IOP Conference Series: Materials Science and Engineering

Calculation of Natural Frequencies on Transverse Vibration for drill strings in Non-Uniform Temperature Field

To cite this article: Ma Jianqing *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **563** 052049

Calculation of Natural Frequencies on Transverse Vibration for drill strings in Non-Uniform Temperature Field

Ma Jianqing*, **Li Junqiang and Zhang Guangwei**

School of Mechanical Engineering, Xi'an Shiyou University, Xi'an, Shanxi, 710065, China

*Corresponding author's e-mail: majianqing2000@163.com

Abstract. It is necessary to consider the effect of temperature which changes the mechanical properties of materials when drill strings work in a non-uniform temperature field. In this paper, the natural frequencies of transverse vibration are studied under axial compression in the non-uniform temperature field. Using an asymptotic solution method, first-order approximation formula of the nonlinear vibration equation is obtained. The effects of temperature changes and axial pressure on the natural frequencies are discussed by an example. The results show that the influence of temperature on the critical load is small; change rate of natural frequency increases with the increase of axial pressure; while the axial pressures become larger, the influence of temperature on the first-order frequency is greater than on the second-order's.

1. Introduction

Drill string structures can be simplified into elastic straight rods or elastic beams in drilling engineering. Because of increasing temperature, structures are usually in a state of thermal expansion which changes natural frequencies of straight rods or beams[1-3]. Domestic research is mainly divided into two categories. Considering the change of temperature in constitutive relations or material elastic modulus, one type obtained the nonlinear motion differential equations of vibration, which were solved in different ways. Another type paid attentions to linear vibration of straight rods or beams and discussed the effect of elastic modulus and geometry change on solution. Li G. Q. et al. [4] introduced the influence of axial force on the beam-column unit displacement function, and derived the geometric nonlinear beam-column unit considering the temperature influence. Based on the Hamilton variational principle, Zhao Y. B. et al. [5] obtained the nonlinear differential equation of motion which introduced temperature variation to the stress-strain relationship of beams and investigated temperature effects on the vibration characteristics to three different boundary conditions. Zhou D.[6] studied the natural frequency of transverse, longitudinal and torsional free vibrations of elastic straight bars in non-uniform temperature field using the asymptotic solution method, but did not consider the axial compression.

Drill strings are large-volume and high-quality pipes in oil drilling. It is extremely important to study their dynamic characteristics [7-9]. As temperature increases with the depth of the well, temperature change has a critical impact on drill strings and downhole equipments [10]. Li J. Q. et al. [11] analyzed the influence of high temperature on the natural frequency of drill string transverse vibration, and got the calculation formula of the natural frequency under axial force at any temperature. Temperature distribution and influence on the vibration of drill strings were researched by Dong Z. et al. [12] when their temperature of gas drilling was higher than that of fluid drilling. In this paper, the lateral nonlinear vibration of drill strings subjected to axial compression in a non-uniform temperature

IOP Conf. Series: Materials Science and Engineering **563** (2019) 052049 doi:10.1088/1757-899X/563/5/052049

field is studied and the first-order approximation formula of natural frequency is given. The influence of temperature on the natural frequency is discussed.

2. Basic equation of dynamics

The drill string is simplified into Eular beam model for the non-uniform temperature fields, so the basic dynamic equation of lateral vibration is

$$\frac{\partial^2}{\partial x^2}\left[E(x)I\frac{\partial^2 u}{\partial x^2}\right]+N'\frac{\partial^2 u}{\partial x^2}+\rho A\frac{\partial^2 u}{\partial t^2}=0 \qquad (0\le x\le l) \tag{1}$$

Suppose there is a separate variables solution, let $u(x,t)=\mathrm{U}(x)e^{i\omega t}$, Dimensionless quantity: $X=\frac{x}{l}$.

The vibration equation of beams can be written as

$$\frac{\partial}{\partial X^2}\left(E(X)I\frac{\partial^2 U}{\partial X^2}\right)+N'l^2\cdot\frac{\partial^2 U}{\partial X^2}-\rho A\cdot l^4\cdot\omega^2\cdot U=0 \quad \left(0\le X\le 1\right) \tag{2}$$

For the general temperature distribution, it is expressed as: $T=T_1+\Delta TX^m$, ΔT is the temperature difference between the ends of the beam, T_1 is the temperature at $X=0$, T is the temperature at $X=1$. Experiments had shown that for most metallic materials, the relationship between elastic modulus and temperatures is expressed as: $E=E_0(1-\alpha_E\Delta T)$, α_E is the thermoelastic coefficient.

