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Determination of the rational profile of fine-module ratchet teeth when cutting with a rack-type tool

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Abstract. The article analyzes the mathematical expressions describing the process of shaping the edges of the outer fine-module ratchet teeth when cutting with a rack-type tool. Mathematical dependencies are received for calculating the length of straight segments of the front and back edges of ratchet teeth, allowing one to determine their rational profile at the design stage. The initial data for the calculations are: the radius of the workpiece circumference, passing through the dedendum of the internal teeth; a gradient angle of the front and back edges of the tooth and their grinding angle; ratchet engagement module. The use of fine-module ratchet teeth with a rational profile makes it possible to increase the load capacity of the ratchet engagement.

1. Introduction

Ratchet engagement is used in various mechanisms for transferring load by normal forces: gear drives [1–4], chain drives [3–6], freewheel mechanisms [7–11], etc. [12–14]. For example, in the eccentric freewheel mechanisms of the non-friction type, fine-modular engagement with the module $m_t=0.3$ –1.0 mm is used [10].

The choice of method for cutting fine-module ratchet teeth is of great importance, since it determines the technological possibility and economic feasibility of their making.

Modern methods of making teeth are quite diverse and include more than 50 names [15–25]. One of the most productive and widely used methods of making teeth of various profiles on cylindrical surfaces is tooth cutting carried out mainly by the generating process and, in some cases, by the template method.

Modern gear-shaping machines have the ability to cut teeth on cylindrical workpieces starting from a diameter of 1.5 mm and a 0.1 mm module that is especially important in the production of fine-module teeth [15–17].

The making of external teeth by the generating process is possible in two ways – by a cutter and a rack. As was shown in [26], the use of rack-type tool for shaping external ratchet teeth makes the most sense. In this case, the largest possible length of the front edge of the tooth is generated.

When cutting fine-module ratchet teeth by the generating process, except straight segments of the profile, which are formed as an intrinsic curve to the sequential position of the contour of cutting tool teeth, there are also sections that are transition curves.

To improve the load and kinematic characteristics of the ratchet engagement, it is necessary to manufacture ratchet teeth with a rational profile, which ensures the greatest ratio of the length of



straight segments of the front and back teeth edges to their theoretically possible lengths, contact along smooth cylindrical surfaces.

2. Ratchet tooth profile determination

2.1. The front edge of the ratchet tooth profile

Figure 1 shows the geometric characteristics of fine-module ratchet engagement. In the preparation of the design scheme adopted: L_{t1} and L_{t2} – theoretical lengths of the front and back teeth edges; H_t – theoretical tooth height; l_{t1} and l_{t2} – the length of the straight segments of the front and back edges of the teeth; r – circumference radius of the workpiece, passing through the dedendum of the internal ratchet teeth; γ_1 and γ_2 – gradient angles of the front and back edges of the teeth; γ_3 – grinding angle of the front edge of the tooth; τ – pitch arc.

The extreme point of the straight segment of the front edge of the outer ratchet teeth l_{t1} is profiled by the top of the rack tooth, the coordinate y of which is equal to the theoretical height of the tooth H_t [26], i.e. $y = H_t$.

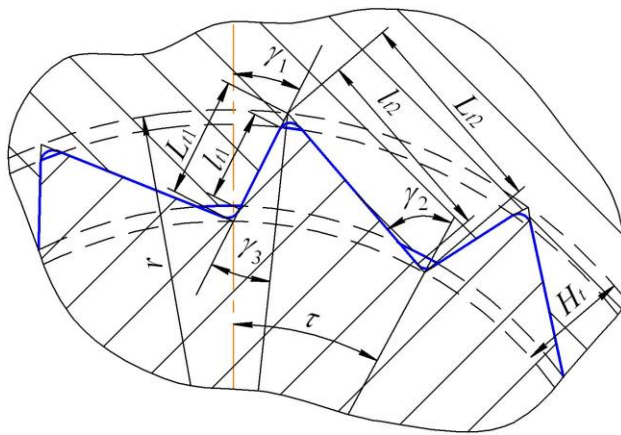


Figure 1. The location of straight segments on the front and back edges of the external fine-module ratchet tooth.

$$\begin{cases} x_{s1} = \frac{\pi r \Theta'}{180^\circ} - r \sin \Theta' + x' \sin(\gamma_3 + \Theta'); \\ y_{s1} = r [1 - \cos \Theta'] + x' \cos(\gamma_3 + \Theta'), \end{cases} \quad (1)$$

where Θ' – the angle of workpiece rotation at the time of shaping of the front edge of the ratchet tooth.

