

PAPER • OPEN ACCESS

## The mathematical description of the process of material milling in the zone of intersecting flows in a centrifugal counter-flow mill

To cite this article: I A Semikopenko *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **560** 012120

View the [article online](#) for updates and enhancements.



**IOP | ebooks™**

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

# The mathematical description of the process of material milling in the zone of intersecting flows in a centrifugal counter-flow mill

I A Semikopenko, D V Vavilov and S V Vyalych

Belgorod State Technological University named after V.G. Shoukhov, 46, Kostukov Street, Belgorod, 308012, Russia

E-mail: Semickopenko.i@yandex.ru

**Abstract.** This article presents the analytical expression for the determination of the degree of milling of particles in a centrifugal counter-flow mill with a selective effect on the material to be milled. The degree of milling is one of the most important factors characterizing the efficiency of particle collisions in the opposite intersecting flows in the tangential sleeve. The projection  $w_x$  of the velocity of interaction of particles with an oblique impact is determined on the basis of the value of the tangential stresses in the zone of opposing intersecting flows. In this case, the density of counter-intersecting flows should be such that the collision of particles occurs in the zone of milling. The structural design of the mill rotors provides separation of the particle fluxes in the tangential sleeve depending on their size. This design of the rotors improves the efficiency of grinding and fineness of the finished product. A design diagram is given in order to describe the process of milling of a material in the zone of opposing intersecting flows. As a result of theoretical studies, the analytical expression was obtained that allows determining the degree of milling of material particles in the zone of tangential collisions depending on the design and technological parameters of a centrifugal counter-flow mill.

## 1. Introduction

The degree of milling is one of the most important indicators characterizing the efficiency of particle collisions in the tangential sleeve of a centrifugal counter-flow mill. The density of counter-intersecting flows should be such that the collision of particles occurs in the zone of milling. When particles collide in opposite flows, the uniformity of the grain composition of the interacting particles is of great importance. The frontal collision of the largest particles and the collision of small particles in intersecting flows are considered to be the most effective ones [1].

In order to solve the problem of providing a selective effect on the particles of the comminuted material, depending on their size, a new design of a centrifugal counter-flow mill was developed (Figure 1).

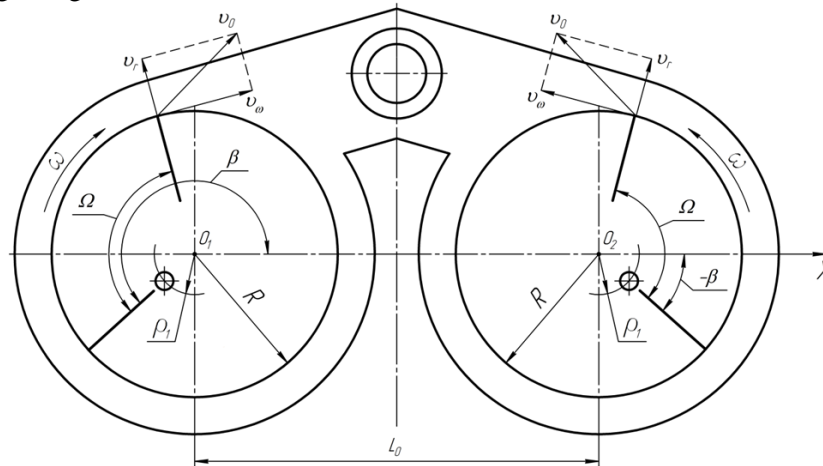
## 2. Materials and methods

The proposed design of a centrifugal counter-flow mill has features that increase the efficiency of the milling process as compared to other centrifugal counter-flow mills which are used in the process of milling. The milling chamber contains a tangential sleeve with slopes for colliding small particles in intersecting dispersed flows. The installation contains rotors 1 with curved blades 2 and straight blades 3, loading p sleeves 4, discharge sleeve 5 evenly spaced from the axes of rotation of the rotors 1. The



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

rotors 1 are rotated in opposite directions. The location of the boot sleeves 4 in the plan ensures the coaxial alignment of the counter-flows of large particles directed towards each other using curvilinear blades 2 and the angular mutual arrangement of the counter-flows of small particles directed towards each other using straight-blades 3.



