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# Analysis of transient and steady-state processes in multi-pulse rectifiers by the finite-difference method

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**Abstract.** Rectifiers are featured by periodic processes depending on the source frequency and rectification circuit. The output current of each diode is discrete even if the load current is continuous. The relationship between the current values at the end point of the periodical working cycle of the rectifier is determined by the finite-difference method. The difference equations describing the working cycle contain constant coefficients depending on the rectification circuit type and load parameters. The analytic dependence of the current on the lattice function argument is obtained. The dependences of the current on the lattice function argument are stated for the eight-pulse rectifier with the active inductive load. The load current envelope reflects the type of the transient process and makes it possible to compare rectification circuits and to analyze steady-state processes.

## 1. Introduction

The analysis of processes in multi-pulse rectifiers is very useful for theoretical and practical tasks. The characteristic feature of a rectifier is the periodicity of the processes depending on the source frequency and rectification circuit type [1-4]. The output current of each diode is discrete even if the load current is continuous. The analytical relationship between the current values at the end point of the periodical working cycle of the rectifier can be determined by the differential equations. The numerical value of the output current can be obtained either by serial transition from one period to another or by difference equations with constant coefficients which depend on the rectification circuit type and load parameters. The problem is solved simpler when the diodes are switched at the constant frequency of a power source.

## 2. Theory

The differential equation for the  $n$ -th half-period of the rectified voltage (Figure 1) is:

$$R_d i + L_d \frac{di}{dt} = U_k, \quad (1)$$

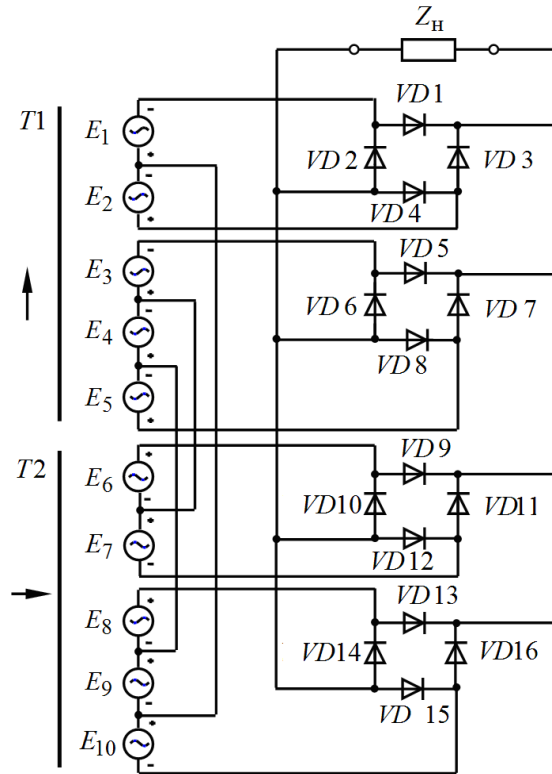
where  $U_k$  is the output voltage of the bridge at the interval  $t = \frac{2\pi}{m\omega}$ ,  $R_d, L_d$  are the inductance and load resistance,  $i$  is the instantaneous current.



The moment when the diodes are switched is taken as an origin. The component of the output voltage produced by the  $k$ -th source for the  $n$ -th half-period is [5, 6]:

$$U_k = (-1)^n U_m \sin \left[ \omega t + \frac{3\pi}{2} + \frac{\pi}{m}(1-2k) \right],$$

where  $U_m$  is the amplitude of phase voltage for the  $k$ -th power supply ( $k=1, 2, \dots$ ).



**Figure 1.** The eight-pulse rectification circuit

The derivation of the difference equation requires the solution of (1). This solution expresses the load current  $i[n+1]$  at the end of the  $n$ -th state through the current  $i[n]$  at the beginning of the  $n$ -th state for an inter-commutation interval [7, 8].

Multi-pulse systems ( $m > 2$ ) based on the bridge single-phase rectifying circuits have the number of rectified voltage half-periods which do not correspond to the unspecified interval.

As the current is continuous, its value at the end of the  $n$ -th state is equal to the initial value of the current at the  $n+1$  state. The current values at the time intervals  $t = \frac{2\pi}{m\omega}$  depend on switching moments.

When the continuous function is transformed to the lattice function for the time points  $t = \frac{2\pi}{m}n$ , it is easy to show that the following identical equation is realized for any unspecified  $n$ -th state at the interval  $0 \leq \omega t \leq 2\pi$ :

$$(-1)^n U_m \sin \left[ \omega t + \frac{3\pi}{2} + \frac{\pi}{m}(1-2k) \right] = U_m \sin \left[ \frac{2\pi}{m}n + \frac{\pi}{2} - \frac{\pi}{m} \right]. \quad (2)$$

If a rectification circuit is  $m$ -pulse, the differential equation describing the state of the lattice function for the rectified current at the equal time intervals of the  $n$ -th state follows from (1) and (2):

$$R_d i + L_d \frac{di}{dt} = U_m \sin \left( \frac{2\pi}{m} n + \frac{\pi(m+2-4k)}{2m} \right). \quad (3)$$

The equation (1) allows one to determine the current at the fixed time interval  $t = \frac{2\pi n}{m\omega}$  for the integer number  $n$ .

