_{it} / P_{it-1})$ [5; 14]. The return data model estimation is then performed on mean and volatility as follows.

2.1 Modeling of mean and volatility

Modeling of mean. Identification of long memory effects on stock returns data r_{it}. Identification is done using the rescale (R / S) method or the Geweke and Porter-Hudak (GPH) methods. Estimation of fractional differentiation parameters d_i ($i = 1,...,N$ and N the number of stocks analyzed), carried out using the maximum likelihood method [11; 13]. Confidence interval $(1-\alpha)100\%$ for d_i is $\hat{d}_i - z_{\alpha/2}.\sigma_{d_i} < d_i < \hat{d}_i + z_{1-\alpha/2}.\sigma_{d_i}$ with $\hat{d}_i$ is estimator of d_i, and $z_{\alpha/2}$ standard normal distribution percentile if given a level of significance α. Suppose d_i fractional differentiation to be tested by the hypothesis. Suppose also σ_{d_i} standard deviation of d_i. Hypothesis testing is carried out against $H_0 : \hat{d}_i = 0$ and $H_1 : \hat{d}_i \neq 0$ use $z_{d_i} = d_i / \sigma_{d_i}$. Test criteria are reject H_0 if value $z_{d_i} < z_{\alpha/2}$ atau $z_{d_i} > z_{1-\alpha/2}$ [12; 14].

Fractional differentiation processes are defined as:

$$(1-B)^{d_i} r_{it} = a_{it}, \; -0.5 < d_i < 0.5 ; \tag{1}$$

with $\{a_{it}\}$ is residual white noise series, and B stated the backshift operator. If fractional differentiation series $(1-B)^{d_i} r_{it}$ follow the model of ARMA (p,q), then r_{it} called the autoregressive fractionally integrated moving average degree process p, d and q, or ARFIMA(p,d,q) [14]. Model equation of ARMA(p,q) is:

$$r_{it} = \psi_{i0} + \sum_{g=1}^{p} \psi_{ig} r_{it-g} + a_{it} + \sum_{h=1}^{q} \theta_{ih} a_{it-h} , \tag{2}$$

with ψ_{i0} constants, and ψ_{ig} $(g = 1,..., p)$,and θ_{ih} $(h = 1,..., q)$ parameter coefficient of the mean stock return model i ($i = 1,..., N$ and N the number of stocks analyzed). Assumed $\{a_{jt}\}$ residual white noise sequence with zero mean and variance $\sigma^2_{a_j}$ [14][15].

The stages of the average modeling process include: (i) Model identification, (ii) Estimation of parameters, (iii) Test diagnosis, and (iv) Prediction [14].

Modeling of volatility. Stock return volatility modeling is performed using generalized autoregressive conditional heteroscedastic (GARCH) models. Suppose μ_{it} and σ^2_{it} successive mean and volatility of stock returns i ($i = 1,..., N$ and N the number of stocks analysed), at the time t ($t = 1,..., T$ and T data observation period). Residual a_{it} the above has an equation $a_{it} = r_{it} - \mu_{it}$ [11; 14]. Volatility σ^2_{it} will follow the GARCH model with degrees m and n or written as GARCH(m,n), when:

$$a_{it} = \sigma_{it}\varepsilon_{it}\ ,\ \ \sigma^2_{it} = \alpha_{i0} + \sum_{k=1}^{m} \alpha_{ik} a^2_{it-k} + \sum_{l=1}^{n} \beta_{il}\sigma^2_{it-l} + \varepsilon_{it}\ , \tag{3}$$

with α_{i0} constants, and α_{ik} $(k = 1,..., m)$,and β_{il} $(l = 1,..., n)$ parameter coefficient of stock return volatility model i ($i = 1,..., N$ and N the number of stocks analyzed). Assumed $\{\varepsilon_{it}\}$ squence of random variables are mutually independent and have identical distributions (iid) with an average of 0 and variance 1 $\alpha_{i0} > 0$, $\alpha_{ik} \geq 0$, $\beta_{il} \geq 0$, and $\sum_{k=1}^{\max(m,n)} (\alpha_{ik} + \beta_{ik}) < 1$ [14][16].

