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Assessment of shear capacity of concrete bridge deck slabs using theoretical formulations and FEM analysis

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Abstract. Reinforced concrete (RC) slabs without shear reinforcement are typical cases of bridge deck slabs. For such structures, shear has been a challenging problem in the assessment based on the current standards. This paper deals with the comparison of the methods for assessment of bridge deck slabs subjected to a concentrated load. Two experimental campaigns were selected as case studies and analysed by the simplified theoretical formulations and linear finite element analysis (LFEA). The differences between analysis methods were discussed regarding one-way shear behaviour of the bridge deck slabs.

1. Introduction

Shear in RC slabs is a challenging problem in the assessment based on the current standards. Verifications used in the past provided higher shear resistance than current codes of practice and this often leads to the requirement to provide shear reinforcement in the areas where it was not necessary in the past. This also raises the question of reliability of existing structures and therefore it has become the major motivation for the experimental campaigns following with theoretical and numerical assessments of the shear resistance. The greatest differences between outcomes of the past and current models for prediction of shear capacity are in the case of slabs subjected to a concentrated load. The typical example are RC bridge deck slabs loaded by wheel pressure located close to the support. In this case two models have to be merged, the first one is model for one-way shear and the second one is for the distribution of a concentrated load. The obtained product, method for the assessment of shear capacity has to be verified by the test results from experiments. An article is focused on the verification of two methods; the first method is based on the design equations from relevant codes of practice and the second method is based on the linear analysis using FEM models. Test results from two experimental campaigns are used for the verification of the methods.

2. Experimental campaigns

2.1. Campaign by R Vida and J Halvonik

The geometry of the tested specimens by R Vida and J Halvonik [1] are introduced in figure 1(a). The slabs were loaded on both sides of cantilevers with a concentrated load on a 250 x 250 mm steel plates, generated by a two synchronized hydraulic jacks supported by a strong steel frame. The position of the loaded area was constant, at the distance of 335 mm (twice the effective depth of the slab) from the face of the central beam under the slab. The measured concrete strength before tests varied between 30 and 38 MPa. Three different reinforcement ratios of 0.73%, 1.27% and 1.69% with standard B500B grade of steel were used.

2.2. Campaign by G Rombach and L Henze
The geometry of the cantilever slabs tested by G Rombach and L Henze [2] can be seen in figure 1(b). The load was applied in the symmetry axis through a 400 x 400 mm steel plate and the cantilever on the backside was fixed by means of four vertically prestressed bars against the laboratory strong floor. Grade B500S of steel was used and a reinforcement ratio of 1.17% was the same for all slabs. The measured concrete strength varied between 38 and 56 MPa. Five different loading locations were investigated with a_v/d ratio from 2 to 6, where a_v is the distance between the edge of a support and the inner edge of the loading plate and d is the effective depth of slab.
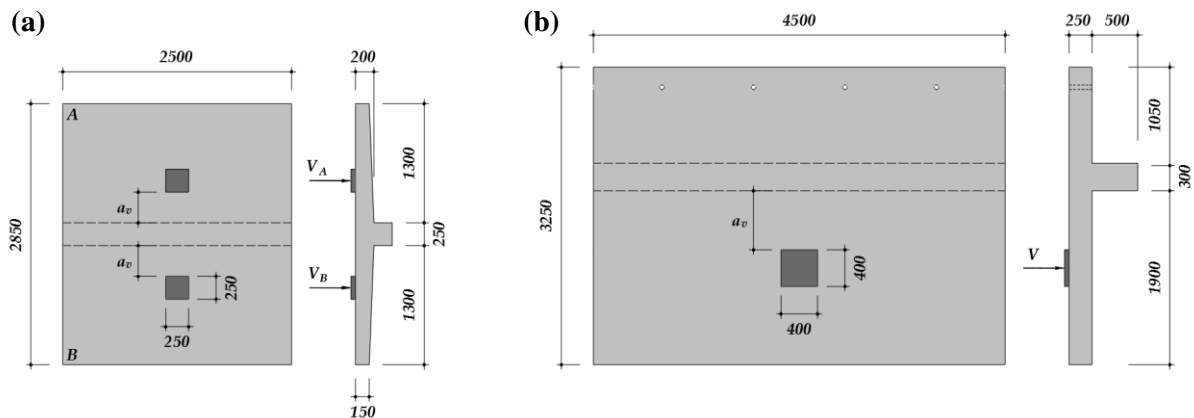

Figure 1. Investigated slab specimens tested by: (a) R Vida and J Halvonik [1]; and (b) G Rombach and L Henze [2].

