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Computer simulation of crossing line based geometric illusions

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Abstract. Computer modelling of visual illusions may offer useful insights into visual illusions from different perspectives. Focusing on crossing line based geometric illusions where target lines illusorily change their shapes under the influence of context elements, we show that, by taking into account the crossing angles between target and context lines and the distance between intersection points along the target lines, it is possible to simulate some typical crossing line based geometric illusions with a simple model. Compared with a recent work which requires the context lines to be differential, our model eliminates this restriction so that it is applicable to simulating many more crossing line based geometric illusions.

1. Introduction

Visual illusions exhibit a discrepancy between perceived and physical image. The best known and most studied of all illusions are the geometrical illusions. The number of illusory patterns that fall in this class is very large, and the perceptual phenomena seem to be quite diverse.

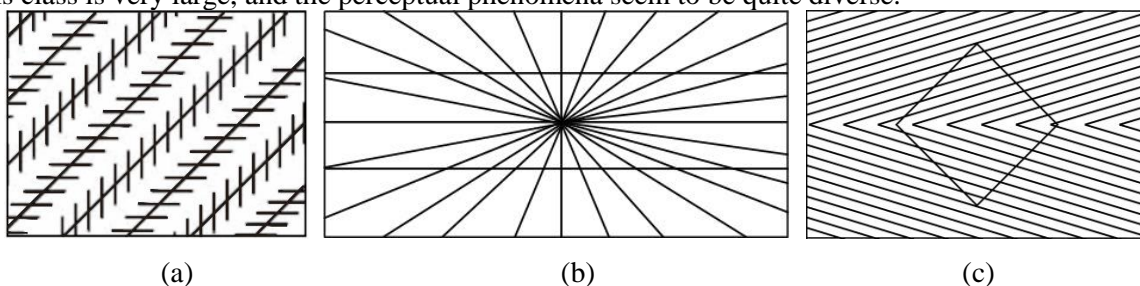


Figure 1. Examples of crossing line based geometric illusions. (a) Zollner illusion, (b) Hering illusion, (c) Orbison illusion.

In this study, we focus on crossing line based geometric illusions - a subset of geometric illusions, which are produced by the interaction between target elements which illusorily change their shapes and context elements. Figure 1 shows three examples of crossing line based geometric illusions: (a) Zollner illusion, where parallel lines intersected by a pattern of short lines appear to be unparallel; (b) Hering illusion, where two horizontal parallel lines look as if they were bow outwards; (c) Orbison illusion, where the bounding rectangle and inner square both appear distorted in the presence of the radiating lines.

In the past the study of illusions has mostly been carried out by psychologists. For a comprehensive review of several categories of illusion that have been more traditionally explored in history and in modern neuroscience please refer to [1]. Glass [2] considered perception of Poggendorff illusion as a Gaussian blurring effect in foveal vision. Ginsburg [3] showed that the low spatial frequency attenuation characteristics of the visual system did aid the formation of the illusory triangle in Kanizsa triangle.

In recent decades the study of illusions was joined by scientists in neurology, physiology, philosophy, and the computational sciences. Let us mention here a few examples that attempt to simulate illusions by computational means. Morgan and Moulden [4][5] have shown that the cafe wall illusion can be explained by the effects of processing with a difference-of-Gaussians filter. Their account leads to the prediction of the reverse tilt illusion if the width of the mortar lines is suitably increased [6]. Bulatov et al [7] applied the neurophysiological model of spatial information filtering to the Muller-Lyer and Oppel-Kundt figures, and the output patterns obtained by the model contain distortions equivalent to the illusions observed in psychophysical experiments. Fermler and Malm [8] proposed that many geometrical optical illusions are due to the statistics of visual computations. Gutasukas et al [9] conducted psychophysical measurements of illusion of the puffy circle. Focusing on crossing line based geometric illusions, article [10] explained them qualitatively as the result of diffraction.