The change of elasticity modulus along length is: $E=E_0\left[1-\alpha X^m\right]$, $\alpha=\alpha_E\Delta T$.

Using the asymptotic solution method, consider α as a small parameter.

Let $\lambda=\frac{\rho A}{E_0 I}\cdot l^4\cdot\omega^2$, $N=\sqrt{\frac{N'}{E_0 I}}\cdot l$

$$\begin{cases}\mathrm{U}=\mathrm{U}_0+\alpha\mathrm{U}_1+\alpha^2\mathrm{U}_2+\cdots\\ \lambda=\lambda_0+\alpha\lambda_1+\alpha^2\lambda_2+\cdots\end{cases} \tag{3}$$

Substituting Eq. (3) into Eq. (2) , the coefficients α^i $(i=0,1,2\cdots)$ on both sides of the equation are equal. So

$$\begin{cases}\frac{\partial^4 U_0}{\partial X^4}+N^2\cdot\frac{\partial^2 U_0}{\partial X^2}-\lambda_0\cdot U_0=0 & (4)\\ \frac{\partial^4 U_1}{\partial X^4}+N^2\cdot\frac{\partial^2 U_1}{\partial X^2}-\lambda_0\cdot U_1-\lambda_1\cdot U_0=m(m-1)X^{m-2}\frac{\partial^2 U_0}{\partial X^2} & \\ \qquad +2mX^{m-1}\frac{\partial^3 U_0}{\partial X^3}+X^m\frac{\partial^4 U_0}{\partial X^4} & (5)\end{cases}$$

If only considering the temperature linear distribution along the beam $(m=1)$, the first order approximation solution of the natural frequency is:

The solution of Eq. (4) is:

$$U_0(X)=C_1\sin\beta_1 X+C_2\cos\beta_1 X+C_3\,\mathrm{sh}\,\beta_2 X+C_4\,\mathrm{ch}\,\beta_2 X$$

$$\beta_1 = \sqrt{\frac{N^2}{2} + \sqrt{\frac{N^4}{4} + \lambda_0}} \qquad \beta_2 = \sqrt{-\frac{N^2}{2} + \sqrt{\frac{N^4}{4} + \lambda_0}}$$

Substituting U_0 and λ_0 into Eq. (5), w can get

$$\frac{\partial^4 U_1}{\partial X^4} + N^2 \cdot \frac{\partial^2 U_1}{\partial X^2} - \lambda_0 \cdot U_1 = \lambda_1 \cdot U_0 + 2\frac{\partial^3 U_0}{\partial X^3} + X\left(\lambda_0 \cdot U_0 - N^2 \cdot \frac{\partial^2 U_0}{\partial X^2}\right) \tag{6}$$

The general solution of Eq. (6) is:

$$U_{1G}(X) = C_1 \sin(\beta_1 X) + C_2 \cos(\beta_1 X) + C_3 \operatorname{sh}(\beta_2 X) + C_4 \operatorname{ch}(\beta_2 X)$$

The special solution of Eq. (6) is:

$$U_{1S}(X) = A_1 X \sin(\beta_1 X) + B_1 X^2 \sin(\beta_1 X) + A_2 X \cos(\beta_1 X) + B_2 X^2 \cos(\beta_1 X)$$
$$+ A_3 X \operatorname{sh}(\beta_2 X) + B_3 X^2 \operatorname{sh}(\beta_2 X) + A_4 X \operatorname{ch}(\beta_2 X) + B_4 X^2 \operatorname{ch}(\beta_2 X)$$

$B_1 = -C_2\beta_1^3 / (8\beta_1^2 - 4N^2)$, $B_2 = C_1\beta_1^3 / (8\beta_1^2 - 4N^2)$, $B_3 = C_4\beta_2^3 / (8\beta_2^2 + 4N^2)$,