Stages of the volatility modeling process include: (i) Estimation of the average model, (ii) ARCH effect test, (iii) Model identification, (iv) Estimation of the volatility model, (v) Test diagnosis, and (vi) Prediction [14].

Using the mean model (2) and volatility (3), the prediction is carried out aimed at calculating the mean $\hat{\mu}_{it} = \hat{r}_{iT}(1)$ and variance $\hat{\sigma}^2_{it} = \hat{\sigma}^2_{iT}(1)$, namely the prediction of the l-step forward after the time period to T [14].

2.2 Portfolio modal and Value-at-Risk

In the formation of an investment portfolio w, will relate to the proportion of funds allocated to each of the shares analyzed. Suppose w_i is the proportion of funds allocated to stocks i, where $\sum_{i=1}^{N} w_i = 1$, then portfolio return can be expressed as:

$$R_{wt} = \sum_{i=1}^{N} w_i R_{it}\ , \tag{4}$$

where R_{wt} portfolio return w at time t, and N the number of stocks in the formation of a portfolio[10; 14].

Based on (5), the mean (expectation) portfolio is obtained with weights w_i can be stated as:

$$\hat{\mu}_{wt} = \sum_{i=1}^{N} w_i \hat{\mu}_{it} \tag{5}$$

While portfolio variance can be expressed as:

$$\sigma^2_{wt} = \sum_{i=1}^{N} w_i^2 \sigma^2_{it} + 2\sum_{i=1}^{N} \sum_{j=1}^{N} w_i\, w_j \sigma_{ij}\ ;\ i \neq j \tag{6}$$

where $\sigma_{ij} = Cov(r_{it}, r_{jt})$.

Suppose the amount of the initial investment is one unit, and the level of significance of the risk of loss is α , then VaR for portfolios with weights w_i is:

$$VaR_{wt} = z_\alpha \hat{\sigma}_{wt} - \hat{\mu}_{wt} \tag{7}$$

where z_α is the percentile value of a standard normal distribution with a level of significance α [10; 11; 14].

2.3 Mean-Vary Portfolio Optimization

Suppose that the vector values of expectations and covariance matrices are given successively by: $\boldsymbol{\mu}^T = (\hat{\mu}_{1t},...,\hat{\mu}_{it})$, with $\hat{\mu}_{it} = E[R_{it}]$, $i = 1,...,N$, and $\boldsymbol{\Sigma} = (\hat{\sigma}_{ij})_{i,j=1,...,N}$, with $\hat{\sigma}_{ij} = Cov(r_{it}, r_{jt})$, $i, j = 1,...,N$. Refer to the previous discussion, the weight of stock returns in a portfolio $\mathbf{w}^T = (w_1,...,w_N)$, where $\sum_{i=1}^{N} w_i = 1$ or $\mathbf{e}^T\mathbf{w} = 1$ with $\mathbf{e} = (1,...,1)^T$ vector with one-on-one element. Referring to equation (5) can be rewritten as

$$\mu_{wt} = E[R_{wt}] = \boldsymbol{\mu}^T\mathbf{w}, \tag{8}$$

and equation (6) is rewritten as:

$$\sigma_{wt}^2 = Var(R_{wt}) = \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}. \tag{9}$$

Use the level of significance α, the percentile z_α obtained from the standard normal distribution table. So the Value-at-Risk investment portfolio equation (13) can be rewritten as:

$$VaR_{wt} = z_\alpha \sigma_{wt} - \mu_{wt} = z_\alpha (\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2} - \boldsymbol{\mu}^T\mathbf{w}. \tag{10}$$

