Fatigue Estimation based on Stress RMS under Stochastic Excitation

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Abstract. The thesis presents a case study of fatigue estimation based on stress response root mean square under stochastic excitation. The cantilever beam with a hole is subjected to a stochastic vibration test firstly, during which measured the time history of stress at critical locations using strain gauges. Then the characteristics of measured locations' stress response RMS is discussed and the stress RMS curve of fatigue failure location is fitted. Finally, fatigue estimation is taken out based upon the stress RMS curve compared with experimental test life. The result indicates that it is practical to use stress RMS to estimate fatigue life under stochastic excitation, providing a method for fatigue analysis in engineering applications.

1. Introduction

Vibration under random excitation is common problems in aerospace, railway and shipping transportation areas. Most vibration is caused by random excitation in working environment such as impact force from weapons' ejection and delivery, aircrafts' taken-off or landing. Vertical tail of aircraft in flight, spacecraft in launch phase often subject to severe random vibrations finally leading to fatigue failure.

Time-domain method is widely used in fatigue simulation under stochastic excitation, the first step is to obtain structure's stress-time relationship at critical locations by strain gauge or finite element analysis, and then calculate stresses (amplitudes, mean, peak and vale) in statistical means, finally estimate fatigue life through proper damage accumulation criterion[1]. This method performs better in narrow-band and wide-band stochastic vibration, while it is difficult to be done in finite element analysis for describing a random vibration process requiring huge signal data which leads to heavy workload[2]. The time history of structural stress response is irregular under stochastic excitation, so it could only be described in the perspective of mathematical statistics[3]. For a stochastic excitation, root-mean-square(RMS) is a key feature related to mean energy distribution，including both static component and dynamic component of vibration amplitudes[4]. Hu[5] figured out that stress RMS can directly reflect change of structures' response amplitudes in stochastic vibration and has laws to go by.

Based upon existing research, the thesis applies stress response RMS into the time-domain method of fatigue life estimation under stochastic excitation.

The thesis focuses on stress response RMS characteristics of cantilever beam with a circular hole under stochastic excitation applied to fatigue life estimation in time-domain method. It is organized as follows：carrying out a stochastic vibration test to get stress-time history of critical locations on spaceman through strain gauges, converting the stress-time data into stress RMS-time data by mathematical statistical, making a curve to fit stress RMS-time data that could be used for calculating

stress after strain gauges broken down due to the accumulated damage in spaceman，applying rain-flow method to count all stress RMS amplitudes and corresponding frequencies by filtering low amplitude loading invalid for accumulative damage from the time history of stress RMS. Then calculating fatigue cyclic numbers of the stress amplitudes under the material's dynamic S-N relationship, calculating the damage by Miner linear damage accumulative rules until total damage satisfied fatigue criterion, the time experienced is the fatigue life of the specimen under stochastic excitation. Figure 1 shows technical route of the thesis.