2.3. Material properties
Material properties for all specimens from both experimental campaigns can be seen in table 1.

Table 1. Material properties of the tested specimens [1] and [2].

Authors	Specimen	d	a_v/d	ρ_l	f_c	E_c	d_g
R Vida and J Halvonik (2018) [1]	**SL0.1**	168	2		36.05	35.20	16
	SL0.2			0.73	38.18	38.20	
	SL0.3				34.49	40.50	
	SL1.1			1.69	34.80	38.30	
	SL1.2				32.15	36.80	
	SL2.1			1.27	30.47	34.50	
	SL2.2				29.41	36.30	
G Rombach and L Henze (2017) [2]	**P-2d-1**	215	2	1.17	47.30	34.45	16
	P-2d-2				55.50	36.45	
	P-3d-1		3		37.80	28.32	
	P-3d-2				49.50	34.29	
	P-4d-1		4		40.00	32.75	
	P-5d		5		46.40	30.18	
	P-6d		6		43.00	29.87	

3. Models for assessment of shear resistance

3.1. Eurocode 2
The shear strength formulation according to current EN 1992-1-1 (Eurocode 2) [3], for a member without shear reinforcement was empirically calibrated with experimental data through a statistical approach. The shear strength is given by:

$$V_{R,c} = \left[C_{R,c} \cdot k \cdot \left(100 \cdot \rho_l \cdot f_c \right)^{\frac{1}{3}} \right] \cdot b_w \cdot d \geq 0.035 \cdot k^{\frac{3}{2}} \cdot \sqrt{f_c} \cdot b_w \cdot d$$

(1)

where $C_{R,c}$ is an empirical factor (0.18 MPa), f_c is the concrete compressive strength (in MPa), d is the effective depth (in mm), ρ_l is the longitudinal reinforcement ratio and b_w is the effective shear width (in mm). The size-effect parameter k can be determined as:

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \quad d \text{ in [mm]}$$

(2)

The effective shear width b_w usually depends on the national practices [4]. French and Dutch models are introduced in figure 2(a) and 2(b), respectively. The distribution under 45° is measured from the outer edges of the loaded area in the case of French and from the center of the loaded area in the case of Dutch model.
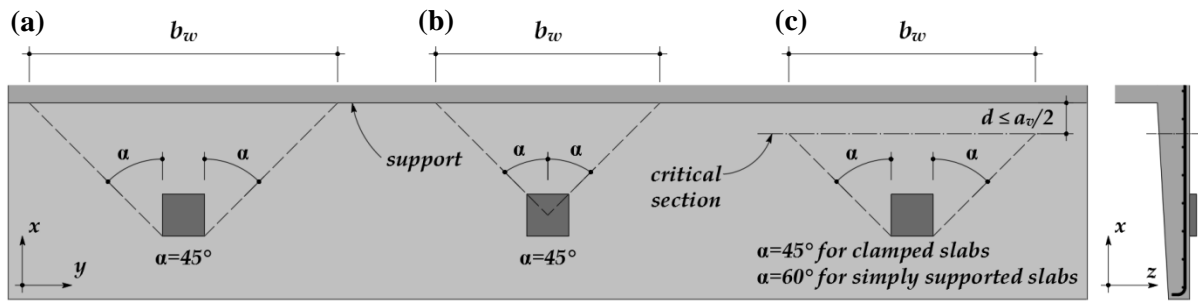

Figure 2. Effective shear width according to the: (a) French practice; (b) Dutch practice; and (c) Model Code 2010.