A portfolio $\mathbf{w}^*$ *called (Mean-VaR) efficiently if there is no portfolio* $\mathbf{w}$ *with* $\mu_{wt} \geq \mu_{wt^*}$ *and* $VaR_{wt} < VaR_{wt^*}$ [3; 7; 9]. To get an efficient portfolio, the objective function is determined maximally $\{2\tau\mu_{wt} - VaR_{wt}\}$, $\tau \geq 0$ where τ is investor risk tolerance. So, for investors with risk tolerance $\tau \geq 0$ must solve optimization problems [13; 5]:

$$\begin{aligned} &\text{maks}\{2\tau\boldsymbol{\mu}^T\mathbf{w} - z_\alpha(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2} + \boldsymbol{\mu}^T\mathbf{w}\} \\ &\qquad \text{kendala } \mathbf{e}^T\mathbf{w} = 1 \end{aligned} \tag{11}$$

Because of the covariance matrix $\boldsymbol{\Sigma}$ semidefinite positive, the objective function is quadratic concave. Therefore, (12) is a concave quadratic optimization problem. The Lagrange function is given by:

$$L(\mathbf{w},\lambda) = (2\tau+1)\boldsymbol{\mu}^T\mathbf{w} - z_\alpha(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2} + \lambda(\mathbf{e}^T\mathbf{w} - 1).$$

Using the Kuhn-Tucker theorem, the optimality requirement is:

$$\partial L / \partial\mathbf{w} = (2\tau+1)\boldsymbol{\mu} - z_\alpha\boldsymbol{\Sigma}\mathbf{w} / (\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2} + \lambda\mathbf{e} = 0 \text{ dan } \partial L / \partial\lambda = \mathbf{e}^T\mathbf{w} - 1 = 0. \tag{12}$$

Based on algebraic calculations, if for example $A = \mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}$, $B = (2\tau+1)\,(\boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\mathbf{e} + \mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu})$ and $C = (2\tau+1)^2(\boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}) - z_\alpha^2$, then the ABC formula is obtained:

$$\lambda = \{-B + (B^2 - 4AC)^{1/2}\} / 2A \tag{13}$$

For $\tau \geq 0$, solving equation (12) obtained a portfolio with weight vector $\mathbf{w}^*$

$$\mathbf{w}^* = \frac{(2\tau+1)\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\boldsymbol{\Sigma}^{-1}\mathbf{e}}{(2\tau+1)\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}} \tag{14}$$

If vector $\mathbf{w}^*$ substituted into equations (8) and (10), then the average portfolio return and optimum Value-at-Risk will be obtained [2; 9].

2.4 Reward-to Value-at-Risk (RVaR)

Because in this case portfolio risk is measured based on Value-at-Risk (*VaR*), the size of the Reward to Volatility (*RVAR*) portfolio performance is expanded to *RVaR*. Meaning *RVaR* measures the comparison between portfolio risk premiums and Value-at-Risk (*VaR*). The mathematical equation of *RVaR* is:

$$RVaR = \frac{\mu_{wt} - \mu_f}{VaR_{wt}}. \tag{15}$$

Where μ_{wt} mean portfolio return w at time t, μ_f mean of risk-free asset return, and VaR_{wt} Value-at-Risk portfolio w at time t. Measures of good portfolio performance are determined based on the greatest value of *RVaR* [11].

3. Result and Analysis

The analyzed stock data is accessed through the website http://www.finance.go.id//. The data consists of 10 (ten) selected shares, for the period January 2, 2015 up to June 4, 2018, which includes shares: INDF, DEWA, AALI, LSIP, ASII, TURB, HDMT, BMRI, UNTR, BBRI . Next, it is called sequentially S_1 up to S_{10}. Share prices include the opening price, the highest price, the lowest price, and the closing price, but only the closing price is closed.

3.1 The results of mean and volatility modeling

In this section ten shares are analyzed S_1 up to S_{10}. The analysis starts with calculating the return of each stock, then identifying the long memory effect, estimating the average model and the volatility model.

*Identification of long memory effects.*To identify long memory effects, it is done by estimating fractional differentiation parameters d_i ($i = 1,...,10$) in equation (1). Estimates were made using the Gewek and Porter-Hudak methods, with the help of software R. The estimation results obtained fractional differentiation values $\hat{d}_1$=0.333742; $\hat{d}_7$= 0.016421 and $\hat{d}_{10}$= -0.062398. Based on the results of the hypothesis test shows that stocks S_1, S_7 and S_{10} there is a significant long memory effect, with fractional differentiation $\hat{d}_1$, $\hat{d}_7$ and $\hat{d}_{10}$. Whereas other stocks do not have long memory effects. Results of fractional differentiation estimates for stocks S_1 up to S_{10} given in Table-1 column $\hat{d}_i$.