**Figure 1.** The design scheme for the description of the process of milling material in the zone of opposing intersecting flows.

### 3. Results and discussion

The installation works as follows. The source material through the boot sleeves 4 evenly (for example, using vibro-trays) is fed into the milling chamber, where large particles of the material are captured by curvilinear blades 2 of greater height, bypassing rectilinear blades 3, and small particles by rectilinear blades 3 of smaller height, move along the surface of these blades and is thrown into the tangential sleeve. Since the length of rectilinear blades 3 is less than the length of curved blades 2, small particles pass straight sections of blades 3 faster than large particles pass curvilinear sections of blades 2, detach from rectilinear blades 3 before large particles. The coordinates of the loading sleeve 4 in the plan are common to the whole material. Therefore, small particles come off at a certain angle to the axis passing through the centers of rotation of the rotors 1.

The design of each loading sleeve and its location are designed to ensure the frontal collision of large particles and the collision of small particles in intersecting flows. The crushed material is carried by the air stream into the discharge sleeve 5 and is separated from the air stream in the cyclone filter (it is not shown in the figure).

Due to the separation of the flow of the crushed material by size before its impact in the tangential sleeve and the selective impact on the material, depending on its size, the efficiency of milling in a centrifugal counter-flow mill increases. Let us determine the degree of milling particles of the material in the zone of intersecting flows.

The change in the velocity  $w$  of a material particle in the zone of opposing intersecting flows in a centrifugal counter-flow mill can be described on the basis of the second law of dynamics:

$$m \frac{dw}{dt} = F, \quad (1)$$

where  $m$  – the mass of material particle equal to:

$$m = \frac{\pi d_H^3}{6} \rho_0, \quad (2)$$

here  $\rho_0$  is the density of material particle;  $F$  – action force on material particle in the zone of intersecting flows, which we express through the value of the resulting tangential stresses  $\sigma$ :

$$\sigma = \frac{F}{S_0}, \quad (3)$$

where  $S_0$  –the contact area of material particles at oblique impact.

Taking into account (2) and (3) a formula (1) may have the following form:

$$\frac{\pi d_H^3}{6} \rho_0 \frac{dw}{dx} \frac{dx}{dt} = \sigma S_0. \quad (4)$$

On the basis of the design scheme presented in Figure 1, we find that

$$x = L + 2R_p \cos \varphi. \quad (5)$$

According to the result of the work [1], in the zone of counter-intersecting flows, the value of the action of tangent stresses can be determined on the basis of the following expression:

$$\sigma = \mu \frac{w_x}{x}, \quad (6)$$

where  $\mu$  –The coefficient of pseudo-viscosity milling, the value of which is equal to 2618 Pa·s

Let us calculate the projection  $w_x$  of the velocity of interaction with an oblique impact. According to the design scheme presented in Figure 1, we find:

$$w_x = w_0 \cos(\varphi - \varphi_0) + w_0 \cos(\varphi + \varphi_0) = 2w_0 \cos \varphi_0 \cos \varphi, \quad (7)$$

here  $w_0$  –the total velocity of a material particle with a radial flat blade, the value of which is equal to:

$$w_0 = \sqrt{w_r^2 + w_\omega^2}. \quad (8)$$

According to the work [2] we have:

$$w_r = \frac{\omega \rho_1}{2f}, \quad (9)$$

and

$$w_\omega = \omega R_p. \quad (10)$$

Taking into account (9) and (10) the expression (8) has the following form:

$$w_0 = \omega R_p \sqrt{1 + \frac{1}{4f^2} \left( \frac{\rho_1}{R_p} \right)^2}. \quad (11)$$