Then the coordinates of the profile in view of $x' = l_{t1}$ can be written as:

$$y_{s1} = r[1 - \cos \Theta'] + x' \cos(\gamma_3 + \Theta') \text{ or } r(1 - \cos \Theta') + l_{t1}(\cos \gamma_3 \cos \Theta' - \sin \gamma_3 \sin \Theta') = H_t. \quad (2)$$

According to [26] values $\cos \Theta'$ and $\sin \Theta'$ are defined by expressions:

$$\cos \Theta' = \frac{\cos \gamma_3 (r_1 \cos \gamma_3 - l_{t1}) + A_1 \sin \gamma_3}{r};$$

$$\sin \Theta' = \frac{A_1 \cos \gamma_3 - \sin \gamma_3 (r_1 \cos \gamma_3 - l_{t1})}{r},$$

where $A_1 = (r_1^2 \sin^2 \gamma_3 + 2r l_{t1} \cos \gamma_3 - l_{t1}^2)^{1/2}$.

$$\text{Then } r(1 - \cos \Theta') = r - r \cos^2 \gamma_3 - l_{t1} \cos \gamma_3 + A_1 \sin \gamma_3; \quad (3)$$

$$l_{t1} \cos \gamma_3 \cos \Theta' = l_{t1} \cos \gamma_3 (r \cos^2 \gamma_3 - l_{t1} \cos \gamma_3 + A_1 \sin \gamma_3) r^{-1}; \quad (4)$$

$$l_{t1} \sin \gamma_3 \sin \Theta' = l_{t1} \sin \gamma_3 (A_1 \cos \gamma_3 - r \cos \gamma_3 \sin \gamma_3 + l_{t1} \sin \gamma_3) r^{-1}. \quad (5)$$

After the conversion (1), taking into account expressions (3)–(5), we obtain:

$$r^2 \sin^2 \gamma_3 + 2rl_{t1} \cos \gamma_3 - l_{t1}^2 - A_1 r \sin \gamma_3 = rH_t. \quad (6)$$

$$\text{We introduce the substitution } z_1^2 = A_1^2 = r_1^2 \sin^2 \gamma_3 + 2r_1 l_{t1} \cos \gamma_3 - l_{t1}^2. \quad (7)$$

We rationalize expression (6) by substitution (7) and reduce it to the form of a quadratic equation $z_1^2 - z_1 r \sin \gamma_3 - rH_t = 0$.

The roots of quadratic equation determine [27]:

$$z_1 = 0.5[r \sin \gamma_3 \pm (r^2 \sin^2 \gamma_3 + 4rH_t)^{1/2}]. \quad (8)$$

We represent the expression (8) in the form of a quadratic equation:

$$l_{t1}^2 - 2r_1 l_{t1} \cos \gamma_3 + z_1^2 - r_1^2 \sin^2 \gamma_3 = 0. \quad (9)$$

The roots of the quadratic equation (9) determine [27]:

$$l_{t1} = r_1 \cos \gamma_3 \pm (r^2 \cos^2 \gamma_3 - z_1^2 + r^2 \sin^2 \gamma_3)^{1/2}. \quad (10)$$

Because of the expression (8) and taking the height of the ratchet tooth equal to the module $H_t = m_t$, the formula (10) for determining the length of the straight segment of the front edge of the outer fine-module ratchet teeth when cutting with a rack-type tool can be represented as:

$$l_{t1} = r \cos \gamma_3 \pm \sqrt{0.5[r^2(2 - \sin^2 \gamma_3) - 2r_1 m_t - r \sin \gamma_3(r^2 \sin^2 \gamma_3 + 4rm_t)^{1/2}]}. \quad (11)$$

The sign before the root in expression (11) must be taken based on the calculation results.

2.2. The back edge of the ratchet tooth profile

The task of determining the length of the straight segment of back edge l_{t2} is solved by analogy with the previous one.

The coordinates of the profile of the back edge of teeth of rack-type tool are determined taking into account $y' = l_{t1}$ from the system of mathematical expressions [26]:

$$\begin{cases} x_{s2} = \frac{\pi \Theta''}{180^\circ} - r \sin(\tau - \Theta'') - y' \sin(\gamma_2 + \tau - \Theta''); \\ y_{s2} = r(1 - \cos \Theta'') + y' \cos(\gamma_2 + \tau - \Theta''), \end{cases} \quad (12)$$

where Θ'' – the angle of workpiece rotation at the time of shaping of the back edge of the ratchet tooth.