Mean model estimates. In this section Eviews 9 software is used for estimating the mean model. The fractional stock return data will be estimated for the mean model. The first stage is the identification and estimation of the mean model. Identification was carried out through the fractional function (ACF) and partial autocorrelation function (PACF) samples. Based on the ACF and PACF patterns, tentative models are possible for the return data of each stock S_1 up to S_{10} determined. Then the model estimation is done by referring to equation (2). From the model estimation and diagnostic test can be obtained a significant mean model of stocks S_1 up to S_{10}, the results are given in Table-1 below.

Estimation of volatility models. In this section Eviews 9 software is also used to estimate the volatility model. First, the detection of the existence of an element of autoregressive conditional heteroscedasticity (ARCH) against residuals a_{it} ($i = 1,...,10$) from the mean model. Detection is done using the ARCH-LM test method. Detection results indicate that the calculation value of χ^2 (obs * *R-Square*) on each stock S_1 up to S_{10} generate a probability of 0.0000 or 5% smaller, which means there are elements of ARCH.

Table 1. The results of estimation of mean and volatility models of stocks S_1 up to S_{10}

Stocks (S_i)	Time Series Model	**Frac.Dif.** ($\hat{d}_i$)	**Mean** ($\hat{\mu}_{it}$)	**Variance** ($\hat{\sigma}_{it}^2$)
S_1	ARFIMA(1, $\hat{d}$,0)-GARCH(1,1)	0.333742	0.015399	0.002643
S_2	ARFIMA(2, $\hat{d}$,2)-ARCH(1)-M	0	0.039007	0002797
S_3	ARFIMA(0, $\hat{d}$,1)-GARCH(3,3)	0	0.003315	0.001331
S_4	ARFIMA(1, $\hat{d}$,1)-GARCH(1,1)	0	0.008672	0.001921
S_5	ARFIMA(0, $\hat{d}$,1)-GARCH(1,1)	0	-0.000262	0.001873
S_6	ARFIMA(1, $\hat{d}$,1)-FIGARCH(1,1)	0	0.022085	0.001181
S_7	ARFIMA(1, $\hat{d}$,1)-GARCH(1,1)	0.016421	0.003564	0.001073
S_8	ARFIMA(0, $\hat{d}$,1)-EGARCH(1,1)	0	0.001594	0.001411
S_9	ARFIMA(2, $\hat{d}$,2)-TGARCH(1,1)	0	0.020709	0.013362
S_{10}	ARFIMA(0, $\hat{d}$,1)-GARCH(1,1)	-0.062398	-0.000865	0.001237

Second, identification and estimation of volatility models are carried out. The volatility model used is the generalized autoregressive conditional heteroscedasticity (GARCH) model referring to equation (3). Based on correlogram residual squares a_{it}^2 ($i = 1,...,10$), set a tentative volatility model for each stock S_1 up to S_{10}. Estimation of the volatility model is carried out simultaneously with the average model.

In the volatility modeling process it is also shown that based on the ARCH-LM test, residuals ε_{it} ($i = 1,...,10$) from the volatility model of each stock S_1 up to S_{10} is white noise. The estimation results of the average model and volatility for stocks S_1 up to S_{10} given in Table-1 column of "Time Series Model". Furthermore, the mean and volatility equations are used to estimate values $\hat{\mu}_{it} = \hat{r}_{it}(1)$ and $\hat{\sigma}_{it}^2 = \hat{\sigma}_{it}^2(1)$ ($i = 1,...,10$); is a recursive 1-step forward prediction. The results for each stock S_1 up to S_{10} given in Table-1 column of $\hat{\mu}_{it}$ and $\hat{\sigma}_{it}^2$.