$B_4 = C_3\beta_2^3 / (8\beta_2^2 + 4N^2)$,

$$A_1 = \left[-C_2\lambda_1 + 2C_1\beta_1^3 + B_2(2N^2 - 12\beta_1^2)\right] / (4\beta_1^3 - 2N^2\beta_1)$$
$$A_2 = \left[C_1\lambda_1 + 2C_2\beta_1^3 - B_1(2N^2 - 12\beta_1^2)\right] / (4\beta_1^3 - 2N^2\beta_1)$$
$$A_3 = \left[C_4\lambda_1 + 2C_3\beta_2^3 - B_4(2N^2 + 12\beta_2^2)\right] / (4\beta_2^3 + 2N^2\beta_2)$$
$$A_4 = \left[C_3\lambda_1 + 2C_4\beta_2^3 - B_3(2N^2 + 12\beta_2^2)\right] / (4\beta_2^3 + 2N^2\beta_2)$$

The solution of Eq. (6) is: $U_1(X) = U_{1G}(X) + U_{1S}(X)$

Therefore, the first order approximate solution of lateral vibration when the temperature is linearly distributed along the beam is

$$\begin{cases} U = U_0 + \alpha U_1 \\ \lambda = \lambda_0 + \alpha\lambda_1 \end{cases} \tag{7}$$

3. Result analysis

Consider a simply supported beam, the parameters are: $T_1 = 25^\circ C$,inner diameter 5.08cm, outer diameter 12.07cm, length 100m, section moment of inertia $1007 cm^4$, elastic modulus $E_0 = 2.171 \times 10^{11} Pa$, the mass density of the material $\rho = 7385 kg/m^3$, the thermoelastic coefficient $\alpha_E = 3.95412 \times 10^{-4} /^\circ C$, and the temperature difference between ends of the beam $\Delta T = 200^\circ$.

The boundary conditions are:

$$\begin{cases} U(X)\big|_{X=0} = U(X)\big|_{X=1} = 0 \\ \left.\dfrac{d^2U}{dX^2}\right|_{X=0} = \left.\dfrac{d^2U}{dX^2}\right|_{X=1} = 0 \end{cases}$$

We can get λ_0 from Eq. (4). Substituting λ_0 into Eq. (5), and λ_1 can be obtained by boundary conditions. So $\lambda = \lambda_0 + \alpha\lambda_1$.

According to the formula $\omega = \sqrt{\dfrac{\lambda E_0 I}{\rho A l^4}}$, the first-order approximate solution of the natural frequency can be obtained.

Fig. 1 shows the curve of first-order natural frequency with axial pressure. It can be seen that the natural frequency of the beam decreases with axial pressure when the temperature is constant, namely $T_1 = 25^{\circ}C$. The buckling occurs and axial pressure is the critical load when the natural frequency is zero. When the temperature changes, namely temperature difference between the two ends of the beam is 200°, the natural frequency is slightly smaller than that of constant temperature with same axial pressure. It also decreases with axial pressure and the critical load is same with constant temperature.

Fig. 2-3 show the change rate curves of first two natural frequencies with temperature. As axial pressure increases, the natural frequency change rate increases gradually. When the axial force is zero, the first two orders of natural frequency change rates are identical. When axial pressure becomes large, the influence of temperature on the first-order natural frequency of the lateral vibration is much greater than it on the second-order. Compared with the paper[11], the influence of temperature on natural frequency in the non-uniform temperature field is smaller than that in the uniform temperature field, which is more suitable for actual engineering conditions.

