Thermal Effect on Band Gaps of 1D Metallic Phononic Crystals

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Thermal Effect on Band Gaps of 1D Metallic Phononic Crystals

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Abstract. We have investigated thermal effect on band gaps of shear horizontal (SH) waves propagating through a layered metallic periodic structure. For a metallic material, the lattice length varies slightly with temperature while the Young's modulus is strongly thermosensitive. Firstly, the dispersive equation without thermal effect was derived by the transfer matrix method. Secondly, the thermal effect on the material coefficients was investigated. Finally, the dispersive curves at different temperatures with changing incident angles were drawn. It reveals that the band gaps get a greater impact in high-frequency regions than those in low-frequency regions. The results can provide theoretical basis for the production of phononic crystals.

1. Introduction

Due to the stop band characteristic, phononic crystal (PC) is useful in anti-vibration, noise reduction, wave guides, and so on, showed by Xiao et al [1], Ji et al [2] and Qin et al [3]. Many researches were focused on various factors that influence the band gaps of PCs. Wang et al [4] studied the stop band properties of elastic waves in three-dimensional piezoelectric phononic crystals with initial stress taking the mechanical and electrical coupling into account. Ding et al [5] investigated ways for controlling and adjusting the longitudinal wave band structures of one dimensional rod phononic crystals with magneto-strictive material by the plane wave expansion method. Guo et al [6] studied the effects of mechanically and dielectrically imperfect interfaces on dispersion relations of elastic waves in a one-dimensional piezoelectric phononic crystal.

The temperature also has evident effect on band gaps. There has been some research [7-9] on the thermal tuning of band structures. Of course, although there have been many studies about thermal tuning of band structures, the approaches are still not thoroughly investigated. In this paper, the thermal effect on the band gaps of the one-dimentional metallic phononic crystal composed of two different metal materials are consided. The dispersive equation is derived and the band gaps of metallic PCs are showed in figures 1.

2. Band Gaps of Phononic Crystals

2.1. Dispersion Relation by the Transfer Matrix Method

When the elastic waves propagate through a periodic structure band gaps will occur. The layered metallic structure will be chosen here. The PC system consists of two different materials denoting by A and B. In one unit cell, the length of phase A is a while it of phase B is b. Hence, the lattice length $d = a + b$. Let the z- axis is the poling direction and the slab is transversely isotropic in the oxy coordinates plane. The motion equation of isotropic elastic media is

$$\mu\nabla^2\mathrm{u} + (\lambda + \mu)\nabla(\nabla \cdot \mathrm{u}) = \rho\frac{\partial^2\mathrm{u}}{\partial t^2} \tag{1}$$

where λ and μ are Lamé constants, ρ is the mass density, u is the displacement vector, and u={0,0, $w(x,y)$} for obliquely propagating SH waves. The displacement $w^A(x,y)$ in the component A can be expressed as

$$w^A(x,y) = [I_n^A\exp(ik_x^A x\cos\theta_1) + R_n^A\exp(-ik_x^A x\cos\theta_1)]e^{(ik_y y - i\omega t)} \tag{2}$$

where I_n^A and R_n^A are the amplitudes of the incident and the reflected SH-waves in the n-th unit cell, respectively; k is the wave number, $k_x(=k\cos\theta_1)$ is the transverse wave number, and $k_y(=k\sin\theta_1)$ is the apparent wave number.

Then the stress $\tau_{zx}^A(=\mu^A\,\partial w^A/\partial x)$ can be expressed as

$$\tau_{zx}^A(x,y) = iM^A[I_n^A\exp(ik_x^A x\cos\theta_1) - R_n^A\exp(-ik_x^A x\cos\theta_1)]\,e^{(ik_y y - i\omega t)} \tag{3}$$

where $M^A = \mu^A k_x^A cos\theta_1$. The displacement $w^B(x,y)$ and the stress $\tau_{zx}^B(x,y)$ in the component B can be gotten in the same way as follows,

$$w^B(x,y) = [I_n^B\exp(ik_x^B x\cos\theta_2) + R_n^B\exp(-ik_x^B x\cos\theta_2)]e^{(ik_y y - i\omega t)} \tag{4}$$

$$\tau_{zx}^B(x,y) = iM^B[I_n^B\exp(ik_x^B x\cos\theta_2)) - R_n^B\exp(-ik_x^B x\cos\theta_2)]\,e^{(ik_y y - i\omega t)} \tag{5}$$

where $M^B = \mu^B k_T^B cos\theta_2$.