The current at the steady state mode ( $n \rightarrow \infty$ ) resulting from the operation of phase voltage sources would only reproduce its form during the time interval  $t = \frac{2\pi}{m\omega}$ .

The general solution of (3) can be represented as follows:

$$i = \frac{U_m}{\sqrt{R_d^2 + (\omega L_d)^2}} \sin \left( \frac{2\pi}{m} n + \frac{\pi(m+2-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right) + A e^{-\frac{R_d}{L_d} \frac{2\pi}{m\omega} n}. \quad (4)$$

The integration constant  $A$  in (4) can be determined from the initial conditions at the beginning of the  $n$ -th state:

$$i[n] = \frac{U_m}{\sqrt{R_d^2 + (\omega L_d)^2}} \sin \left( \frac{\pi(m+2-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right) + A. \quad (5)$$

In accordance with (5) the integration constant is determined as follows:

$$A = i[n] - \frac{U_m}{\sqrt{R_d^2 + (\omega L_d)^2}} \sin \left( \frac{\pi(m+2-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right). \quad (6)$$

The current value at the end of the  $n$ -th state at the  $t = \frac{2\pi}{m\omega}$  is:

$$i[n+1] = \frac{U_m}{\sqrt{R_d^2 + (\omega L_d)^2}} \sin \left( \frac{2\pi}{m} n + \frac{\pi(m+2-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right) + A e^{-\frac{R_d}{L_d} \frac{2\pi}{m\omega} n}. \quad (7)$$

The first-order difference equation with the constant coefficients can be obtained by fixing the initial conditions at the current (6) and subsequent (7) states with respect to the condition of current continuity:

$$\begin{aligned} i[n+1] = i[n] e^{-\frac{R_d}{L_d} \frac{2\pi}{m\omega}} + \frac{U_m}{\sqrt{R_d^2 + (\omega L_d)^2}} & \left[ \sin \left( \frac{\pi(m+6-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right) - \right. \\ & \left. - \sin \left( \frac{\pi(m+2-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right) \right] e^{-\frac{R_d}{L_d} \frac{2\pi}{m\omega} n}. \end{aligned} \quad (8)$$

Hence, the equation (8) represents the recurrence formula for the subsequent determination of current values  $i[n]$  for any  $n$ .

The current at the end of the  $n$ -th state is presented as follows:

$$i[n+1] = i[n]h + B, \quad (9)$$

where  $h = e^{-\frac{R_d}{L_d} \frac{2\pi}{m\omega}}$ ;

$$B = \frac{U_m}{\sqrt{R_d^2 + (\omega L_d)^2}} \left[ \sin \left( \frac{\pi(m+6-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right) - \sin \left( \frac{\pi(m+2-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right) \right] e^{-\frac{R_d}{L_d} \frac{2\pi}{m\omega}}.$$

### 3. Methods

The solution of difference equation (9) is implemented using the discrete Laplace transformation for the equations with the lattice functions. The original and the image in this transformation are connected [6]:

$$D\{i[n+1]\} = e^q [I^*(q) - i[0]], \quad (10)$$

where  $I^*(q)$  is the image of function  $i[n]$ ;  $i[0]$  is the initial value of the rectified current at the beginning of the process.

With respect to the initial process at the time of the switching-on  $i[0] = 0$ , (11) can be reduced to:

$$i[n+1] = e^q I^*(q). \quad (11)$$

Taking into consideration (9) and (10), the following expression is obtained:

$$I^*(q) = \frac{e^q B}{(e^q - 1)(e^q - h)}. \quad (12)$$

A new variable  $z = e^q$  is introduced for the simplification of (12). The discrete Laplace transformation  $i[n]$  (12) results in the unilateral forward  $z$ -transformation of the lattice function [9, 10]:

$$I(z) = \frac{zB}{(z-1)(z-h)}. \quad (13)$$

The original for (14) is found using the residues [7]:

$$i[n] = \sum_{k=1}^n \text{Res}[I(z) z^{n-1}].$$

The function  $I(z)$  has two simple poles  $z_1 = 1$  and  $z_2 = h$ . Thus, the residues of  $I(z) z^{n-1}$  are:

$$\text{Res}_{z \rightarrow 1} I(z) = B \lim_{z \rightarrow 1} \frac{z^n}{z-h} = \frac{B}{1-h}; \quad \text{Res}_{z \rightarrow h} I(z) = B \lim_{z \rightarrow h} \frac{z^n}{z-1} = \frac{B h^n}{h-1}.$$