Recently, Ehm and Wackermann [11] assumed local target-context interactions in a vector field representation of the context, and modeled the perceptual distortion of the target as the solution to a minimization problem in the calculus of variations. This model requires however the vector field to be continuous and differentiable (as one given in figure 1 (b)), thus is unable to model those illusions where the fields are un-differentiable (as those in figure 1 (a) and (c)). Our goal is to remove the restriction of context line being differential and simulate many more crossing line based geometric illusions with a simple model.

2. Relationship between the acute crossing angle and illusory rotation angle

One property associated with crossing line based geometric illusions is the angular expansion, the illusory distortion of the target lines in figure 1 always acts to enlarge the acute angles at the intersection points [12], as illustrated locally by a intersection point in figure 2 (for illustrative purpose, we just draw one context line here), where α is the acute crossing angle at an intersection point between the context and target line, $\Delta\alpha$ is its angular expansion which equals to the illusory rotation angle β .

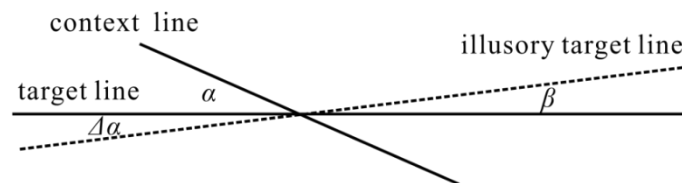


Figure 2. The acute crossing angle α and the illusory rotation angle β .

In order to explore the quantitative relationship between α and β , we designed an UI in the window of size 1024X768 pixel where Zollner illusion is drawn with four horizontal parallel target lines, as shown in figure 3, and short diagonal context lines with different orientations are added over every other target lines. The distance between two neighboring intersection points a long target lines is copied from original Zollner illusion and α is set initially to be 5 degree in relation to the target lines.

Users first view Zollner illusion defined with this setting and perceive somewhat un-parallel target lines (say, the first and second target line from the top in figure 3 are illusorily rotated a little bit clockwise and anti-clockwise respectively), and then they click the *High* button under *Rotation Angle* in the UI, the system correspondingly rotates each target line slightly in the opposite direction of perceived illusory rotation of the target line (say, the first and second target line from the top in figure 3 are rotated anti-clockwise and clockwise respectively, thus the illusory rotation angles become

correspondingly smaller). Users click the *High* button repeatedly until they feel perceived target lines become parallel.

In the cases where the target lines are over rotated, users can alternatively click the *Low* button repeatedly until they feel perceived target lines become parallel. When users perceive illusorily parallel target lines stably, they click the *Submit* button on the UI and our system takes down values of both α and β . We continue this process by increasing α with an incremental value of 5 degree each time to get corresponding β until α reaches 90 degree finally.

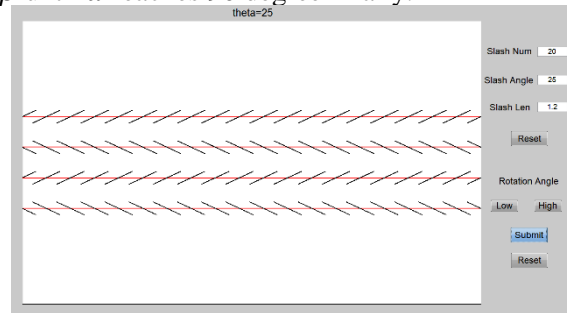


Figure 3. The experiment UI.

We asked 30 students (17 male and 13 female, ages 18 - 26) to do the experiment aforementioned, and show the averaged data by dots in figure 4, where the horizontal axis is α , and the vertical axis is β , blue lines indicate the variance of recorded β corresponding to each α value. We next fit the averaged data by a Spline, and the fitted curve has a peak at $\alpha = 26.875$ and overall it appears like being composed of two exponential curves with different slopes before and after the peak, we finally fit the averaged data with following models, as shown by the red line in figure 4:

$$\beta_1(\alpha) = \begin{cases} 0.6 \exp(-0.0067(\alpha - 26.875)^{1.94}) & (\alpha \leq 26.875) \\ 0.6 \exp(-0.0106(\alpha - 26.875)^{1.39}) & (\alpha > 26.875) \end{cases} \quad (1)$$