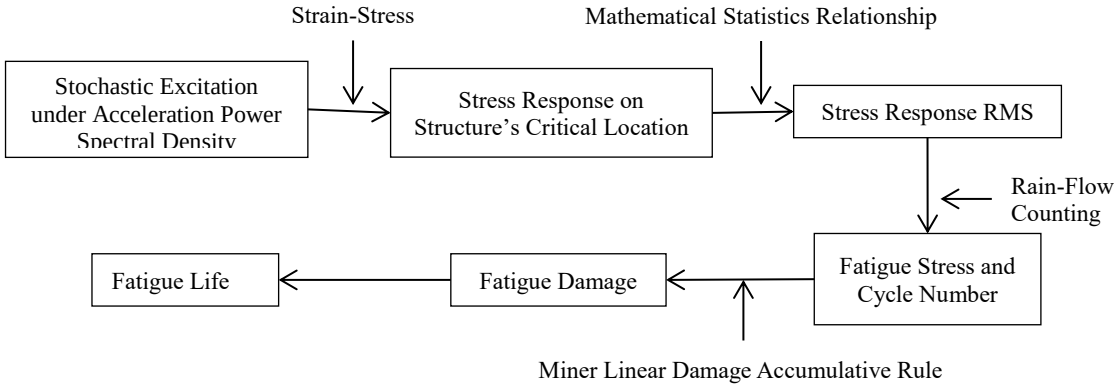


Figure 1. Technical route of the thesis

2. Experiment Research on Stochastic Vibration Fatigue

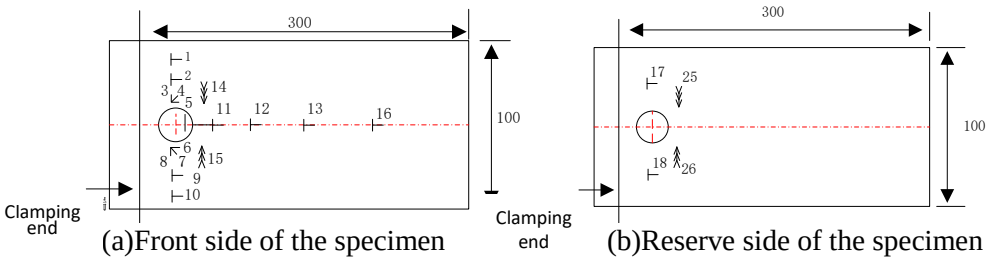
2.1. Experiment scheme

Equipment used in the test includes vibration-control system, data acquisition device and data analysis software. The specimen is Aluminium LY12, a cantilever beam with a hole. Properties are shown in Table 1.

Table 1. Mechanical Behavior of Aluminium LY12

Elastic modulus (GPa)	Shear Modulus (GPa)	Poisson's Ratio	Density (g/cm^3)	Yield Strength (MPa)	Failure Strength (MPa)
71	26	0.33	2.75	288	390

Figure 2 shows strain gauges arrangement. There are six strain gauges on one symmetrical side of circle hole edge to measure tension-compression strains，two strain gauges near the circle hole to measure torsional strains. On the symmetrical axis of specimen three strains are placed at modal peak or trough, and one strain are placed at a random location along the axis. Specimen photo is shown in figure2. Stochastic excitation accelerated PSD is shown in figure3.



(a)Front side of the specimen (b)Reserve side of the specimen

Figure 2. Strain gauges arrangement

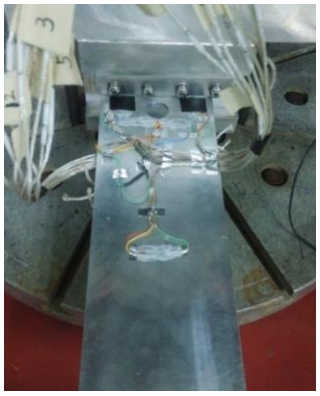


Figure 3. Specimen in experiment

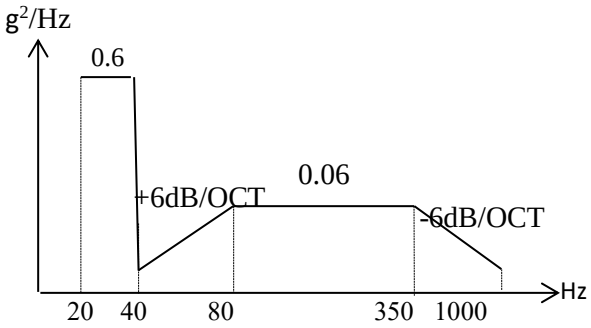


Figure 4. Stochastic excitation PSD

2.2. Vibration Fatigue Experiment Result

It is carried out the test on three specimens. During the test, vibration-control system pauses every two minutes to detect visual initiate crack. Three specimens fatigue life are 18min, 26min and 22min respectively.

3. Stress Analysis

3.1. Stress Analysis

In software MATLAB，strain is converted into stress through LY12 static stress-strain relationship after removing drifts and noise. Table 2 presents the statistical stress results of 1# specimen.