3.2 The results of portfolio optimization A and B

In this section analyzes two portfolios, namely portfolios A and B. portfolio of stocks that were analyzed include stocks S_1 up to S_{10}, which is divided into two portfolios. Portfolio A consists of stocks S_1 up to S_5, while portfolio B consists of stocks S_6 up to S_{10}.

Portfolio Optimization A

A portfolio of five stocks, namely stock S_1 up to S_5. Estimator based on the mean stock return S_1 up to S_5, in Table-1 the average vector is arranged as follows:

$$\boldsymbol{\mu}_A^T = (0.015399 \ \ 0.039007 \ \ 0.003315 \ \ 0.008672 \ \ -0.000262)$$

Portfolio A consists of five stocks, so the unit vector $\mathbf{e}^T = (1 \ \ 1 \ \ 1 \ \ 1 \ \ 1)$. Next, use a volatility estimator in Table-1 and a return covariance estimator between stocks S_1 up to S_5 covariance matrix is formed as:

$$\mathbf{\Sigma}_A = \begin{pmatrix} 0.002643 & -2.49868\times10^{-7} & -9.11399\times10^{-9} & -3.79169\times10^{-8} & -3.92689\times10^{-8} \\ -2.49868\times10^{-7} & 0.002797 & 8.90245\times10^{-8} & -1.27711\times10^{-7} & -2.23112\times10^{-8} \\ -9.11399\times10^{-9} & 8.90245\times10^{-8} & 0.001331 & 9.74434\times10^{-8} & 6.30434\times10^{-7} \\ -3.79169\times10^{-8} & -1.27711\times10^{-7} & 9.74434\times10^{-8} & 0.001921 & 7.96921\times10^{-8} \\ -3.92689\times10^{-8} & -2.23112\times10^{-8} & 6.30434\times10^{-7} & 7.96921\times10^{-8} & 0.0001873 \end{pmatrix}$$

Based on the matrix $\mathbf{\Sigma}_A$, then the inverse matrix can be calculated $\mathbf{\Sigma}_A^{-1}$.

Optimization is carried out based on portfolio problems in equation (11). Next, vector $\boldsymbol{\mu}^T$ and $\mathbf{e}^T$ and matrix $\mathbf{\Sigma}_A^{-1}$, used to calculate the optimum weight vector using equation (14). Where risk tolerance τ on condition $\tau \geq 0$ in portfolio optimization here is simulated by taking several values that meet the requirements $\mathbf{e}^T\mathbf{w} = 1$. Assuming short sale is not permitted, taking the risk tolerance value is only for value $0 \leq \tau \leq 0.486$. This is due to the risk tolerance value $\tau > 0.486$ produce a negative weight.

For each risk tolerance value $0 \leq \tau \leq 0.486$ generate portfolio mean return $\hat{\mu}_A$ and the level of risk VaR_A different. Curved lines between pairs $\hat{\mu}_A$ and VaR_A it forms an efficient surface. Where the mean minimum portfolio return is 0.015736 with a minimum VaR of 0.019629, and the mean highest portfolio return is 0.025191 with a maximum VaR of 0.025022.

Ratio between $\hat{\mu}_A$ and VaR_A the biggest is 0.90099912 or obtained when risk tolerance τ =0.486. Ratio between $\hat{\mu}_A$ and VaR_A continue to experience an increase in risk tolerance intervals $0 \leq \tau \leq 0,486$. Based on Mean-VaR portfolio optimization analysis, the optimal portfolio composition of stocks S_1 up to S_5 produce a weight vector $\mathbf{w}^T$ = (0.21701 0.51389 0.09856 0.17038 0.00016). Where the composition of the optimal portfolio produces $\hat{\mu}_A$ = 0.025191 with VaR_A = 0.025022 which is also a maximum portfolio.

