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## Three Calculation Methods of the WGM Cylindrical Cavity Diameter

To cite this article: Zhou Li *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **563** 052012

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# Three Calculation Methods of the WGM Cylindrical Cavity Diameter

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**Abstract.** WGM mode in cylindrical cavity can be rapidly calibrated with analytical approximate formulas based on cylindrical micro-cavity WGM resonance peaks position. On this basis, the diameter of circular cavity can be calculated by applying the three methods, which are the first-order approximation method, the second-order approximation method and mode field radius. When comparing the results which are got from the three methods with the actual geometric diameter, we find that the deviation is within 3%, 2%, 1% respectively. As a result, the diameter of circular cavity can be correctly worked out by making use of these three ways mentioned.

## 1. Three calculation methods and comparisons of micro-cavity diameter

### 1.1 Calculating Micro-cavity Diameter by applying First-order Approximation Method

The analytical approximate formula can be derived by applying characteristic equation <sup>[1]</sup> satisfied by WGM mode in cylindrical cavity, and the polynomial expansion method <sup>[2]</sup> for the position and spacing of WGM mode in spherical micro-cavity. And the formula can quickly calibrate the WGM mode in cylindrical micro-cavity. The analytical approximate formula is as following:

$$\frac{m_1}{\lambda_{l,n}} = \frac{1}{2\pi a} \left[ n + 2^{-1/3} a_l n^{1/3} - \frac{p}{(m^2 - 1)^{1/2}} + \frac{3}{10} 2^{-2/3} a_l n^{-1/3} - \frac{2^{-1/3} p(m^2 - 2p^2/3)}{(m^2 - 1)^{3/2}} a_l n^{-2/3} + O(n^{-1}) \right] \quad (1)$$

For TM mode,  $p = \frac{m_1}{m_2} = m$ , for TE mode,  $p = \frac{m_2}{m_1} = \frac{1}{m}$ ;  $m_1$  is the refractive index of the cylindrical micro-cavity,  $m_2$  is the refractive index of the external medium;  $\lambda_{l,n}$  is the resonance wavelength of WGM,  $a_l$  is the root of the Airy function, which values respectively are  $a_1 = 2.338107$ ,  $a_2 = 4.087949$ .

Formula (1) shows that WGM of cylindrical micro-cavity can be calibrated by three independent parameters ( $p, l, n$ ). By doing this, the interval (wave number difference) formula of adjacent angular modes ( $\Delta n=1$ ) is available:

$$\frac{1}{\lambda_{l,n+1}} - \frac{1}{\lambda_{l,n}} = \frac{1}{2\pi a m_1} \left\{ 1 + \frac{2^{-1/3}}{3} a_l n^{-2/3} - \frac{2^{-2/3}}{10} a_l^2 n^{-4/3} \right\}$$



$$+ \frac{2^{2/3}}{3} \frac{p(m^2 - 2p^2/3)}{(m^2 - 1)^{3/2}} a_l n^{-5/3} + O(n^{-2}) \} \quad (2)$$

According to the wave optics theory of WGM and the first-order approximation of formula (1), it can be deduced:

$$\delta\lambda = \frac{\lambda^2}{2\pi r m_1} \quad (3)$$

For the mode spacing of different cylindrical cavities, the diameters of different cylindrical cavities can be calculated from formula (3). Based on cylindrical micro-cavity WGM laser spectral wavelength which diameter varied respectively from 85  $\mu\text{m}$ , 91  $\mu\text{m}$ , 195  $\mu\text{m}$  to 289  $\mu\text{m}$ , as shown in Figure 1.