Table 2. Statistical Stress Results of 1# specimen

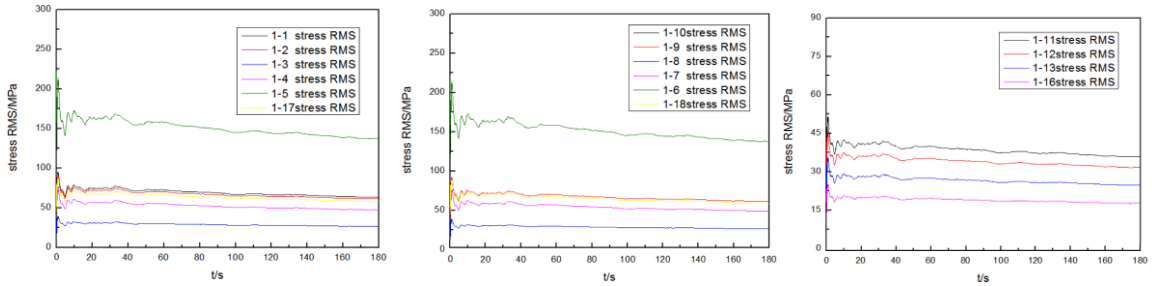
Gauge NO.	MaxStress (MPa)	MinStress (MPa)	AverStress (MPa)	StressRMS (MPa)
1	204.28	-204.66	1.67	61.91
2	200.82	-194.27	2.22	59.18
3	79.65	-80.87	-0.79	25.42
4	170.38	-164.83	1.60	47.18
5	334.47	-375.62	6.83	138.26
6	331.72	-378.00	6.57	138.57
7	174.02	-168.43	1.69	48.21
8	76.77	-78.76	-0.94	25.03
9	194.92	-197.64	1.41	59.49
10	195.52	-189.94	1.94	57.99
11	92.60	-89.16	0.40	35.74
12	82.71	-77.80	0.30	31.62
13	65.00	-62.10	0.21	24.77
16	45.37	-47.32	-0.61	17.68
17	173.75	-199.08	-12.44	57.71
18	179.95	-203.60	-12.07	58.81
19	47.42	-42.77	-0.34	17.09

It is shown that, taking cantilever beam's symmetry axis as a reference, stress levels in symmetrical position (1# and 10#, 2# and 9#, 5# and 6# etc.) are almost equal, and so do the stress levels in the same region of the specimen's front side and the corresponding reverse side(2# and 17#, 9# and 18#). The maximum stress level appears at the edge of the hole. The farther away from the hole, the lower the stress level is. Compared to former tension-compression stress, the torsional stress levels keep a relative low magnitude with a maximum value of 17MPa, which proves the specimen performs mainly bending deformation during the stochastic vibration test. About 3 percent number of stress exceeds the material yield strength (288MPa) locating at the edge of the hole (5# and 6#).

3.2. Stress RMS Analysis

Figure 5 exhibits stress RMS-time curves at different locations, which converted from stress-time curves by MATLAB. Stress RMS of all the test points show similar variation in fluctuating at the beginning, soon falling to a certain level and then declining slowly in the following time. In figure 5(a), 5# at the circle hole has the largest stress RMS, starts with fluctuation, then slowly decline in the remain time after falling to about 150MPa. The stress RMS level of 1#, 2# and 7# are almost same for 1#, 2# both on one side of the hole and the same distance from the clamping end, 17# on the reversal side of the specimen corresponding to 1#.

The stress RMS of 6#~10# and 18# on the other side of symmetry axis also follow similar variation just as 1#~5# and 17#.



(a)Point 1#~5#, 17# (b) Point 6#~10#, 18# (c) Point11#~13#, 16#

Figure 5. Stress RMS of test points on 1# specimen

In figure 5(c), the stress RMS of points (11#~13#, 16#) on the symmetry axis of the specimen also shows a similar pattern. The largest stress RMS occurs on 11# that experience fluctuation at the beginning, then slowly declines when RMS falls to about 40MPa. The stress RMS of 12#, 13# and 16# decrease slowly from 35MPa, 25MPa and 18MPa respectively after fluctuating initially. The interval between points and clamping end is correlated to the stress RMS level, the farther away from the clamping end, the lower the RMS level of stress at the points.

