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Abstract: Unmodeled dynamics and external disturbances are the main factors affecting system performance. In this paper, a robust design method based on backstepping control and observer is proposed to overcome the problem of the system unmodeled parts and external disturbances. To improve the performance, the controller uses adaptive function to eliminate the effect of the unmodeled dynamics. For the external disturbances, this paper using observer to accurately estimated and effectively compensated. The simulation results show that the controller designed in this paper has good performance and can accurately response the input commands in the case of the unmodeled part of the system and the external disturbance.

1. Introduction

There are always unmodeled parts and external disturbances in a practical plant, which have brought a great challenge for the design of the controller. In this case, the robust design method has received great attention [1~3].

To improve the robustness of the system, many control methods have been proposed and studied extensively [4~6]. Among these methods, the back-stepping control method has attracted the attention of researchers in various fields, which has displayed good performance to deal with nonlinear problems and a rigorous mathematical proof process [7,8]. In [9], an adaptive neural network controller is designed using the back-stepping method. A method of preset performance robust controller is proposed based on error transformation in [10]. A robust method combining the backstepping method and sliding mode method is proposed in [11,12]. To better eliminate the influence of external disturbances, the observer is a feasible method which can accurately estimate disturbance and then compensate it effectively. Backstepping method and the observer can provide feasible solutions for improving the performance of the system.

Although these methods can improve the robustness of the system to a certain extent, mainly aimed at the impact of the unmodeled part, the impact of external disturbances on system performance has not been fully considered. To better eliminate the influence of external disturbances, the observer is a feasible method which can accurately estimate disturbance and then compensate it effectively in [13~18]. The integration of the back-stepping method and the observer can provide feasible solutions for improving the performance of the system. For example, [19] proposed an attenuation control strategy using generalized extended state observer which is used to estimate the disturbances and uncertainties in the plant. In [20], nonlinear disturbance observer and back-stepping finite-time sliding mode control are integrated. In [21], a finite dimensional backstepping technique with boundary control is designed for the accurate control of a flexible riser system, the robustness of disturbance rejection and vibration abatement is improved by integrating with a boundary disturbance observer.

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Although these methods use various observers to estimate and compensate for disturbances, the effects of the unmodeled parts on the system are not considered. Unmodeled parts and external disturbances are two main factors that affect system performance and must be taken seriously. For this problem, the present paper proposes a robust control method by integrating backstepping technology and observer on the bias of preceding literature considering the unmodeled parts and external disturbances. The controller uses an adaptive function to eliminate the influence of the unmodeled part on the system. For the objective of disturbance rejection, the control strategy based on the designed observer is used to estimate and effectively compensate for disturbances accurately.

2. Problem Formulation

In practical systems, there exit unmodeled parts and external disturbances bringing a challenge for the design of the controller and limit the practical implementation. Therefore it's necessary to design a robust controller. The influence of the unmodeled part of the system and the external disturbance should be fully considered when constructing the system model. Therefore, this paper establishes the following model:

$$\dot{x}_1 = f_1(x_1) + b_1(x_1, x_2)x_2 \tag{1}$$

$$\dot{x}_2 = f_2(x_1, x_2) + b_2 u - d \tag{2}$$

In the formula, x_1, x_2 is the state of the system, d is the external disturbance.

Considering the influence of the unmodeled part of the system, the outer loop part of the system is expressed as follows:

$$f_1(x_1) = f_{10}(x_1) + \Delta f_1(x_1)$$

$$b_1(x_1, x_2) = b_{10}(x_1, x_2) + \Delta b_1(x_1, x_2)$$

where, $f_{10}(x_1), b_{10}(x_1, x_2)$ is the nominal parameter of the system, and the rest are uncertainties of the system.

3. The design of Out-loop Controller and stability analysis

The controller is designed to make the state converge to a specified infinitely small neighborhood of origin. Therefore, we introduce error state as follows

$$\begin{cases} z_1 = x_1 - x_{1d} \\ z_2 = x_2 - x_{2d} \end{cases} \tag{3}$$

where, x_{1d}, x_{2d} is the desired system state trajectory. The dynamic of the state error is as follows

$$\dot{z}_1 = f_1(x_1) + b_1(x_1, x_2)x_2 - \dot{x}_{1d} \tag{4}$$

$$\dot{z}_2 = f_2(x_1, x_2) + b_2 u - d - \dot{x}_{2d} \tag{5}$$

The outer loop controller of the system is designed in the control scheme of backstepping control technology. For nominal systems, there exits the following lemma:

Lemma 1: There exit real constants $\alpha_m, \beta_m, \phi_m$ that make $b_1(x_1, x_2)$ reversible for all that satisfy the condition $|\alpha| \le \alpha_m, |\beta| \le \beta_m, |\phi| \le \phi_m$.