After solving the system (12), we obtain:

$$y_{s2} = r[1 - \cos(\tau - \Theta'')] + y' \cos(\psi - \Theta'') = H_t \text{ or}$$

$$r - r(\cos \tau \cos \Theta'' + \sin \tau \sin \Theta'') + l_{t2}(\cos \psi \cos \Theta'' + \sin \psi \sin \Theta'') = H_t, \quad (13)$$

where ψ – complementary angle.

According to [26], values $\cos \Theta''$ and $\sin \Theta''$ are defined by expressions:

$$\cos \Theta'' = \frac{r \cos \tau - l_{t2} \cos(\gamma_2 + \tau) - \sin(\gamma_2 + \tau)(r \sin \gamma_2 - A_2)}{r};$$

$$\sin \Theta'' = \frac{r \sin \tau - l_{t2} \sin(\gamma_2 + \tau) + \cos(\gamma_2 + \tau)(r \sin \gamma_2 - A_2)}{r},$$

where $A_2 = (r^2 \sin^2 \gamma_2 + 2rl_{t2} \cos \gamma_2 - l_{t2}^2)^{1/2}$.

$$\text{Then } \cos \tau \cos \Theta'' = r^{-1}[r \cos^2 \tau - l_{t2} \cos \tau \cos(\gamma_2 + \tau) - \cos \tau \cdot \sin(\gamma_2 + \tau)(r \sin \gamma_2 - A_2)]; \quad (14)$$

$$\begin{aligned} \sin \tau \sin \Theta'' &= r^{-1}[r \sin^2 \tau - l_{t2} \sin \tau \sin(\gamma_2 + \tau) - \sin \tau \cos(\gamma_2 + \tau) \\ &\quad - \sin \tau \cos(\gamma_2 + \tau)(r \sin \gamma_2 - A_2)] \end{aligned} \quad (15)$$

$$\cos \psi_4 \cos \Theta'' = r^{-1}[r \cos \tau \cos \psi - l_{t2} \cos \psi_4 \cos(\gamma_2 + \tau) - \cos \psi \sin(\gamma_2 + \tau)(r \sin \gamma_2 - A_2)]; \quad (16)$$

$$\sin \psi_4 \sin \Theta'' = r^{-1}[r \sin \tau \sin \psi - l_{t2} \sin \psi_4 \sin(\gamma_2 + \tau) + \sin \psi \cos(\gamma_2 + \tau)(r \sin \gamma_2 - A_2)]. \quad (17)$$

Taking into account the expressions (14)–(17), we can write:

$$\cos \tau \cos \Theta'' + \sin \tau \sin \Theta'' = r^{-1}[r - l_{t2} \cos \gamma_2 - \sin \gamma_2 (r \sin \gamma_2 - A_2)], \quad (18)$$

and

$$\begin{aligned} \cos \psi_4 \cos \Theta'' + \sin \psi_4 \sin \Theta'' &= r^{-1}[r \cos(\tau - \psi) - l_{t2} \cos(\gamma_2 + \tau - \psi) \\ &\quad - \sin(\gamma_2 + \tau - \psi)(r \sin \gamma_2 - A_2)] \end{aligned} \quad (19)$$

In view of $\psi = \gamma_2 + \tau$ the expression (19) is written:

$$\cos \psi \cos \Theta'' + \sin \psi \sin \Theta'' = r^{-1}[r \cos \gamma_2 - l_{t2}]. \quad (20)$$

After the conversion (20), taking into account expressions (18) and (19), we obtain:

$$r^2 \sin^2 \gamma_2 + 2rl_{t2} \cos \gamma_2 - l_{t2}^2 - rA_2 \sin \gamma_2 = rH_t. \quad (21)$$

$$\text{We introduce the substitution } z_2^2 = A_2^2 = r^2 \sin^2 \gamma_2 + 2rl_{t2} \cos \gamma_2 - l_{t2}^2. \quad (22)$$

We rationalize expression (21) by substituting (22) and reduce it to the form of a quadratic equation $z_2^2 - z_2 r \sin \gamma_2 - rH_t = 0$.