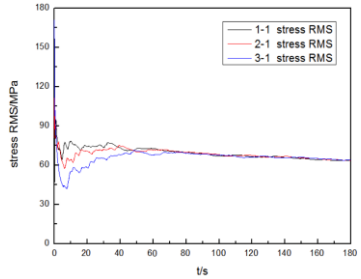


Figure 6. Stress RMS of 1#, 2#, 3# specimens

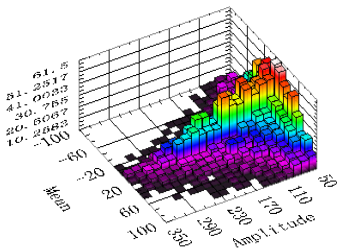


Figure 7. 0~120s stress RMS counted by rain-flow method

Figure 6 shows the stress RMS of 1#, 2# and 3# specimens. The RMS level slowly descend from 75MPa right after volatility at the beginning. The longer the time is, the closer the RMS of these specimens.

From above analysis, it could be considered that the stress RMS follows a certain rule in time history.

Fitting time history of max stress RMS at the hole in MATLAB, the results is：

$$s = 0.011t^2 - 1.584t + 157.1 \quad (1)$$

Strain gauges at the hole failed around 550 seconds under stochastic excitation，after that no useful data is acquisited. The following stress predicted by the fitted curve in equation(1).

4. Fatigue Life Estimation under Stochastic Excitation

Stress RMS processed by rain-flow method is also normal distributed around the symmetric axis-mean value, obeyed Gauss random distribution.

$$P(S_{RMS}) = \frac{1}{\sqrt{2\pi}\sigma}\exp(\frac{S_{RMS}-\mu}{2\sigma^2}),\ -\infty < S_{RMS} < +\infty \quad (2)$$

μ, σ represent the mean, variance value，respectively.

Dynamic S-N relationship[6] is expressed as follow

$$lgN = 20.2053 - 6.8432lgS \quad (3)$$

S represents stress amplitude, N is the corresponding cyclic numbers.

Accumulative damage under Miner linear damage accumulative rules could be calculated as

$$D = \int_0^{\infty} \frac{n(s)}{N(s)} ds \quad (4)$$

n(s), N(s) represent actual cyclic numbers and failure cyclic numbers respectively under stress amplitude S.

Miner linear accumulative damage rule gives a simple way to estimate fatigue life under stochastic excitation, thus used widely in engineering fatigue predicting. However，the theory does not take effect of loading sequence into consideration which has been proven to have influences on fatigue life because some high value loads generating residual stresses harmful to fatigue life[7,8]. In paragraph 3.1, it has figured out only 3percent stresses gets into the material's plastic zone, other stresses is below elastic limit, so loading sequence effect can be ignored for there exists few high value residual stresses.

Table 3. Accumulative damage of specimen

time/s	stress RMS/MPa	Accumulative Damage
0-120	157	0.0093
121-240	151	0.0082
241-360	148	0.0077
361-480	144	0.0073
481-600	142(fitted)	0.0071
601-720	140(fitted)	0.007
721-840	137(fitted)	0.0069
…	…	…
…	…	…
1021-1025		0.0052
Total Accumulative Damage		1.5024

In stochastic excitation, fatigue analysis is more accord with engineering applications when D=1.5[9].

In table 3, the contribution of the stresses recorded by the strain gauges to the accumulative damage comes up to 0.0324, following damage is calculated by the fitted curve mentioned in equation(1). When the damage sums up to 1.5, fatigue life reaches about 1025s, namely 17min.

From previous analysis, fatigue life estimated by considered stress RMS response amplitude variation is closer to test results, but there still exists some differences.

5. Conclusion

The thesis discusses how to use stress RMS to estimate fatigue life of cantilever beam with a hole. It is critical to measure stresses at dangerous location and fit a proper curve on those stress data. At present, the theoretical and experimental research on this method is still in the exploratory stage. Compared to traditional fatigue life estimation under stochastic excitation, this method requires less measured data by using fitted stress RMS curve that saves huge workload, considering more damage under various stress levels by means of rain-flow counting at the meantime, both of which make it approximate to real fatigue life. But when the stress RMS fitted curve exists large error, the accuracy of fatigue life estimation results will be cut down.

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