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# Definition of distortion power in AC network and analysis of its reasons

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**Abstract.** This paper studies the conditions under which such non-active power components as reactive power and distortion power appear in the alternating current power network. By simulating with MatLab Simulink, the validity of formulas obtained by analytical way was tested. The formulas define the components of non-active power based on one of the most well-known theories. The results obtained will be useful for mode optimization in the alternating current networks and for developing effective non-active power compensating technologies.

## 1. Introduction

Modes optimization in alternating current networks requires the compensation of all power components that cannot perform useful work, i.e. they are not active power.

Compensation issues are more studied for ideal conditions when electric networks have symmetrical and sinusoidal voltages and balanced phase loads and the loads have linear voltage-current characteristics (V-I). In these situations, an excess of the apparent power ( $S$ ) over the active power ( $P$ ) is due to the emergence of the non-active power component, known as reactive power ( $Q$ ).

In actual practice, the AC power networks do not have sinusoidal voltages, and a lot of modern loads have considerable non-linear V-I characteristics. In these situations, non-sinusoidal currents flow through circuit components with non-sinusoidal voltages. It leads to such excess of the apparent power  $S$  over the active power  $P$  which cannot be explained only by the reactive power  $Q$ . In scientific papers this fact is explained by the presence of another component of non-active power which is known as the distortion power and which is designated by  $T$  [1] or  $D$  [2]. Under some specific conditions the distortion power  $D$  can appear in the AC power network as the only component of non-active power.

## 2. Problem statement

This study aims to analyze the most common static modes which lead to the separate occurrence of  $Q$  and  $D$  power in the AC electrical network. The results obtained may be useful to study the electrical networks operation modes by means of simulation tools and to create more efficient technical devices for non-active power compensation.

## 3. Theory

The reasons for the reactive power  $Q$  occurrence and its negative impact on the electrical networks with sinusoidal currents and voltages are well described by the basic electrical theory. Therefore, today technical tools are used with the compensation power  $Q_K$  which makes it possible to reduce the



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value  $Q$  to the minimum value  $Q - Q_K \rightarrow 0$ , and thus bringing the value  $S$  closer to the numerical value  $P$  of the consumer.

In this case, one of the obvious criteria forming the target function is shown as:

$$S = \sqrt{P^2 + (Q - Q_K)^2} \rightarrow \min. \quad (1)$$

The task of the power compensation is more complex if there are non-sinusoidal currents  $i$  and voltages  $u$  in the electrical network. In this case they are usually represented as Fourier series [3]:

$$\begin{aligned} u &= U_0 + U_{1m} \sin(\omega t + \alpha_1) + U_{2m} \sin(2\omega t + \alpha_2) + \dots + U_{Nm} \sin(N\omega t + \alpha_N), \\ i &= I_0 + I_{1m} \sin(\omega t + \alpha_1 + \varphi_1) + I_{2m} \sin(2\omega t + \alpha_2 + \varphi_2) + \dots + I_{Nm} \sin(N\omega t + \alpha_N + \varphi_N). \end{aligned} \quad (2)$$

The constant component of series (2) can be considered as a zeroth harmonic, and in this case, the apparent power is defined as the product of currents harmonics and voltages with the same subscripts:

$$S = U \cdot I = \sqrt{\sum_{i=0}^N U_i^2 \cdot \sum_{i=0}^N I_i^2}, \quad (3)$$

where  $U_i = U_{im}/\sqrt{2}$ ,  $I_i = I_{im}/\sqrt{2}$  – RMS of voltage and current,  $i$  – a serial number of a harmonic.

In this case, the active component of the power is calculated as an average value of the product of voltage and current for all harmonics (2) for period  $T$ :

$$P = \frac{1}{T} \int_0^T (u \cdot i) dt = U_0 I_0 + U_1 I_1 \cos \varphi_1 + U_2 I_2 \cos \varphi_2 + \dots + U_N I_N \cos \varphi_N. \quad (4)$$

It proves that useful work can be done by all harmonics in the non-sinusoidal AC network, and the active power is presented as a sum of active powers for each harmonic:

$$P = P_0 + P_1 + P_2 + \dots + P_N = \sum_{i=0}^N P_i. \quad (5)$$

By analogy with the power  $P$ , there is a theory [4] that the power  $Q$  is the sum of reactive powers for each harmonic in the electrical network:

$$\begin{aligned} Q &= U_1 I_1 \sin \varphi_1 + U_2 I_2 \sin \varphi_2 + \dots + U_N I_N \sin \varphi_N, \\ Q &= Q_1 + Q_2 + \dots + Q_N = \sum_{i=1}^N Q_i. \end{aligned} \quad (6)$$

The component  $D$  of the non-active power which can be found in the network with non-sinusoidal currents and voltages [5] is usually defined by the unbalance of the previously found power  $P$  and  $Q$  in the inequality  $S > \sqrt{P^2 + Q^2}$  using their orthogonal property:

$$D = \sqrt{S^2 - P^2 - Q^2}. \quad (7)$$

It should be noted that in scientific community there is no commonly accepted equation to define the distortion power as well as generally accepted interpretation of its physical meaning in contrast to the power  $P$  and  $S$ . Moreover, among various scientists there is no clear understanding of physical meaning for the total reactive power defined by (6) in the electrical networks with non-sinusoidal voltages and currents. It can be seen from the accepted standard IEEE 1459-2010 [6] which introduced the concept of non-active power, but the definition of power  $Q$  is based only on the first harmonic.