Supposed a state matrix

$$\mathbf{V}^j = \{w^j, \tau_{zx}^j\}\mathbf{T} \tag{6}$$

where $j = A, B$. Supposed that the interfaces between the components A and B are connected perfectly, the following relationships will be gotten,

$$\mathbf{V}_{Rt}^A = \mathbf{T}_1\mathbf{V}_{Lt}^A,\ \mathbf{V}_{Rt}^A = \mathbf{V}_{Lt}^B,\ \mathbf{V}_{Rt}^B = \mathbf{T}_2\mathbf{V}_{Lt}^B \tag{7}$$

where the subscripts Rt and Lt denote the right and left sides of the sub-layers, respectively. By equations (6) and (7) it can be derived that

$$\mathbf{T}_1 = \left[P_{ij}\right] \tag{8}$$

where $P_{11} = P_{22} = cos\alpha,\ P_{12} = cos\alpha/M^A, P_{21} = -M^A cos\alpha, \alpha = k_T^A acos\theta_1$, and

$$\mathbf{T}_2 = \left[Q_{ij}\right] \tag{9}$$

where $Q_{11} = Q_{22} = cos\beta,\ Q_{12} = cos\beta/M^B, Q_{21} = -M^B cos\beta,\ \beta = k_T^B bcos\theta_2$.

Because of the perfectly connected interfaces, we obtain the following result from equation (7)

$$\mathbf{V}_{Rt}^B = \mathbf{T}_2\mathbf{T}_1\mathbf{V}_{Lt}^A \tag{10}$$

Meanwhile, the Bloch theorem requires the displacement and stress in a periodic structure satisfy

$$\mathbf{V}_R^B = e^{i\kappa d}\mathbf{V}_L^A \tag{11}$$

Equations (10) and (11) yield the following dispersive equation

$$\left|\mathbf{T}_2\mathbf{T}_1 - e^{i\kappa d}\mathbf{I}\right| = 0 \tag{12}$$

that is,

$$\cos(\kappa d) = \cos\left(k_T^A acos\theta_1\right)\cos(k_T^B bcos\theta_2) - (F^B/F^A + F^A/F^B)/2\ldots$$
$$\times\sin\left(k_T^A acos\theta_1\right)\sin(k_T^B bcos\theta_2) \tag{13}$$

where $F^j = \rho^j c_T^j\cos\theta_i, i = 1,2,\ k_T^B\sin\theta_2 = k_T^A\sin\theta_1$, $k_T^j = \omega/c_T^j$, $c_T^j = \sqrt{\mu^j/\rho^j}$ is the shear wave velocity, $j = A, B$, therefore $\sin\theta_2 = \sin\theta_1\frac{c_T^B}{c_T^A}$.

2.2. Material Coefficients with Changing Temperature

We assume that the expansion coefficient α of hard Aluminum alloy (LY) is independent of temperature between 0 and 800 degrees centigrade, because Al alloy has good thermal resistance property. To model the effect of temperature more accurately, the change of lattice length should also be taken into account. The density ρ changes with the volume affected by the temperature. The PC system is infinite on the y-direction, and the change of the length Δd on the x-direction is very tiny, because that

$$\Delta \mathrm{d} = \alpha d_0 \Delta T. \tag{14}$$

For the metals the size of α is about 1e-6 and that of d_0 is about 1e-2. In the temperature scope of 0~800°C $\Delta \mathrm{d}/d_0 = \alpha \Delta T < 0.02$ which is too tinny to make influence on dispersive curves. So Δd can be omitted. Hence, the density ρ is supposed to be unchanged with temperature.