For the system identified by equation (4), the system model can be expressed as:

$$\begin{aligned} \dot{z}_1 &= f_{10}(x_1) + b_{10}(x_1, x_2)x_2 - \dot{x}_{1d} + (\Delta f_1(x_1) \\ &+ \Delta b_1(x_1, x_2)x_2) = f_{10}(x_1) + b_{10}(x_1, x_2)x_2 - \dot{x}_{1d} + \Delta_1 \end{aligned} \tag{6}$$

where, $\Delta_1 = \Delta f_1(x_1) + \Delta b_1(x_1, x_2)x_2$ is the unmodeled part of the system. The impact of this part requires the robustness of the system to be eliminated.

For the plant, suppose that there exit a positive real number ρ_1 make:

$$\|\Delta_1\| \le \rho_1\delta_1(x_1,x_2) \tag{7}$$

where, $\delta_1(x_1,x_2)$ is a known non-negative smooth function. This assumption defines the boundedness of the unmodeled part of the system. This assumption is satisfied for most systems.

Take x_2 as the virtual control input of the outer loop of the system; an ideal virtual control amount can be seen from Lemma 1.

$$x_2^* = -b_{10}^{-1}(x_1,x_2)[f_{10}(x_1) - \dot{x}_{1d} + k_1 z_1 + \Delta_1] \tag{8}$$

$$\dot{z}_1 = -k_1 z_1 + b_{10}(x_1,x_2)(x_2 - x_2^*) \tag{9}$$

In the formula, according to the system performance requirements, the given design parameters is required.

Due to the existence of the unmodeled part of the system Δ_1, x_2^* can't be determined by the Eq.(8). Suppose that there eixts an ideal amount of virtual control as follows:

$$x_{2d} = -b_{10}^{-1}(x_1,x_2)[f_{10}(x_1) - \dot{x}_{1d} + k_1 z_1 - \eta_1] \tag{10}$$

where, η_1 is the robust function coefficient to needs to be designed to eliminate the effects of unmodeled parts of the system Δ_1. This coefficient can be determined by the Lyapunov stability theory as follows.

Assume that the estimated value of the unknown real number, choose the Lyapunov function candidate as follows

$$V_1 = \frac{1}{2}z_1^T z_1 + \frac{1}{2r_1}\bar{\rho}_1^2 \tag{11}$$

where, $r_1 > 0$ is the parameter that needs to be choosen appropriately, $\bar{\rho}_1 = \hat{\rho}_1 - \rho_1$ is the estimated error.

$$\begin{aligned} \dot{V}_1 &= z_1^T\dot{z}_1 + \frac{1}{r_1}\bar{\rho}_1\dot{\bar{\rho}}_1 \\ &= z_1^T(f_{10}(x_1) + b_{10}(x_1,x_2)x_2 - \dot{x}_{1d} + \Delta_1) \\ &\quad + z_1^T(b_{10}(x_1,x_2)x_{2d} - b_{10}(x_1,x_2)x_{2d}) + \frac{1}{r_1}\bar{\rho}_1\dot{\bar{\rho}}_1 \end{aligned} \tag{12}$$

x_{2d} has been determined by the Eq. (10) bring in the method (12) can obtain the following equation:

$$\begin{aligned} \dot{V}_1 &= -k_1\|z_1\|^2 + z_1^T b_{10}(x_1,x_2)(x_2 - x_{2d}) + z_1^T(\Delta_1 - \eta_1) + \frac{1}{r_1}\bar{\rho}_1\dot{\bar{\rho}}_1 \\ &= -k_1\|z_1\|^2 + z_1^T b_{10}(x_1,x_2)z_2 + z_1^T(\Delta_1 - \eta_1) + \frac{1}{r_1}\bar{\rho}_1\dot{\bar{\rho}}_1 \end{aligned} \tag{13}$$