Portfolio Optimization B

Portfolio B consists of five stocks, namely stocks S_6 up to S_{10}. From the average estimator of stock returns S_6 up to S_{10}, in Table-1 the average vector is arranged as follows:

$$\boldsymbol{\mu}_B^T = (0.022085 \;\; 0.003564 \;\; 0.001594 \;\; 0.020709 \;\; -0.000865)$$

Portfolio B consists of five stocks, so the unit vector $\mathbf{e}^T$ = (1 1 1 1 1). Next, from the volatility estimator in Table-1 and the return covariance estimator between stocks S_6 up to S_{10} covariance matrix are formed as follows:

$$\mathbf{\Sigma}_B = \begin{pmatrix} 0.001181 & 2.05888\times10^{-7} & 2.63104\times10^{-7} & 1.60945\times10^{-7} & 1.60497\times10^{-7} \\ 2.05888\times10^{-7} & 0.001073 & -2.65181\times10^{-8} & -4.58792\times10^{-8} & -2.38116\times10^{-8} \\ 2.63104\times10^{-7} & -2.65181\times10^{-8} & 0.001411 & 7.18467\times10^{-7} & 7.62223\times10^{-7} \\ 1.60945\times10^{-7} & -4.58792\times10^{-8} & 7.18467\times10^{-7} & 0.0013362 & 6.47969\times10^{-7} \\ 1.60497\times10^{-7} & -2.38116\times10^{-8} & 7.62223\times10^{-7} & 6.47969\times10^{-7} & 0.001233 \end{pmatrix}$$

Based on the matrix $\mathbf{\Sigma}_B$, then the inverse matrix can be calculated $\mathbf{\Sigma}_B^{-1}$.

The portfolio optimization problem is based on equation (11). Using vectors $\boldsymbol{\mu}^T$ and $\mathbf{e}^T$ and matrix $\boldsymbol{\Sigma}_B^{-1}$, Optimal weight vector is calculated using equation (14). Risk tolerance τ on condition $\tau \geq 0$ in portfolio optimization here is simulated by taking several values that meet the requirements $\mathbf{e}^T \mathbf{w} = 1$. Assuming short sale is not permitted, taking the risk tolerance value is only for value $0 \leq \tau \leq 0.409$. Because for the risk tolerance value $\tau > 0.409$ produce a negative weight.

Each risk tolerance value $0 \leq \tau \leq 0.409$ produce $\hat{\mu}_B$ and VaR_B different. Curved lines between pairs $\hat{\mu}_B$ and VaR_B it forms an efficient surface. Where the mean minimum portfolio return is generated $\hat{\mu}_B$ = 0.013436 with a minimum risk level VaR_B = 0.014542. The highest mean return portfolio is 0.018832 with a maximum VaR of 0.017021.

Ratio between $\hat{\mu}_B$ and VaR_B the biggest is 0.9509312 or obtained when risk tolerance τ =0.409. Ratio between $\hat{\mu}_B$ and VaR_B continue to experience an increase in risk tolerance intervals $0 \leq \tau \leq 0,409$. Based on Mean-VaR portfolio optimization analysis, the optimal portfolio composition of stocks S_6 up to S_{10} is a stock weight vector $\mathbf{w}^T$ = (0.46850 0.09989 0.04210 0.38933 0.00018). Where the composition of this optimal portfolio produces $\hat{\mu}_B$ = 0.018832 dan VaR_B = 0.017021 which is also $\hat{\mu}_B$ and VaR_B maximum.

3.3 Performance of portfolio A and B

In this section an assessment of the performance of portfolios A and B is carried out, the aim is to determine the performance of each portfolio at various risk tolerances that meet both portfolios. Performance appraisal is carried out referring to equation (15). Calculation $RVaR_A$ for portfolio A and $RVaR_B$ for portfolio B, the results is summarized in the following Table-2. Based on the RVaR values presented in Table 2, it appears that for the risk tolerance value $0 \leq \tau \leq 0.50$ the values $RVaR_A$ greater than values $RVaR_B$.

Table 2. The values of $RVaR_A$ and $RVaR_B$

No	τ	$RVaR_A$	$RVaR_B$
1	0.00	0.666721	0.387433
2	0.05	0.697884	0.405700
3	0.10	0.728374	0.423855
4	0.15	0.757918	0.441876
5	0.20	0.786218	0.459663
6	0.25	0.812914	0.477149
7	0.30	0.835589	0.494258
8	0.35	0.859494	0.510899
9	0.40	0.878353	0.526912
10	0.45	0.893241	0.542234
11	0.50	**0.900935**	0.556655
12	0.55	-	0.570065
13	0.60	-	0.582166
14	**0.658**	-	**0.594192**

For more details, RVaR values in Table-2 can be presented in graphical form as given by Figure 1.