In a word, the effect of temperature on the wave velocity is mainly on the Young's modulus E.The Young's modulus E of Al alloy can be expressed as

$$E = E_0(1 - 25\alpha \mathrm{T}), \tag{15}$$

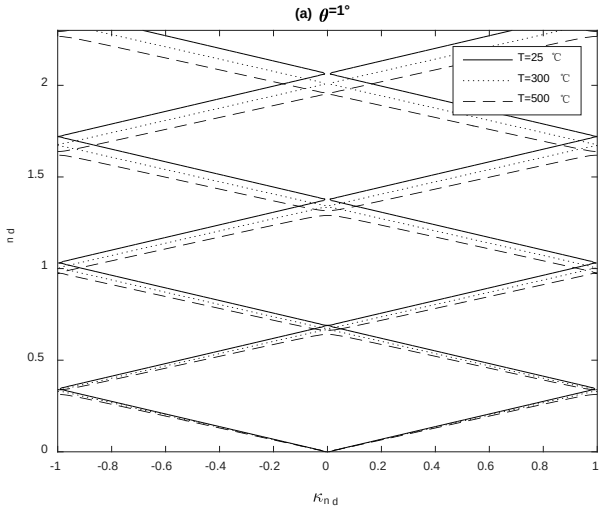
where $\alpha = 23.03 \times 10^{-6}$ is the expansion coefficient. From equation (15), it can be found that elastic modulus decreases linearly with the increasing temperature if the expansion coefficient keeps constant. The material coefficients of Al Alloy are ρ =2700 kg/m^3, E = 71GPa and σ = 0.42 at 25°C.

Supposed that the temperature hardly has effect on Poisson Ratio σ, then E_0 =72GPa by equation (16). The wave velocity changing with temperature can be derived from equation (14), c_T^A = 3168*m/s* at 25°C, 2912*m/s* at 300°C and 2685*m/s* at 500°C.

We choose the phononic crystal with the Pb as phase B to observe the thermal effect on band gaps, because the temperature hardly has effect on material coefficients of Pb. The material constants of Pb are ρ =11600 kg/m^3, E = 17GPa and σ = 0.42 at various temperatures. Therefore, the wave velocity of the material Pb c_T^B = 718*m/s* is derived.

2.3. Band Gaps with Temperature

Now the band gaps of the phononic crystals consisting of different components will be drawn. In the figures the thermal effect is the main object. The material Pb which has the excellent thermal impedance is chosen as the phase B. We have to widen the temperature scope to 25~500°C, because the influence of temperature on band gaps is too small to be observed in the range of 0~100°C. In fact the temperature around 25°C (room temperature) don't have a large enough effect to change the material properties. The length ratio of the phase A and phase B is 2:1. κ_{nd} and ω_{nd} are respectively dimensionless x and y labels where $\kappa = \kappa_{nd}\,\pi/d$ and $\omega = \omega_{nd}\,2\pi c_T^B/b_0$.