Choose adaptive law

$$\dot{\hat{\rho}}_1 = r_1[l_1 - \sigma_1(\hat{\rho} - \rho_1^0)] \tag{14}$$

where, σ_1 and ρ_1^0 are the design parameters. ρ_1^0 can be determined according to the uncertain characteristic information of the system. Then Eq. (12) can be expressed as

$$\begin{aligned} \dot{V}_1 &= -k_1\|z_1\|^2 + z_1^T b_{10}(x_1,x_2)z_2 + z_1^T(\Delta_1 - \eta_1) \\ &\quad + \bar{\rho}_1 l_1 - \sigma_1\bar{\rho}_1(\hat{\rho}_1 - \rho_1^0) \end{aligned} \tag{15}$$

Choose the robust function coefficients η_1 and functions l_1 as shown below:

$$\eta_1 = \varepsilon_1 z_1 \hat{\rho}_1^2 \delta_1^2 \tag{16}$$

$$l_1 = \varepsilon_1 \|z_1\|^2 \delta_1^2 \tag{17}$$

where, ε_1 is the design parameter.

$$-\sigma_1 \bar{\rho}_1 (\hat{\rho}_1 - \rho_1^0) = -\frac{1}{2}\sigma_1 \bar{\rho}_1^2 - \frac{1}{2}\sigma_1 (\hat{\rho}_1 - \rho_1^0)^2 + \frac{1}{2}\sigma_1 (\rho_1 - \rho_1^0)^2 \tag{18}$$

Substitute Eq. (16) and (17) into Eq. (15) , we can derive

$$\begin{aligned} \dot{V}_1 = & -k_1 \|z_1\|^2 + z_1^T b_{10}(x_1, x_2) z_2 - \frac{1}{2}\sigma_1 \bar{\rho}_1^2 \\ & + z_1^T (\rho_1 \delta_1 - \varepsilon_1 z_1 \hat{\rho}_1^2) + \bar{\rho}_1 \varepsilon_1 \|z_1\|^2 \delta_1^2 \\ & - \frac{1}{2}\sigma_1 (\hat{\rho}_1 - \rho_1^0)^2 + \frac{1}{2}\sigma_1 (\hat{\rho}_1 - \rho_1^0)^2 \end{aligned} \tag{19}$$

$$\begin{aligned} & z_1^T (\Delta_1 - \varepsilon_1 z_1 \hat{\rho}_1 \delta_1^2) + \bar{\rho}_1 \varepsilon_1 \|z_1\|^2 \delta_1^2 \\ & \le \|z_1\| \delta_1 \rho_1 - \|z_1\|^2 \varepsilon_1 \hat{\rho}_1 \delta_1^2 + \bar{\rho}_1 \varepsilon_1 \|z_1\|^2 \delta_1^2 \\ & \le (\varepsilon_1 \|z_1\|^2 \delta_1^2 + \frac{1}{4\varepsilon_1}) \rho_1 - \|z_1\|^2 \varepsilon_1 \hat{\rho}_1 \delta_1^2 \\ & - \frac{\varepsilon_1^2}{4} \|z_1\|^2 \delta_1^2 + \bar{\rho}_1 \varepsilon_1 \|z_1\|^2 \delta_1^2 \\ & = -\frac{\varepsilon_1^2}{4} \|z_1\|^2 \delta_1^2 + \frac{\rho_1}{4\varepsilon_1} \le \frac{\rho_1}{4\varepsilon_1} \end{aligned} \tag{20}$$

$$\begin{aligned} \dot{V}_1 \le & -k_1 \|z_1\|^2 + z_1^T b_{10}(x_1, x_2) z_2 - 0.5\sigma_1 \bar{\rho}_1^2 \\ & + 0.5\sigma_1 (\rho_1 - \rho_1^0)^2 + \frac{\rho_1}{4\varepsilon_1} \end{aligned} \tag{21}$$

Then proof of stability is completed.

4. The design of the Inner-loop controller and observer

Similar to the design of the out-loop controller, the design of the inner loop controller is as followed.