The diameters of different cylindrical Chambers can be calculated respectively by substituting ( $\delta\lambda = 0.673 \text{ nm}$ ,  $\lambda = 509 \text{ nm}$ ), ( $\delta\lambda = 0.633 \text{ nm}$ ,  $\lambda = 510 \text{ nm}$ ), ( $\delta\lambda = 0.309 \text{ nm}$ ,  $\lambda = 520 \text{ nm}$ ), ( $\delta\lambda = 0.208 \text{ nm}$ ,  $\lambda = 520 \text{ nm}$ ) and  $m_1 = 1.458$  into equation (3), which are 84.05  $\mu\text{m}$ , 89.71  $\mu\text{m}$ , 191.05  $\mu\text{m}$ , 283.82  $\mu\text{m}$ . Compared the diameter of the cylindrical cavity calculated by using the first-order approximation formula with the actual geometrical diameter of the cylindrical cavity (measured with a reading microscope with the minimum degree of 1 micron), the deviations are less than 3%, as shown in Table 1.

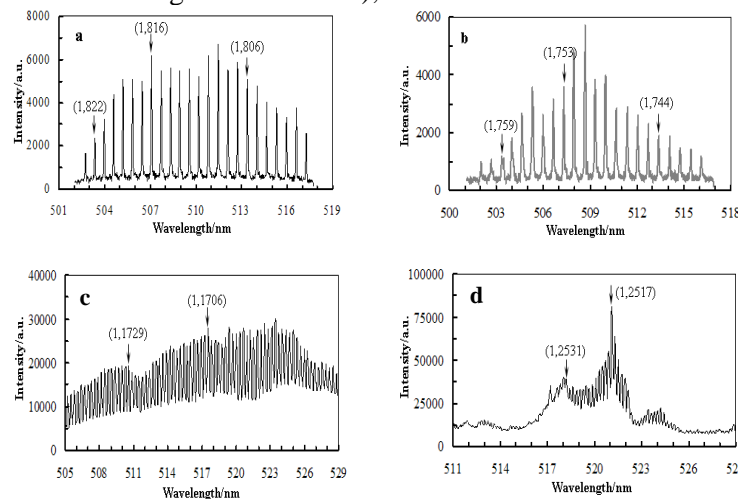


Figure 1, WGM spectra and WGM model calibration of cylindrical micro-cavities with diameters varied from (a) 85  $\mu\text{m}$ , (b) 91  $\mu\text{m}$  (c) ,195  $\mu\text{m}$  and (d) 289  $\mu\text{m}$ .

Table 1 Deviation values of micro-cavity diameter calculated by first-order approximation method and actual geometric measurement diameter. (Diameter units:  $\mu\text{m}$ , deviation units: %)

Actual diameter	Theoretical diameter	D-value	Relative error
85	84.05	0.95	1.1
91	89.71	1.29	1.4
195	191.05	3.95	2.0
289	283.82	5.18	1.8

### 1.2 Calculate Micro-cavity Diameter by applying Second-order Approximation Method

According to analytical approximate formula (1) of WGM resonant peak position of cylindrical micro-cavity, the wave number interval between the same radial mode (1) and adjacent angular mode

(n) can be calculated by formula (2). Given  $\delta\nu = \frac{1}{\lambda_{l=1}^{n-1}} - \frac{1}{\lambda_{l=1}^n}$ , respectively giving the values to  $\delta\nu$ , ( $\delta\nu = 26.15 \text{ cm}^{-1}$ ,  $n = 751$ ), ( $\delta\nu = 24.37 \text{ cm}^{-1}$ ,  $n=810$ ), ( $\delta\nu = 11.44 \text{ cm}^{-1}$ ,  $n = 1697$ ), ( $\delta\nu = 7.66 \text{ cm}^{-1}$ ,  $n = 2517$ ) and  $m_1=1.458$ ,  $a_l=2.338107$ , (substituted them into the forth terms of equation(2), the theoretical diameters by the second-order approximation of the cylindrical micro-cavity of different wave number interval are  $84.11\mu\text{m}$ ,  $90.37\mu\text{m}$ ,  $192.23\mu\text{m}$ ,  $285.97\mu\text{m}$ . Compared it with the actual geometrical diameter of the cylindrical cavity, the author finds that the deviation is less than 2% as shown in table 2:

Table 2 The deviation between the diameter of micro-cavity calculated by the second-order approximation and the actual geometrical measurement. (Diameter units:  $\mu\text{m}$ , deviation units: %)