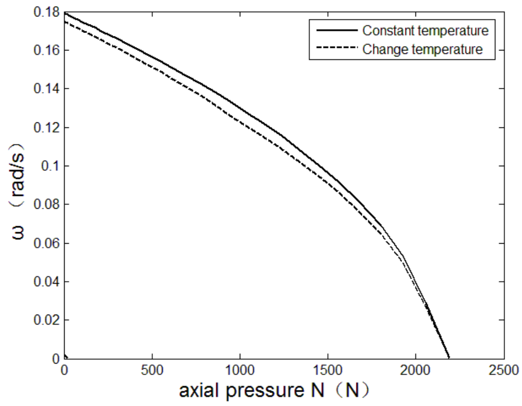

Fig. 1 First-order natural frequency with axial pressure

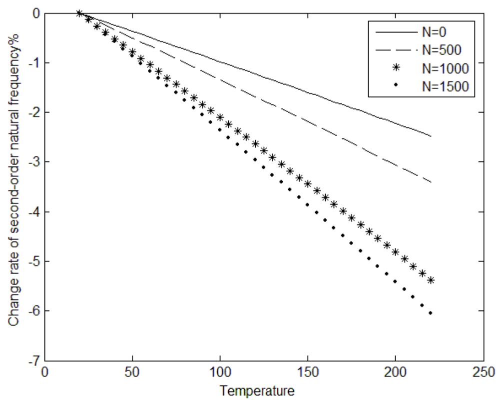

Fig. 2 Change rate of first-order natural frequency with temperature

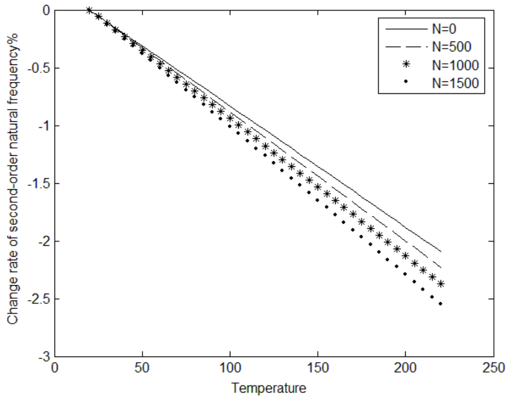

Fig. 3 Change rate of second-order natural frequency with temperature

4. conclusion

For the problem of transverse nonlinear vibration of drill strings subjected to axial compression in non-uniform temperature field, the first-order approximation formula of natural frequency is given by the progressive solution method, and the influence of temperature on the natural frequency is discussed. From the calculation results, the following conclusions can be got:

(1) The critical load of the drill string can not be affected with temperature.

(2) The change rate of natural frequency becomes greater with the axial pressure, but the change is less than it in the uniform temperature field.

(3) The effect of temperature on the first-order frequency is greater than it on the second-order frequency with same axial pressure.

Acknowledgments

The support of the Shanxi Natural Science Foundation "Dynamic Characteristics Research of Closed Loop Controllable Bending Joint System Based on Rotary Steering Drilling Technology" (No. 2018JM5015) is gratefully acknowledged.

References

[1] Shi J., Hao J.P. (2008) Analytic solution for nonlinear vibrations of elastics straight bar in uniform temperature fields. Journal of Chang'an University (Natural Science Edition), , 28(5), 120-123.

[2] Manoach E., Ribero P.(2004)Coupled, thermo-elastic, large amplitude vibrations of Timoshenko beams [J]. International Journal of Mechanical Sciences, 46:1589-1606.

[3] Avsec J., Oblank M.(2007)Thermal vibration analysis for simple supported beam and clamped beam [J]. Journal of Sound and Vibration,308:514-525.

[4] Li G.Q., Wang Z. (2016)An efficient geometric nonlinear beam-column element considering temperature. Journal of Tongji University (Natural Science), 44, (6): 815-821.

[5] Zhao Y.B., Huang C.H., Jin B., Hua W.W. (2017)Temperature variations on nonlinear vibration characteristics of beam with different boundary conditions. Chinese Quarterly of Mechanics, 38(3): 496-502.

[6] Zhou D.(1989) An asymptotic method for natural frequencies of transverse and longitudinal and torsional free vibration of elastic bars in non-uniform temperature fields. Engineering Mechanics, 6(3): 55-62.

[7] Li Z.F.(2016)Research advances and debates on tubular mechanics in oil and gas wells. Acta Petrolei Sinica,37(4):531-556.

[8] Hu Y.B., Di Q.F., Zou H.Y., Wu F.(2006)The new develpments of monitoring technology and researches on drill string dynamics. Petroleum Drilling Techniques, 34(6): 7-10.

[9] Tian J.L., Wu C.M., Yang Y.W., Yao Y., Yang Z., Yuan C.F., Wu B. (2017)Numerical analysis of downhole drill string with a longitudinal and lateral coupled vibration model, Applied Mathematics and Mechanics, 38(6): 685-695.

[10] Li M.B., Xu L.B., Luo H.B., Yan Y.N., Li G.S.(2018)Study on wellbore circulating temperature distribution and control method in deep water high temperature wells, China Offshore Oil and Gas,30(4): 158-162.

[11] Li J.Q., Shi N.N.(2006)The influence of high temperature in deep holes on inherent frequency of drill string lateral vibration. China Petroleum Machinery, 34(6): 14-16.

[12] Dong Z., Han W., Feng Q., Long Y.Y., He Y.X.(2013)The influence analysis of downhole temperature on the vibration of drill string with gas drilling. Chemical Engineering and Equipment,6:111-113.