The substitution of (5), (7) in (6) taking into account, that

$$\frac{dx}{dt} = w$$

Leads to the following expression:

$$\sigma = 2w_0 \cos \varphi_0 \cdot \mu \frac{\cos \varphi}{L + 2R_p \cos \varphi}. \quad (12)$$

After simple mathematical transformations of expressions (5) and (12), differential equation (4) can acquire the following form:

$$\frac{\pi d_H^3}{6} \rho_0 w dw = -4w_0 S_0 \cos \varphi_0 \mu \frac{\cos \varphi \cdot \sin \varphi}{\frac{L}{R_p} + 2 \cos \varphi} d\varphi, \quad (13)$$

Where according to the design scheme in figure 1, the value is the following:

$$\cos \varphi_0 = \frac{w_r}{w_0}. \quad (14)$$

The time of movement of material particle along a radially located blade with a length  $l$  on the basis of (9) is determined by the following relationship:

$$\tau_1 = \frac{l - \rho_1}{\omega \rho_1} 2f = \frac{2f}{\omega} \left( \frac{l}{\rho_1} - 1 \right). \quad (15)$$

During the time interval (15), the radially located blade with the material particle rotates through an angle  $\Omega$  from the moment of contact with the particle [4] (the “x” symbol in Figure 1).

The value of this angle is equal to:

$$\Omega = \omega \cdot \tau_1 = 2f \left( \frac{l}{\rho_1} - 1 \right). \quad (16)$$

The generation of differential equation (13) describes the change in the velocity  $w$  of the interaction of material particle in the zone of oblique collisions.

Let us assume that when the angle of rotation of the radially located rotor blade changes from  $\Omega$  value to  $\pi$ , the speed of the material particle as a result of oblique collisions will change from  $w_0$  to  $u_0$ . The value of  $u_0$  is the speed of the air flow in the area of oblique collisions. The value of the velocity of the air flow is equal to:

$$u_0 = \sqrt{u_p^2 + \omega^2 R_p^2}, \quad (17)$$

Based on the result of the work [3] we have:

$$u_p = \omega R_p \sqrt{\frac{2h}{R_p} - \left(\frac{h}{R_p}\right)^2}. \quad (18)$$

The substitution of (18) in (17) leads to the following result:

$$u_0 = \omega R_p \sqrt{1 + \frac{2h}{R_p} - \left(\frac{h}{R_p}\right)^2}. \quad (19)$$

The integration of equation (13) leads to the following expression:

$$\frac{\pi d_h^3}{12} \rho_0 (u_0^2 - w_0^2) = -4w_0 \cos \varphi_0 \cdot S_0 \cdot \mu \cdot J(\Omega), \quad (20)$$

Where the following notation is introduced:

$$J(\Omega) = \int_{\Omega}^{\pi} \frac{\cos \varphi \cdot \sin \varphi d\varphi}{a + 2\cos \varphi}, \quad (21)$$

here

$$a = \frac{L}{R_p}. \quad (22)$$

The calculation of the integral (21) leads to the following result:

$$\begin{aligned} J(\Omega) &= -\frac{1}{2} \int_{\Omega}^{\pi} \frac{2\cos \varphi d(\cos \varphi)}{a + 2\cos \varphi} = -\frac{1}{2} \int_{\Omega}^{\pi} \frac{a + 2\cos \varphi - a}{a + 2\cos \varphi} d(\cos \varphi) = \\ &= -\frac{1}{2} \int_{\Omega}^{\pi} d(\cos \varphi) + \frac{a}{4} \int_{\Omega}^{\pi} \frac{d(a + 2\cos \varphi)}{a + 2\cos \varphi} = \\ &= -\frac{1}{2} (-1 - \cos \Omega) + \frac{a}{4} \ln(a + 2\cos \varphi) \Big|_{\Omega}^{\pi} = -\frac{1}{2} (-1 - \cos \Omega) + \frac{a}{4} \ln \left| \frac{a - 2}{a + 2\cos \Omega} \right| = \\ &= \frac{1 + \cos \Omega}{2} + \frac{a}{4} \ln \frac{a - 2}{a + 2\cos \Omega}. \end{aligned} \quad (23)$$