3.2. Model Code 2010

The *fib* Model Code 2010 (MC2010) [5] proposes three methods for calculating one-way shear resistance of concrete slabs: Level I, Level II and Level IV approximations, which complexity and accuracy increasing as the level arises. The shear strength formulation is based on The Simplified Modified Compression Field Theory [6] and is given by:

$$V_{R,c} = k_v \cdot \sqrt{f_c} \cdot z \cdot b_w$$

(3)

where the lever arm z can be taken as $0.9d$ and the effective shear width b_w is determined by the load distribution angle of 45° for clamped and 60° for simply supported slabs (figure 2(c)). The critical section is assumed at the smaller distance of d and $a_v/2$ from the face of the support.

For a Level I approximation, k_v is determined as:

$$k_v = \frac{180}{1000 + 1.25z}$$

(4)

For a Level II approximation, k_v is determined as:

$$k_v = \frac{0.4}{1 + 1500\varepsilon_x} \cdot \frac{1300}{1000 + k_{dg} \cdot z}$$

(5)

$$\varepsilon_x = \frac{1}{2E_s \cdot a_s} \cdot \left(\frac{m}{z} + v \right)$$

(6)

where ε_x is the longitudinal strain at the mid-depth of the effective depth in the control section, k_{dg} is a parameter depending on the maximum aggregate size which is 1.0 in this case, since $d_g = 16$ mm.

3.3. Critical Shear Crack Theory

The Critical Shear Crack Theory (CSCT) [7] assumes that the critical crack width w_{cr} is proportional to the product of the longitudinal strain in the control depth ε times the effective flexural depth d:

$$w_{cr} \propto \varepsilon \cdot d \tag{7}$$

The following CSCT failure criterion has been proposed [8][9]:

$$v_c(\varepsilon) = \frac{d \cdot \sqrt{f_c}}{3} \cdot \frac{1}{1 + \dfrac{120 \cdot \varepsilon \cdot d}{16 + d_g}} \tag{8}$$

The longitudinal strain ε and the depth of the compression zone c_{flex} are defined by (9) and (10):

$$\varepsilon = \frac{m}{d \cdot \rho \cdot E_s \cdot \left(d - c_{flex}/3\right)} \cdot \frac{0.6d - c_{flex}}{d - c_{flex}} \tag{9}$$

$$c_{flex} = d \cdot \rho \cdot \frac{E_s}{E_c} \cdot \left(\sqrt{1 + \frac{2E_c}{\rho \cdot E_s}} - 1 \right) \tag{10}$$

where m is the maximum unitary bending moment in the control section and E_c and E_s is the modulus of elasticity of concrete and steel, respectively.

4. Numerical analysis

Numerical analysis based on the linear elastic finite element method (LFEM) is becoming a standard tool for modelling reinforced concrete members like bridge deck slabs. However, the results from this analysis are not always reliable, due to the nonlinear behaviour of the reinforced concrete caused by the cracking and yielding of the reinforcement. Experimental results show that the cantilever slabs subjected to concentrated load have the ability to distribute the load in transverse direction. In order to take into account for this distribution of internal forces when using LFEA, Natário et al. [8] proposed a new method based on the CSCT for the assessment of bridge deck slabs. An average shear force $v_{avg,4d}$ is calculated along a distance $4d$ assuming unitary shear forces obtained from LFEA. The control section of the proposed method is located at the distance of $d/2$ from the support as can be seen in figure 3(a). The reference longitudinal strain (see equation (9)) is calculated at the maximum unitary bending moment m at the control section through an iterative procedure. Finally, the ultimate shear failure force V_R is calculated by equation (11) at the intersection with the failure criterion defined previously in equation (8):

$$V_R = \frac{v_c(\varepsilon)}{\beta} \cdot b \tag{11}$$

where b is calculated from LFEA results as the ratio between the applied load and $v_{avg,4d}$ and in order to take into account the arching action, a factor β is applied as [8]:

$$\beta = \frac{a_v}{2.75d} \leq 1.0 \tag{12}$$
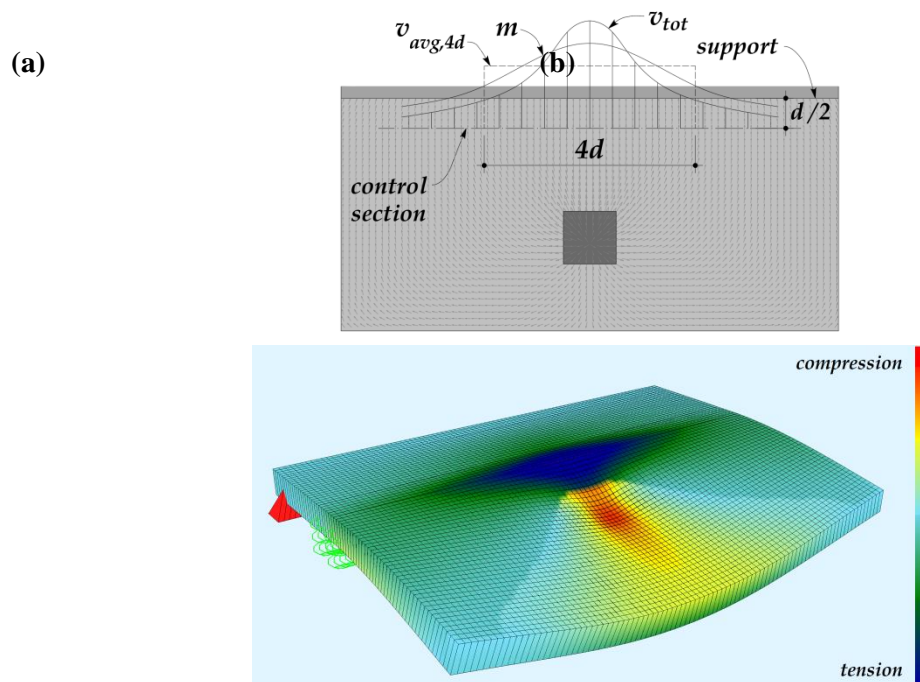
(a)

Figure 3. (a) Definition of reduced shear force $v_{\mathrm{avg,4d}}$ according to Natário et al. [8]; and (b) Deformed numerical model under concentrated load.

Numerical analysis was carried out in SOFiSTiK [10] software, figure 3(b). Quadrilateral 4-node shell elements with shear deformations based on the Mindlin-Reissner theory were used in this study. Self-weight was neglected and the concentrated loads were modelled as surface loads. The central beam of the tested specimens was modelled with linear elastic compression-only springs, representing the stiffness of the steel base in compression and having zero stiffness in tension. Fixed line support was used on the opposite side of the cantilever slabs, allowing rotations only around its axis.

5. Analysis of the test results

Comparisons of the test results with calculated shear resistances according to the relevant models and selected methods are shown in table 2. The best results provided method introduced by Natário et al. [8]. This method is based on the LFEA combined with one-way shear model based on the CSCT. High quality of the method is given by a very low value of CoV $= 0.09$ and average value of the ratio $V_{\mathrm{exp}}/V_{\mathrm{model}}$ equals nearly to one. The model works perfectly for any given a_v/d ratio.

Good results are recorded in the case of the current EC2 model in connection with the model for the distribution of shear forces according to the French recommendation. However, the safety level of the model decreases with increasing of a_v/d ratio. It works well for $a_\mathrm{v}/d \leq 3.0$, then the model safety becomes unacceptable and probably the model for two-way shear (punching) shall be applied or the control section should not be assumed further than $2d$ from the edge of the loading plate. The similar results were observed in the case of the EC2 model with Dutch recommendation. Shorter effective width increases model safety in comparison with the French model but for $a_\mathrm{v}/d \geq 4.0$ the model becomes unsafe too.

Table 2. Comparison between tests and shear strength predictions.