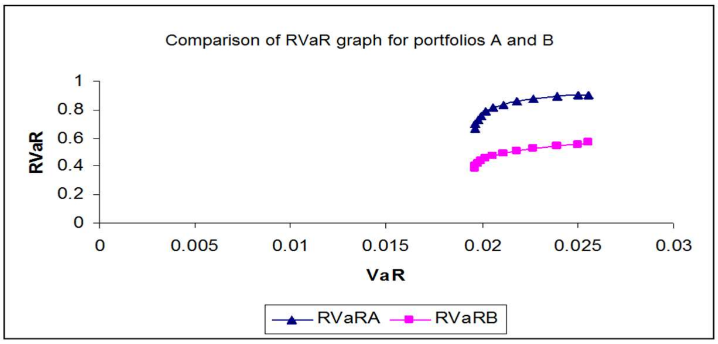

Figure 1. Graph of $RVaR_A$ and $RVaR_B$

In Figure-1 it is also seen that for the risk tolerance value $0 \leq \tau \leq 0.50$ graph of $RVaR_A$ always above than the graphof $RVaR_B$. This situation shows that portfolio A performs better than portfolio B. Therefore, it is advisable for investors to choose portfolio A which consists of stocks S_1, S_2, S_3, S_4 and S_5.

4. Conclusion

In this paper a discussion has been carried out on the expansion of the investment portfolio performance assessment model based on Value-at-Risk using a time series approach. Based on the identification of the long memory effect shows that stock returns S_1, S_7 and S_{10} there is an element of long memory. Average modeling and non-constant volatility indicate that stocks return S_1, S_7 and S_{10} following the ARFIMA-GARCH model; S_2 ARMA-ARCH-M model; S_4, S_4 and S_5 ARMA-GARCH model; S_6 ARMA-FIGARC model; S_8 ARMA-EGARCH model; and S_9 ARMA-TGARCH model. The average model and volatility are used to estimate the average values and variances of each stock. The portfolio A is composed of S_1 up to S_5, whereas portfolio B consists of S_6 up to S_{10}. Portfolio optimization is formed based on the Mean - Value-at-Risk model. Portfolio performance appraisal has been carried out based on the Reward to Value-at-Risk approach, and the results show that portfolio A has better performance than portfolio B. Based on the analysis of investment portfolios which include stocks S_1 up to S_{10}, investors are recommended to choose portfolio A. The limitation of this study is that the comparison of the performance of the investment portfolio is only valid if the risk is measured by Value-at-Risk, and the data follows a time series pattern.

Acknowledgements

Acknowledgments are conveyed to the Rector, Director of Directorate of Research, Community Involvement and Innovation, and the Dean of Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, with whomthe Internal Grant Program of Universitas Padjadjaran was made possible to fund this research.The grant is a means of enhancing research and publication activities for researchers at Universitas Padjadjaran.

References

[1] Hadi, A.S., Naggar, A.A.E., and Bary, M.N.A. 2016. New Model and Method for Portfolios Selection. Applied Mathematical Sciences, Vol. 10, 2016, no. 6, pp. 263–288. doi: 10.12988/ams.2016.58541.