According to Eq. (5) and backstepping technology, there is an ideal control input in the inner loop of the system:

$$u^* = -b_2^{-1}(k_2 z_2 + f_2(x_1, x_2) - d - \dot{x}_{2d}) \tag{22}$$

$$\dot{z}_2 = -k_2 z_2 \tag{23}$$

where, k_2 is a selected parameter greater than zero according to system performance requirements.

According to the control input of equation (22), it guarantees that the error state of the system converges in the exponential form. However, the specific form of the control law cannot be accurately known due to the presence of external disturbances d . To solve this problem, the observer is designed to estimate the external disturbances.

For the outer loop of Eq. (1), it is derived and substituted for Eq. (2):

$$\begin{aligned} \ddot{x}_1 & = \dot{f}_1(x_1) + b_1(x_1 x_2)\dot{x}_2 \\ & = \dot{f}_1(x_1) + b_1(x_1 x_2) f_2(x_1 x_2) + \\ & \quad b_1(x_1 x_2) b_2 u - b_1(x_1 x_2) d \end{aligned} \tag{24}$$

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Express equation (24) as follows:

$$\ddot{x}_1 = -b\dot{x}_1 + c + au - d_1 \tag{25}$$

where, $-b\dot{x}_1$ is the expanded form of $\dot{f}_1(x_1)$;

$$c = b_1(x_1\, x_2) f_2(x_1\, x_2)$$
$$a = b_1(x_1\, x_2) b_2$$
$$d_1 = b_1(x_1\, x_2) d$$

For the system shown in equation (25), an observer of the following form is designed:

$$\dot{\hat{d}}_1 = g_1(\hat{\omega} - \dot{x}_1) \tag{26}$$

$$\dot{\hat{\omega}} = -\hat{d}_1 + au + c - g_2(\hat{\omega} - \dot{x}_1) - b\dot{x}_1 \tag{27}$$

where, $\hat{d}_1$ is the observation of the disturbances term d_1, $\hat{\omega}$ is the observation of $\dot{x}_1$, g_1, g_2 are the observer parameter greater than zero.

The stability of the observer is analyzed below. Select the Lyapunov function as follows:

$$V = \frac{1}{2g_1}\tilde{d}_1^{\,2} + \frac{1}{2}\tilde{\omega}^2 \tag{28}$$

where, $\tilde{d}_1 = d_1 - \hat{d}_1$, $\tilde{\omega} = \dot{x}_1 - \hat{\omega}$.

$$\begin{aligned} \dot{V} &= \frac{1}{g_1}\tilde{d}_1\dot{\tilde{d}}_1 + \tilde{\omega}\dot{\tilde{\omega}} \\ &= \frac{1}{g_1}\tilde{d}_1(\dot{d}_1 - \dot{\hat{d}}_1) + \tilde{\omega}(\ddot{x}_1 - \dot{\hat{\omega}}) \end{aligned} \tag{29}$$

In general, external disturbances are slow time-varying signals; therefore $\dot{d}_1$ it is small. When g_1 taking a larger value $\frac{1}{g_1}\dot{d}_1 \approx 0$.

Substituting the above formulas into equation (29):

$$\begin{aligned} \dot{V} &= \frac{1}{g_1}\tilde{d}_1\dot{\hat{d}}_1 + \tilde{\omega}(\ddot{x}_1 - (-\dot{\hat{d}}_1 + au + c - g_2(\hat{\omega} - \dot{x}_1) - b\dot{x}_1)) \\ &= \frac{1}{g_1}\tilde{d}_1 g_1(\hat{\omega} - \dot{x}_1) + \tilde{\omega}(-b\dot{x}_1 + + au + c - d_1 \\ &\quad -(-\dot{\hat{d}}_1 + au + c - g_2(\hat{\omega} - \dot{x}_1) - b\dot{x}_1)) \\ &= -\tilde{d}_1(\hat{\omega} - \dot{x}_1) + \tilde{\omega}(-d_1 + \dot{\hat{d}}_1 + g_2(\hat{\omega} - \dot{x}_1)) \\ &= -\tilde{d}_1\tilde{\omega} + \tilde{\omega}(-\tilde{d}_1 - g_2\tilde{\omega}) \\ &= -g_2\tilde{\omega}^2 \le 0 \end{aligned}$$

Therefore, effective observation d_1 can be performed by the observer.

Through the control, lows are shown in equations (10) and (22), and the observers are shown in equations (26) and (27), the tracking performance of the system with external disturbance, and uncertainty modeling can be guaranteed.