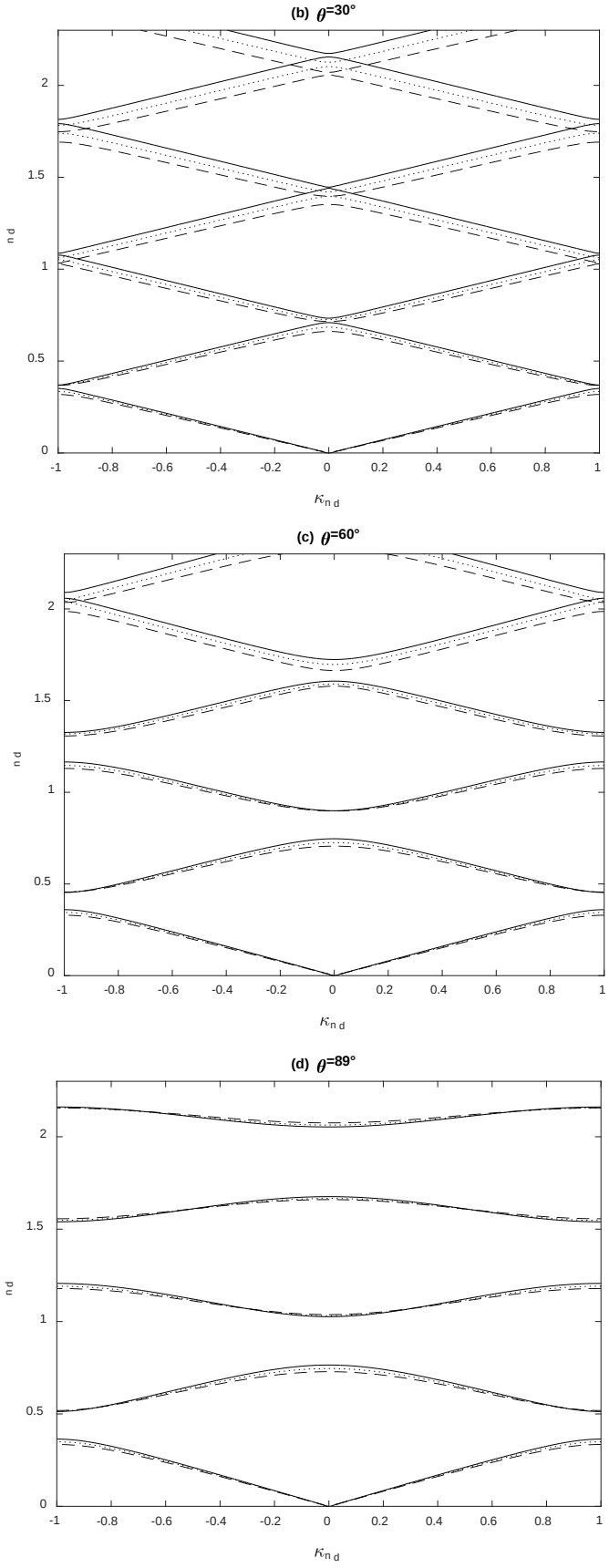


Figure 1. Dispersive curves at different incident angles with various temperatures.

Figures 1(a) to 1(d) show the band gaps for oblique incident SH-wave at four different incident angles with various temperatures. The frequency domains with dispersive curves are pass bands while those without curves are band gaps. To quantify the temperature effect on the band structures, we investigate how the dispersive curves, mid-frequencies and widths of band gaps change with the temperature.

From figure 1, it can be found that:

(1)Temperature influents the dispersive curves more slightly compared with packing ratio, stiffness coefficients, and so on. (2) As temperature increases the curvature of dispersive curves decreases, that is, the curve flattens. (3) With the rise of temperature the dispersive curves shift to lower frequency domain, all of the band gaps become wider, and meanwhile the mid-frequencies turn to lower frequency domain. (4) In the same frequency domain, band gaps in higher temperature region take a larger share than in lower one. (5) With the increasing of incident angle, the temperature effect on dispersive curves gets much smaller. (6) The temperature has more effects on the dispersive curves in higher frequency domain than that in lower frequency domain.

To model the effect of temperature more accurately, the change of lattice length should also be taken into account. But in the temperature scope of 30~300°C $\Delta \mathrm{d}/d_0 = \alpha \Delta T < 0.02$ which is too tinny to make influence on dispersive curves.

3. Conclusion

For the metallic phononic crystals, the temperature has less obvious effect on the dispersive curves compared with the other material parameters like the packing ratio, the stiffness coefficients, and so on. For all incident angles the temperature makes the dispersive curves flatter so that all of the band gaps become wider. It can be predicted that the temperature can be used to enlarge the band gaps of the temperature-sensitive materials. The change range of the Young's modulus with temperature plays a major role, so if a thermosensitive PC is expected at least one of the components must have strongly temperature-depending Young's modulus.

4. References

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