[2] Soeryana, E., Halim, N.B.A., Sukono, Rusyaman, E., and Supian, S. 2016. Mean-Variance Portfolio Optimization on Some Stocks by Using Non Constant Mean and Volatility Models Approaches. *Proceedings of the International Conference on Industrial Engineering and Operations Management*, Vol. 8-10 March 2016, pp. 3124-3131.
[3] Sukono, Hidayat, Y., Lesmana, E., Putra, A.S., Napitupulu, H., and Supian, S. 2018. Portfolio Optimization by Using Linear Programing Models Based on Genetic Algorithm. *IOP Conf. Series: Materials Science and Engineering*, 300 (2018) 012001 doi:10.1088/1757-899X/300/1/012001.
[4] Napitupulu, H., Sukono, Mohd, I.B., Y Hidayat, Y., and Supian, S. 2018. Steepest Descent Method Implementation on Unconstrained Optimization Problem Using C++ Program. *IOP Conf. Series: Materials Science and Engineering*, 332 (2018) 012024 doi:10.1088/1757-899X/332/1/012024.
[5] Sukono, Susanti, D., Najmia, M., Lesmana, E., Napitupulu, H., Putra, A.S., and Supian, S. 2018. Analysis of Stock Investment Selection Based on CAPM Using Covariance and Genetic Algorithm Approach. *IOP Conf. Series: Materials Science and Engineering*, 332 (2018) 012046 doi:10.1088/1757-899X/332/1/012046.
[6] Hasbullah, E.S., Suyudi, M., Halim, N.B.A., Sukono, Gustaf, F., and Putra, A.S. 2018. A Comparative Study of Three Pillars System and Banking Methods in Accounting Long-Term Purposes of Retiree in Indonesian Saving Account. *IOP Conf. Series: Materials Science and Engineering,* 332 (2018) 012017 doi:10.1088/1757-899X/332/1/012017.
[7] Soeryana, E., Halim, N.B.A., Sukono, Rusyaman, E., and Supian, S. 2017. Mean-Variance Portfolio Optimization by Using Non Constant Mean and Volatility Based on the Negative Exponential Utility Function. *AIP Conference Proceedings* **1827**, 020042 (2017); doi: 10.1063/1.4979458.
[8] Soeryana, E., Halim, N.B.A., Sukono, Rusyaman, E., and Supian, S. 2017. Mean-Variance Portfolio Optimization by Using Time Series Approaches Based on Logarithmic Utility Function. *IOP Conf. Series: Materials Science and Engineering* 166 (2017) 012003 doi:10.1088/1757-899X/166/1/012003.
[9] Sukono, Sidi, P., Susanti, D., and Supian, S. 2017. Quadratic Investment Portfolio Without a Risk Free Asset Based on Value-at-Risk. *Journal of Engineering and Applied Sciences,* 12 (19): 4846-4850.
[10] Balibey, M. and Turkyilmaz, S. 2014. Value-at-Risk Analysis in the Presence of Asymmetry and Long Memory: The Case of Turkish Stock Market. *International Journal of Economics and Financial Issues,* Vol. 4, No. 4, 2014, pp.836-848.
[11] Sukono, Lesmana, E., Susanti, D., and Napitupulu, H. 2017. Estimating the Value-at-Risk for Some Stocks at the Capital Market in Indonesia Based on ARMA-FIGARCH Models. *IOP Conf. Series: Journal of Physics: Conf. Series,* 909 (2017) 012040 doi :10.1088/1742-6596/909/1/012040.
[12] Korkmaz, T., Cevic, E.I. & Ozatac, N. (2009). Testing for Long Memory in ISE Using ARFIMA-FIGARCH Model and Structural Break Test. *International Research Journal of Finance and Economics.* Issue 26, 1450-2887.
[13] Turkyilmaz, S. and Balibey, M. 2014. Long Memory Behavior in the Returns of Pakistan Stock Market: ARFIMA-FIGARCH Models. *International Journal of Economics and Financial Issues,* Vol. 4, No. 2, 2014, 400-410.
[14] Tsay, R.S. 2005. *Analysis of Financial Time Series.* Second Edition. Hoboken, New Jersey: John Wiley & Sons, Inc.
[15] Sukono, Hidayat, Y., Suhartono, Sutijo, B., Bon, A.T., and Supian, S. 2016. Indonesian Financial Data Modeling and Forecasting By Using Econometrics Time Series and Neural Network. *Global Journal of Pure and Applied Mathematics*.ISSN 0973-1768 Volume 12, Number 4 (2016), pp. 3745-3757.
[16] Stancu, S. Empirical Results of Modeling EUR/RON Exchange Rate Using ARCH, GARCH, EGARCH, TARCH and PARCH Models. *Romanian Statistical Review,* 1/2017, pp. 57-72.