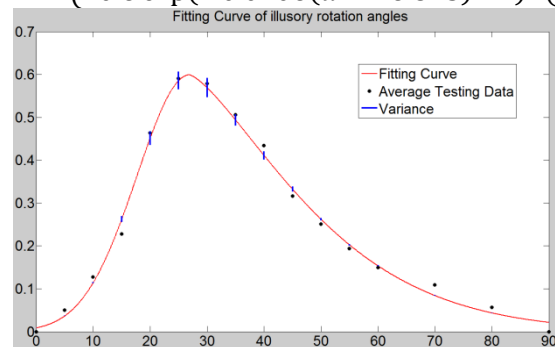


Figure 4. Fitting the data with a model.

3. Relationship between intersection point distance and illusory rotation angle

From figure 1 we notice that the distance between two consecutive intersection points on target lines in Zollner illusion remain fixed while those in Hering illusion do not. The two horizontal parallel lines appearing bow outwards in Hering illusion suggest that strength of geometric illusions on target lines depend on the distance between intersection points.

In this section we explore the relationship between intersection point distance d and β , and start from two examples of Zollner illusion shown in figure 5, where the illusion strength in (a) with a smaller d is larger than that in (b) with a bigger d , although α remains the same in both cases, showing that the strength of illusion is dependent on the intersection point distance.

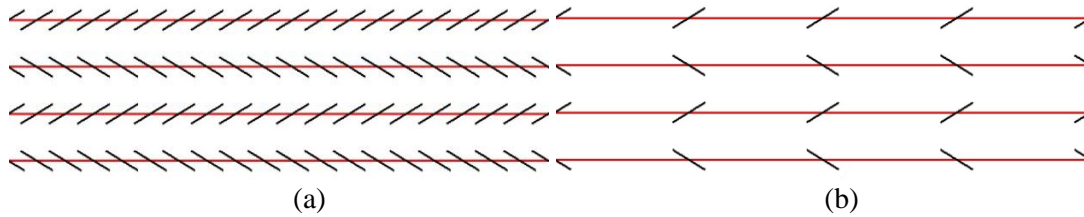
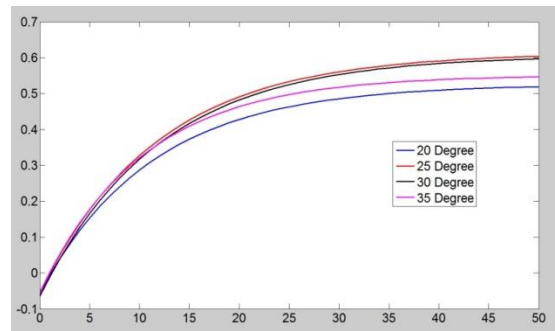


Figure 5. Zollner illusion with different number of context lines

Suppose the length of target lines in figure 5 is L , the number of context lines crossing target lines is n , we draw the left and right extreme context lines on two ends of the target line, thus the distance between two consecutive intersection points is $d = L/(n - 1)$. We first set α to be 20 degree and $n = 5$, and display Zollner illusion with this setting in our UI. Again, we ask former 30 students to do the same thing as done in the previous experiment, and take down values of α , β and n each time. We increase n with an incremental value of 5 to get corresponding β until n reaches 50. The averaged data set was fitted by an exponential model as shown by the blue line in figure 6, where the horizontal axis is n .

Figure 6. Four fitted curves corresponding to different α values.