Authors	Specimen	V_{exp}	$V_{\mathrm{EC2,French}}$		$V_{\mathrm{EC2,Dutch}}$		$V_{\mathrm{MC2010,I}}$		$V_{\mathrm{MC2010,II}}$		$V_{\mathrm{LFEA+CSCT}}$	
			kN	*[-]	kN	*[-]	kN	*[-]	kN	*[-]	kN	*[-]
R Vida and	**SL0.1A**	347	256	1.36	166	2.09	146	2.38	194	1.79	377	0.92
J Halvonik	**SL0.1B**	362		1.42		2.18		2.48		1.87		0.96
(2018) [1]	**SL0.2A**	393	261	1.51	169	2.33	150	2.62	197	1.99	385	1.02

	SL0.2B	360		1.38		2.13		2.40		1.83		0.94
	SL0.3A	390	252	1.55	163	2.39	143	2.73	191	2.04	370	1.06
	SL0.3B	383		1.52		2.35		2.68		2.01		1.04
	SL1.1A	373	334	1.12	216	1.72	143	2.60	251	1.48	457	0.82
	SL1.1B	451		1.35		2.08		3.15		1.80		0.99
	SL1.2A	382	325	1.17	211	1.81	138	2.77	244	1.56	442	0.86
	SL1.2B	419		1.29		1.99		3.04		1.72		0.95
	SL2.1A	380	291	1.31	188	2.02	134	2.83	220	1.73	408	0.93
	SL2.1B	350		1.20		1.86		2.61		1.59		0.86
	SL2.2A	430	287	1.50	186	2.31	132	3.26	217	1.98	401	1.07
	SL2.2B	413		1.44		2.22		3.14		1.90		1.03
	P-2d-1	984	597	1.65	365	2.69	314	3.13	412	2.39	805	1.22
	P-2d-2	897	629	1.42	385	2.33	341	2.63	433	2.07	852	1.05
G Rombach	**P-3d-1**	630	669	0.94	454	1.39	355	1.77	445	1.42	590	1.07
and L Henze	**P-3d-2**	680	732	0.93	497	1.37	406	1.67	484	1.40	643	1.06
(2017) [2]	**P-4d-1**	677	800	0.85	581	1.17	442	1.53	510	1.33	636	1.06
	P-5d	699	964	0.73	734	0.95	558	1.25	588	1.19	708	0.99
	P-6d	656	1061	0.62	836	0.78	616	1.06	624	1.05	693	0.95
*V_{exp}/V_{model}		Average		1.25		1.91		2.46		1.72		0.99
		CoV		0.23		0.27		0.26		0.19		0.09

The similar tendency as for the EC2 model is observed in the case of the Model Code 2010 model, which means greater a_v/d ratio leads to the lower model safety. For $a_v/d = 2.0$ the model safety is very high, the average value is 1.86. It might be due to the direct transfer of shear forces to the support by concrete struts. For greater a_v/d ratio the safety level is still quite high.

6. Summary and future work
The presented analyses of the models for prediction of the shear capacity of bridge deck slabs subjected to a concentrated load allow us to draw following conclusions:
- The safety level of the design models that are based on the calculation of the effective width with distribution of shear forces under 45° in the case of clamped slabs is significantly influenced by a_v/d ratio. With increasing a_v/d the safety level is decreasing.
- All models provide safe solution if $a_v/d \leq 2.0$. The EC2 model in connection with the French recommendation is on the unsafe side for $a_v/d \geq 3.0$ and with the Dutch recommendation for $a_v/d \geq 5.0$. The Model Code 2010 model is very conservative for $a_v/d \leq 2.0$ and for $a_v/d > 3.0$ it provides reasonable solution.
- The best results were obtained for Natário et al.'s CSCT model. The safety of the model is independent on the a_v/d ratio. The average value of the ratio V_{exp}/V_{model} which is close to one, can be increased by simple update of the failure criterion in one-way shear CSCT model.
- Future research should be focused particularly on the cases where a_v/d ratio is higher than two. It is necessary to extend database of the test results, where concentrated load is placed at the distance higher than $2d$ measured from the edge of a support.
- An amendment of the current Eurocode 2 is needed in the case of the design of concrete bridge deck slabs for shear. The minimum and maximum distance of the wheel pressure from the edge of a slab support should be introduced in the standard as well as position of the control section if a_v/d ratio is higher than two.

7. References
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Acknowledgments