Actual diameter	Theoretical diameter	D-value	Deviation
85	84.11	0.89	1.0
91	90.23	0.77	0.85
195	192.23	2.77	1.4
289	285.97	3.03	1.0

### 1.3 Calculating Micro-cavity Diameter by applying "mode field radius"

According to the distribution curve of WGM's field intensity, the peak of light intensity is somewhere which radius  $r < a$ . Therefore, "mode field radius" in cylindrical coordinates is defined by introducing the concept of WGM's mode field radius.

$$RS_{l,n} = \frac{\int_0^{\infty} E_{l,n}(r) E_{l,n}^*(r) r^2 dr}{\int_0^{\infty} E_{l,n}(r) E_{l,n}^*(r) r dr} \quad (4)$$

$RS_{l,n}$  represents a statistical average radius calibrated by (p, l, n), in which the photons in WGM rotates along the  $RS_{l,n}$  surface instead of the cylinder cavity ( $r = a$ ). According to the concept of "mode field radius", the analytical calculation formula for the mode field radius can be obtained by substitute

$$\frac{1}{\lambda_{l,n+1}} - \frac{1}{\lambda_{l,n}} = \frac{1}{2\pi m_1 R_{l,n}} \quad \text{into formula (2)}^{[4]}:$$

$$RA_{l,n} = a \left[ 1 - \left( \frac{2^{-1/3}}{3} a_l n^{-2/3} - \frac{2^{-2/3}}{10} a_l^2 n^{-4/3} + \frac{2^{2/3}}{3} \frac{p(m^2 - 2p^2/3)}{(m^2 - 1)^{3/2}} a_l n^{-5/3} \right) \right] \quad (5)$$

Respectively gave the value  $n=751$ ,  $n = 810$ ,  $n = 1697$  and  $n = 2517$ , according formula (5), the theoretical diameters of cylindrical cavity which actual geometrical diameters are varied from  $84.35 \mu\text{m}$ ,  $91.33 \mu\text{m}$ ,  $194.15 \mu\text{m}$  to  $288.03 \mu\text{m}$  can be calculated. Compared it with the actual geometric diameters of cylindrical cavity, the author found that the deviation is less than 1% as shown in Table 3.

According to the calibration results, the theoretical value of mode field radius, respectively,  $84.35 \mu\text{m}$ ,  $91.33 \mu\text{m}$ ,  $194.15 \mu\text{m}$ ,  $288.03 \mu\text{m}$  is calculated by plugging the cylindrical micro-cavity WGM laser spectral wavelength,  $n = 751$ ,  $n = 810$ ,  $n = 1697$  and  $n = 2517$ , taken from the diameter of cylindrical cavity,  $85 \mu\text{m}$ ,  $91 \mu\text{m}$ ,  $195 \mu\text{m}$ ,  $289 \mu\text{m}$  into formula (5). The deviation is found within 1% when they are compared with the actual geometric diameter of cylindrical cavity, as shown in Table 3.

Table 3 The deviation between the diameter of micro-cavity calculated by the mode field radius and the actual geometrical measurement. (Diameter unit:  $\mu\text{m}$ , deviation unit: %)

Actual diameter	Theoretical diameter	D-value	Deviation
85	84.35	0.65	0.76

91	90.34	0.66	0.70
195	194.15	0.85	0.44
289	288.03	0.97	0.34

## 2. Summary

By comparing the three results of the cylindrical cavity diameter calculated by three calculation methods with the actual geometrical measurement, the deviations are respectively within 3%, 2%, 1%. From this we can draw a conclusion. It can be drawn a conclusion that the diameter of the cylindrical cavity is well calculated by applying the three methods mentioned above and the deviation is small. However, the deviation calculated by WGM mode field radius of cylindrical micro-cavity is the smallest, within 1%. Meanwhile, the theoretical diameters as shown in Table 3, calculated by the mode field radius, is always less than the actual geometrical diameter, which proves that the light in cylindrical micro-cavity cannot rotate along the rim of cylindrical cavity wall, which radius  $r = a$ , but along the cavity wall which radius  $r < a$ . At the same time the deviation gradually decreases with the increases of cylindrical cavity diameter.

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