Next, we set α to be 25, 30 and 35 degree, respectively, to see how those values of α near the peak in figure 4 affect our illusory perception while n varies. Corresponding fitted results are shown by the red, black and pink line in figure 6. We note that magnitudes of red and black lines are bigger than those of the pink and blue line, which agree with figure 4 that α values near the peak correspond to higher β values. Despite of small differences in magnitudes, all curves in figure 6 exhibit similar shapes. We average 4 curves in figure 6 and fit the averaged data with the following model:

$$\beta_2(n) = -\exp(-0.1133 * n) + 0.6606 \quad (2)$$

4. The simulation formulation

The two experiments aforementioned indicate that illusory angle β is the function of both α and n . Since $d = L/(n - 1)$ as given in the previous section, we convert $\beta_2(n)$ (in equation (2)) into $\beta_2(d)$ by substituting n with $(L/d + 1)$, thus β is the function of α and d .

We note that in Hering illusion (figure 1 (b)) the distance between two consecutive intersection points on target lines varies along the target lines, thus an internal intersection point p_i on a target line is associated with two different distance values, i.e, $d_1 = |p_i - p_{i-1}|$ and $d_2 = |p_{i+1} - p_i|$, ($i = 1, 2, 3, \dots, N_j$, where N_j is the number of intersection points on the j th target line, and $j = 1, \dots, M$, where, M is the number of target lines). We use the average of d_1 and d_2 as d_i to calculate $\beta_2(d_i)$ at p_i . Finally we formulate the following expression to simulate the illusory rotation angle $\beta(p_i)$ corresponding to p_i :

$$\beta(p_i) = m * \beta_1(\alpha_i) \beta_2(d_i) \quad (3)$$

where, m is an adjustable coefficient which controls the magnitude of the simulated illusory rotation angle, and we set $m = 0.05$ through experiments.

Note that the crossing angle α_i is restricted to the acute angle in equation (3). When a target line is chosen as the reference axis, α_i maybe over 90 degree, such as those on the left half of the top target

line in Hering illusion (figure 1 (b)). In such cases we simply let the crossing angle be $180 - \alpha_i$, correspondingly $\beta(180 - \alpha_i) = -\beta(\alpha_i)$.

Equation (3) is also valid for Zollner illusion where d_i remains fixed on target lines and Orbison illusion where d_i the same on one side of the target rectangle remains but varies from one side to another on the target rectangle.

Since $\beta(p_i)$ is the slop angle formed by the illusory target line and the context line at p_i , the corresponding slope is then $s(p_i) = \tan(\beta(p_i))$. By integrating $s(p_i)$ over the target line we can obtain the simulated illusory curve $C(p_i)$:

$$C(p_i) = m * \sum s(p_i) |p_{i+1} - p_i| \quad (i = 1, 2, 3, \dots, N_j - 1, j = 1, \dots, M) \quad (4)$$

5. Simulation results

In this section we present several simulated results of crossing line based geometric illusions, as shown in figures 7, 8, 9, 10 and 11. Since illusory changes of target lines are perceived visually rather than measured physically, we add the simulated illusory curves in red over their physical counterpart in grey on the left of each figure and put the original illusory figures on the right for comparison. As shown in figures, the simulated illusory curves look quite similar to those in our perceived illusions.