The roots of quadratic equation determine [27]:

$$z_2 = 0.5[r \sin \gamma_2 \pm (r^2 \sin^2 \gamma_2 + 4rH_t)^{1/2}]. \quad (23)$$

We represent the expression (22) in the form of a quadratic equation:

$$l_{t2}^2 - 2rl_{t2} \cos \gamma_2 + z_2^2 - r^2 \sin^2 \gamma_2 = 0. \quad (24)$$

The roots of the quadratic equation (24) determine [27]:

$$l_{t2} = r \cos \gamma_2 \pm (r^2 \cos^2 \gamma_2 - z_2^2 + r^2 \sin^2 \gamma_2)^{1/2}. \quad (25)$$

Because of the expression (23) and $H_t = m_t$, the formula (25) for determining the length of the straight segment of the back edge of the outer fine-module ratchet teeth when cutting with a rack-type tool can be represented as:

$$l_{t2} = r \cos \gamma_2 \pm \sqrt{0.5[r^2(2 - \sin^2 \gamma_2) - 2r_1 m_t - r \sin \gamma_2(r^2 \sin^2 \gamma_2 + 4r m_t)^{1/2}]} \quad (26)$$

The sign before the root in expression (26) must be taken based on the calculation results.

3. Calculation results and discussion

The influence of module m_t and gradient angle γ_1 on length of straight segment of front l_{t1} and back l_{t2} edges of the ratchet teeth is shown in fig. 2 and 3. Initial data for the calculations are: $r = 30$ mm; $m_t = 0.2 - 0.6$ mm; $\gamma_1 = 0 - 30^\circ$. Calculations are made by formulas (11) and (26).

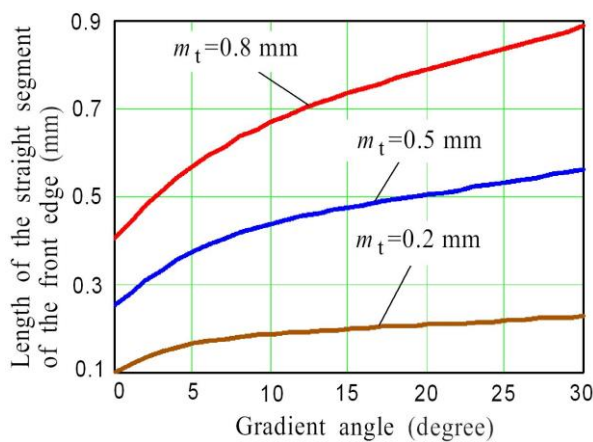


Figure 2. The influence of the gradient angle and module on length of straight segment of front edge of the ratchet tooth.

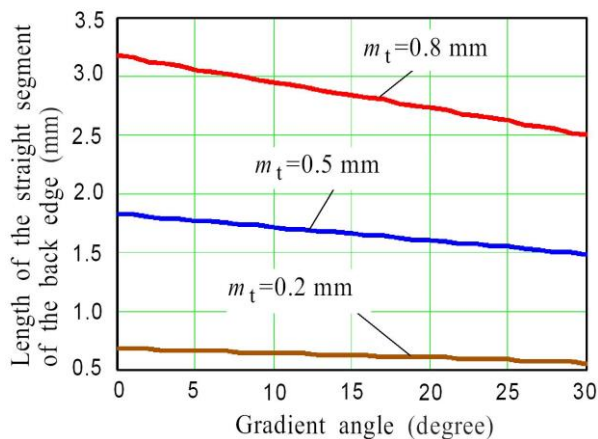


Figure 3. The influence of the gradient angle and module on length of straight segment of back edge of the ratchet tooth.

The calculation results allow us to draw the following conclusions.

The module m_t 3.0-time increase leads to a linear growth (by 2.8-3.0 times) of length of the straight segment of the front l_{t1} and back l_{t2} edges of ratchet teeth, respectively, 2.8-3.0 and 3.0-3.2 times.

Gradient angle change γ_1 in the range from 0 to 30° leads to a nonlinear growth (2.23-2.28 times) of length of the straight segment of front edge and a linear decrease (approximately 1.22 times) of the straight segment of the front edge.

4. Conclusion

The results show that at the design stage you can take the length of the straight segment of the front edge of the ratchet tooth in the range of $l_{t1} = (1.25 - 2.00)m_t$, and the back edge – $l_{t2} = (2.9 - 3.5)m_t$. With that there is the ratio $l_{t2} = (1.75 - 2.25)l_{t1}$.

It should be borne in mind that despite the shorter length of the front edge of the ratchet teeth, it is the one that has the greatest impact on the load capacity of the engagement.

Dependencies (11) and (26) allow us to quantify the possibility of obtaining a rational profile of external fine-module ratchet teeth when cutting with a rack-type tool.

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