According to the relation (20), it is possible to find the value of the contact area of the interacting particles:

$$S_0 = \frac{\pi d_h^3 \rho_0 (w_0^2 - u_0^2)}{\left[ 12(1 + \cos \Omega) + 6 \frac{L}{R_p} \ln \left| \frac{\frac{L}{R_p} - 2}{\frac{L}{R_p} + 2\cos \Omega} \right| \right] \cdot \mu \cdot w_r}. \quad (24)$$

Let us suppose that in the zone of oblique collisions particles of a spherical shape with a diameter  $d_k$  and a surface area  $S_0$  are obtained. Therefore, based on the above mentioned aspects, the following expression can be made:

$$\pi d_k^2 = \frac{\pi d_h^3 \rho_0 (w_0^2 - u_0^2)}{6\mu w_r \left[ 2(1 + \cos \Omega) + \frac{L}{R_p} \ln \left| \frac{\frac{L}{R_p} - 2}{\frac{L}{R_p} + 2\cos \Omega} \right| \right]}. \quad (25)$$

On the basis of (25) it is possible to formulate the following expression [5]:

$$d_k = \xi_0 d_h, \quad (26)$$

The following dimensionless value is introduced here:

$$\xi_0 = \sqrt{\frac{d_n \cdot \rho_0 (w_0^2 - u_0^2)}{6\mu \cdot w_r (2(1 + \cos \Omega) + \frac{L}{R_p} \ln \left| \frac{\frac{L}{R_p} - 2}{\frac{L}{R_p} + 2 \cos \Omega} \right|)}}. \quad (27)$$

Thus, the obtained expression (27) determines the degree of milling of the material particles in the zone of tangential collisions of the milling chamber when the design and technological parameters of the centrifugal counter-flow mill change.

#### 4. Conclusion

In this article the analytical expression is achieved as a result of mathematical transformations, which allows determining the degree of milling of material particle in the zone of tangential interactions in the milling chamber of a centrifugal counter-flow mill associated with the installation of upgraded rotors. The rotors with blades of rectilinear and curvilinear shape make it possible to provide a selective effect on the material depending on the size of the particles to be milled. During the course of the determination of the degree of particle size reduction in the zone of tangential interactions, the coordinates of the particle detachment from the working surface of the rectilinear blade and the trajectory of particle motion in intersecting flows are taken into account.

The obtained mathematical formulas make it possible to associate the constructive, technological and energy parameters of the centrifugal counter-flow mill with a selective effect on the material to be milled.

#### Acknowledgement

The work is realized in the framework of the Program of flagship university development on the base of the Belgorod State Technological University named after V.G. Shukhov, using equipment of High Technology Center at BSTU named after V.G. Shukhov

#### References

- [1] Danilov R G 1997 The mechanism of fine grinding in rotary mills with gear-like engagement *Construction and Road Machines* 32–31
- [2] Voronov V P, Semikopenko I A, Penzev P P 2008 Theoretical studies of the speed of movement of particles of a material along the surface of a percussion element of a disintegrator-type mill *Izvestiya VUZ. Building* 11–12 93–96
- [3] Klochkov N V, Blinichev V N, Bobkov S P, Piskunov A V 1982 Method of calculating air in a centrifugal impact mill *News of Universities. Chemistry and Chemical Technology* 2 230–232
- [4] Semikopenko I A, Voronov V P, Smirnov DV 2015 Conditions of transition of material particles from the grinding chamber to the disintegrator return pipe *Bulletin of BSTU. V.G. Shukhov* 2 99–100
- [5] Bogdanov V S, Eltsov M Y, Shirokova L Y, Khakhalev P A 2017 Engineering design of mechanical equipment for the production of cement on basis of configurators. *ZKG: ZEMENT-KALK-GIPS INTERNATIONAL* 70(5) 64-66