5. Simulation

For systems as follows:

$$\dot{x}_1 = 4x_1^2 + (-4.5x_1 + 2x_2 - 5x_2^2)x_2$$
$$\dot{x}_2 = -6.25x_2 + 5x_1x_2 + (\sin(x_1) - \sin(x_2))u - d$$

Assume that the system's unmodeled parts are:

$$\Delta f(x) = \varepsilon x_1 \sin(x_1)$$
$$\Delta b_1 = \alpha\sqrt{2}/2\sin(x_1x_2)$$

where, ε,α are random numbers in the range $[0,1]$.

5.1. Tracking effect of the observer

Assume that the external disturbance is $d = 150sign(\sin(0.1t))$, $\text{sgn}(\bullet)$ is a sign function. Take the observer parameters $g_1 = 500$, $g_2 = 200$; The tracking effect of disturbances are shown in Fig.1.

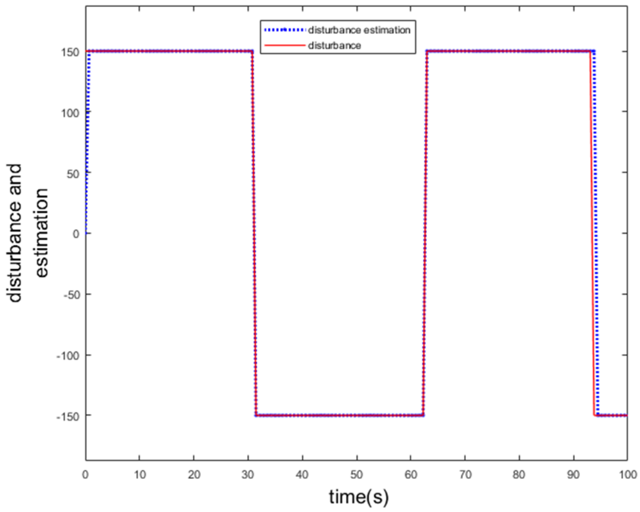

Fig.1 Observational performance of the observer

From the simulation results, the observer designed in this paper can accurately predict slow external disturbances.

5.2. The tracking effect of the controller

If the outer ring command signal $x_{1d} = \sin t$. Under the controller of this paper, the tracking effect of the system is shown in Fig.2.

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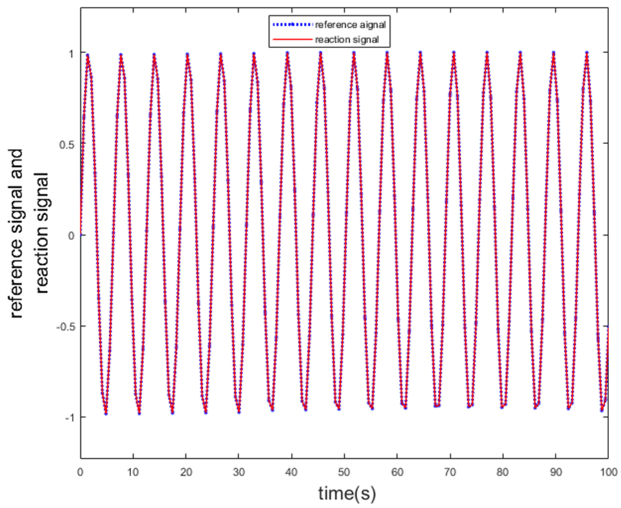

Fig.2 Control effect of the controller

It can be seen from the simulation results that the controller designed in this paper can accurately track the input commands in the case of unmodeled and external disturbances in the system. It is proved that the controller designed in this paper has excellent performance.

6. Conclusions

The system's unmodeled parts and external disturbances are the main factors affecting system performance. In this paper, a robust design method based on backstepping control method and observer is proposed to overcome the shortcomings of current research that cannot fully consider the problem of the system unmodeled parts and external disturbances. To improve the performance of the system, a perturbation model with disturbances is established. The controller uses an adaptive function to eliminate the influence of the unmodeled part of the system. For the external disturbances, this paper designs an observer to accurately estimated and effectively compensated. The simulation results show that the controller designed in this paper has good performance and can accurately response the input commands in the case of the unmodeled part of the system and the external disturbance.

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