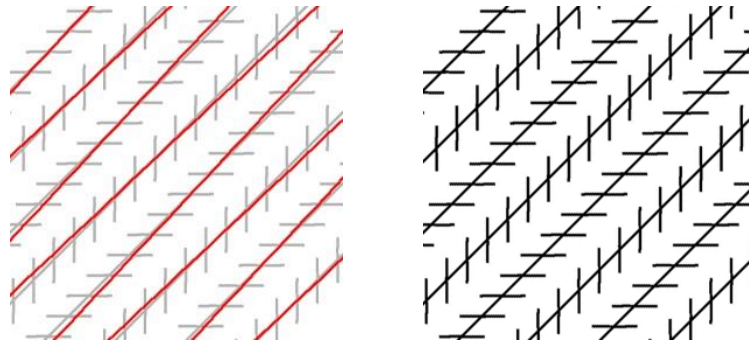


Figure 7. Zollner illusion

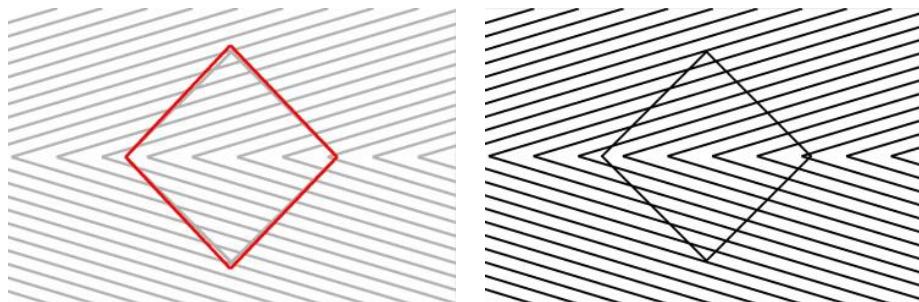


Figure 8. Orbison illusion

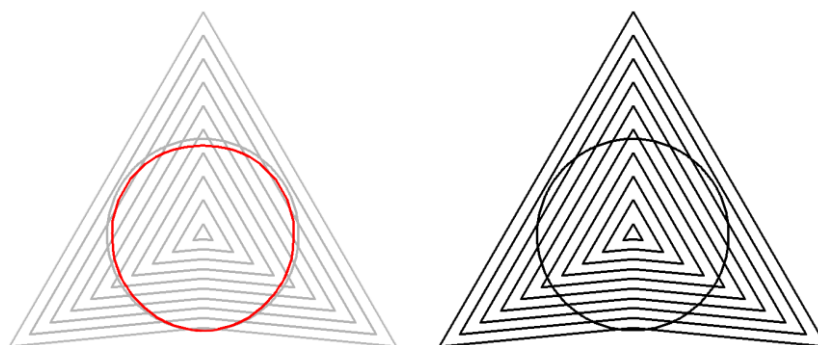


Figure 9. A variant of Orbison illusion

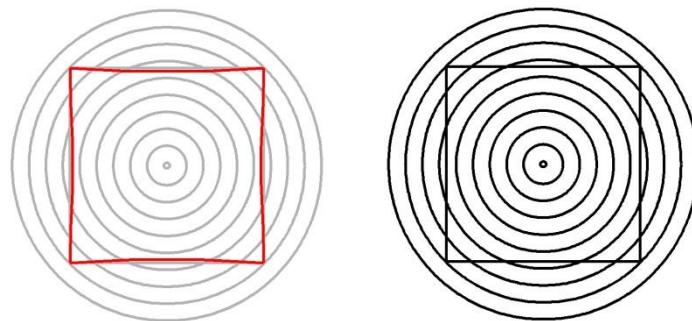


Figure 10. Another variant of Orbison illusion

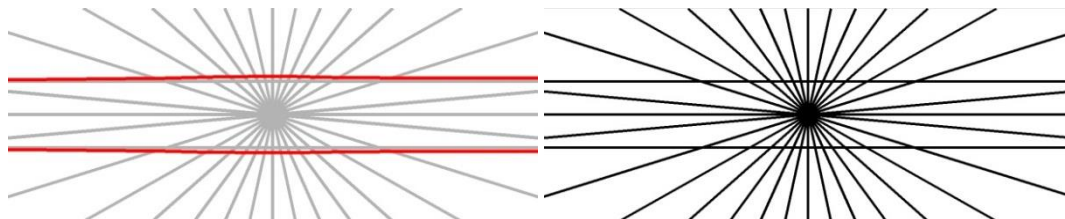


Figure 11. Hering illusion.

6. Conclusions

In this study we propose a model capable of simulating cross line based geometric illusions, based on the relationships between illusory angles and crossing angles as well as the distance between neighboring intersection points formed by context lines over target lines obtained through experiments. Our model suggests that our brain may not rely on complicated variational processing [11] but a simple, straightforward processing on visual stimuli of those illusions. We believe our finding may open up several exciting research avenues in related fields including neurobiology, cognition, etc.

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