Consequently, the sum of residues for the current  $i[n]$  is;

$$\sum_{k=1}^n \text{Res}_{z \rightarrow 1} [I(z) z^{n-1}] = B \left( \frac{1}{1-h} + \frac{h^n}{h-1} \right). \quad (14)$$

Applying the coefficient values  $h$  and  $B$  to (14), the current at the beginning of  $n$ -th state has the form:

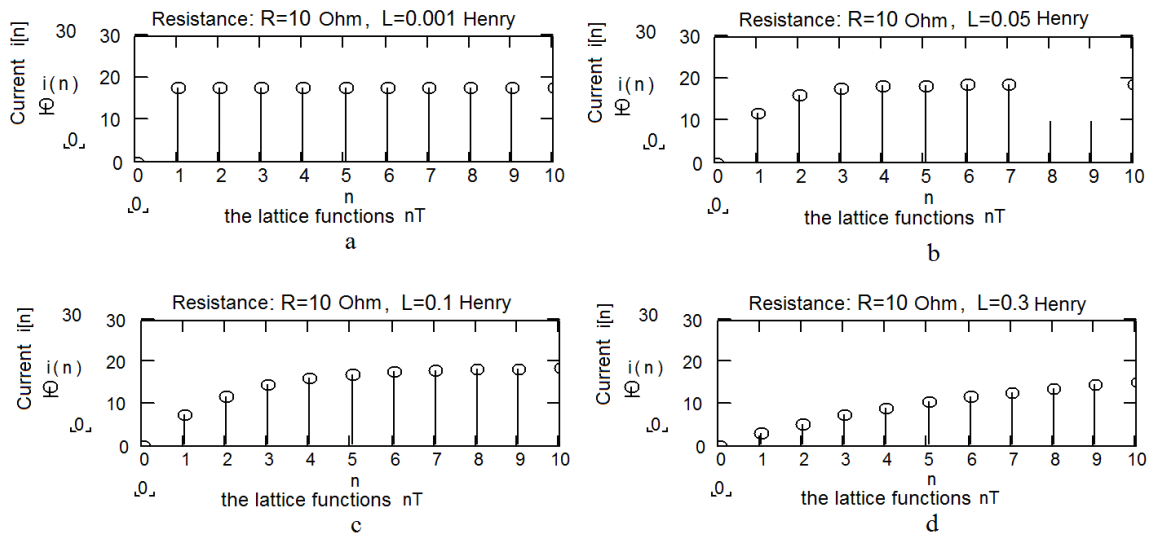
$$i[n] = \frac{U_m}{\sqrt{R_d^2 + (\omega L_d)^2}} \left( 1 - e^{-\frac{R_d 2\pi}{L_d m \omega} n} \right) \left[ \frac{\sin \left( \frac{\pi(m+6-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right)}{1 - e^{-\frac{R_d 2\pi}{L_d m \omega}}} - \frac{\sin \left( \frac{\pi(m+2-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right) e^{-\frac{R_d 2\pi}{L_d m \omega}}}{1 - e^{-\frac{R_d 2\pi}{L_d m \omega}}} \right]. \quad (15)$$

The current  $i[n]$  at the steady state mode ( $n \rightarrow \infty$ ) in the interval  $t = \frac{2\pi}{m\omega}$  is:

$$i[n] \Big|_{n \rightarrow \infty} = \frac{U_m}{\sqrt{R_d^2 + (\omega L_d)^2}} \left( \frac{\sin \left( \frac{\pi(m+6-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right)}{1 - e^{-\frac{R_d 2\pi}{L_d m \omega}}} - \frac{\sin \left( \frac{\pi(m+2-4k)}{2m} - \arctg \frac{\omega L_d}{R_d} \right)}{1 - e^{-\frac{R_d 2\pi}{L_d m \omega}}} \right).$$

#### 4. Main results

The lattice functions for different values of the active inductive load for the eight-pulse rectifier are shown in figure 2.



**Figure 2.** Dependencies of the current on the lattice function argument

As it follows from figure 2, the different values of instantaneous currents  $i[n]$  correspond to different y-coordinates of the lattice functions at the discrete time intervals under various load types. As  $i[n]$  is not absolutely equivalent to the continuous function  $i(t)$ , the transient process analysis can be implemented with the envelopes of the lattice function.

Figure 2 a shows that if the load inductance is relatively small, the steady state mode is achieved at first pulse of the supply voltage.

The plots of instantaneous currents in the case of increase of the load inductance are presented in figures 2 b-d.

## 5. Conclusion

The envelopes of the lattice function illustrate the duration the transient process and the current amplitude dependence on the number of pulses of the supply voltage.

The analytic dependence of current on the argument of the lattice function allows one to determine the value of current from the integer number of pulses of the rectified voltage for the active inductive load.

The load current envelope reflects the type of the transient process and makes it possible to compare rectification circuits and to analyze steady-state processes.

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