Many researchers focus on the non-active power definition. Thus, the theory outlined in [7] describes the equation (6) as the reactive power of all harmonics in the AC network. It was there that the value calculated from the equation (7) was identified as the distortion power for the first time. This theory explains the distortion power occurs due to current form distortion relative to the voltage form. However, there are a lot of questions concerning the definition of distortion power  $D$  and interpretation of its physical meaning. Follow-up papers [8] focused on further study of non-active power properties by means of harmonic components. They were based on the spectral methods as well as research [9] using the sum of instantaneous values of currents and voltages. In other words they were based on integral methods of power calculation. In addition, some methods were developed [10],

[11] such as currents separation into individual components in order to define non-active power in special coordinates or to define individual energy flow elements as orthogonal components. These researches are contradictory regarding the physical meaning of non-active power components and their calculation. It makes This makes it more difficult to search the best compensation by means of technical tools using simple and effective algorithms based on physical meaning.

Therefore, some experiment series should be run simulating the most common static modes of the electrical network. The results of the experiments could define the best theory.

#### 4. Choosing a simulation environment and creation of measuring subsystems

The analysis of the modes resulting in separate occurrence of power  $Q$  and  $D$  in the electrical AC network should be based on results of model experiments for some series of special cases with different harmonics of supply voltages, currents and V-I loads. To achieve the objective of the study, the modelling environment MatLab Simulink is used [12].

It should be noted that according to accepted power definition [6], the functional measurement unit “Power” in Simulink is not convenient as a tool for measuring values  $P$  and  $Q$  because its algorithm is based on calculating the power regarding the fundamental frequency. This require additional adjustments before each experimental run.

As integral methods define the instantaneous power as  $p = u \cdot i$ , the integral of which defines transmitted electrical energy  $W$  at a certain time period limited by integration limits  $t_1$  and  $t_2$ , preliminary consideration of harmonic components is not required. The calculation of power  $P$  in simulation experiments for load in the electrical AC network should be as follows:

$$P = \frac{W_{t_2-t_1}}{t_2-t_1} = \frac{1}{t_2-t_1} \int_{t_1}^{t_2} (u \cdot i) dt. \quad (8)$$

In this case, a simple universal model (Figure 1a) as a measuring subsystem will realize the calculation of the power  $P$  in Simulink.

To simplify the creation of the appropriate measuring subsystem in the further model, the calculation of power  $S$  with regard to the equation (3) is done by the same way:

$$S = \sqrt{\frac{1}{t_2-t_1} \int_{t_1}^{t_2} u^2(t) dt} \cdot \sqrt{\frac{1}{t_2-t_1} \int_{t_1}^{t_2} i^2(t) dt}. \quad (9)$$

The measuring subsystem corresponds to the obtained equation (9) for calculation of value  $S$  which does not require the model presets shown in Figure 1b.

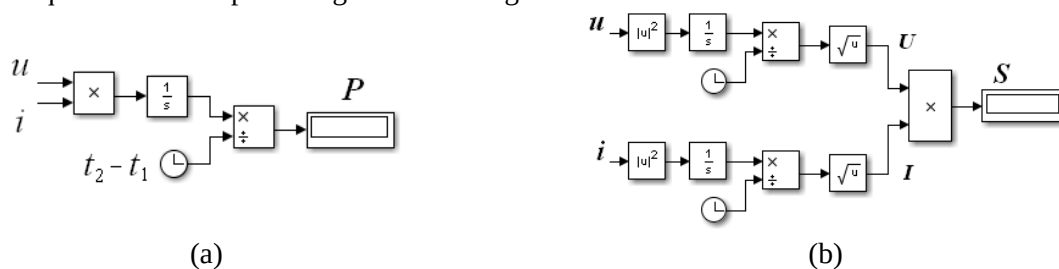


**Figure 1.** Subsystem for measuring in non-sinusoidal AC network: (a) active power; (b) apparent power.

#### 5. Modelling conditions resulting in non-active power $Q$ and $D$

The research is based on correlation of model experiments results which are obtained in conditions which make it possible to separately simulate non-sinusoidal voltage supply, to change the load and its V-I characteristics.

To simplify the analysis of the obtained results, the non-sinusoidal voltage in the AC network is simulated by two previously known harmonics, namely, the fundamental (50 Hz) and the higher, e.g. the tenth (500 Hz) by changing the ratio of their amplitudes.

The type of load in the AC network is determined by the ratio of values of elements  $R$  and  $L$  in parallel connection. To create non-linear V-I characteristics, it is sufficed to add a semiconductor diode to the power supply network using a simple circuit with one half-wave rectifier.

A kind of load is determined by the ratio of values of parallel elements  $R$  and  $L$  in the AC network. To impart a non-linear character of the V-I characteristic, it is enough to add a semiconductor diode in the power supply circuit realizing a simple half-wave rectifier circuit.

Thus, a simple simulation model has been obtained (Figure 2) in Simulink for creating and studying various power components in the AC network.

Fourier measurement series determine the controlled values:

$u_{50}^{\max}$ ,  $u_{500}^{\max}$  – amplitudes of voltage harmonics with 50Hz and 500Hz frequencies;

$\alpha_{50}^u$ ,  $\alpha_{500}^u$  – initial phase angles of voltage harmonics with 50Hz and 500Hz frequencies;

$i_{50}^{\max}$ ,  $i_{500}^{\max}$  – amplitudes of current harmonics with 50Hz and 500Hz frequencies;

$\alpha_{50}^i$ ,  $\alpha_{500}^i$  – initial phase angles of current harmonics with 50Hz and 500Hz frequencies;

$\varphi_{50} = \alpha_{50}^u - \alpha_{50}^i$ ,  $\varphi_{500} = \alpha_{500}^u - \alpha_{500}^i$  – phase shift angle between voltage and current harmonics.

The results of the simulation experiment are shown in Table 1.

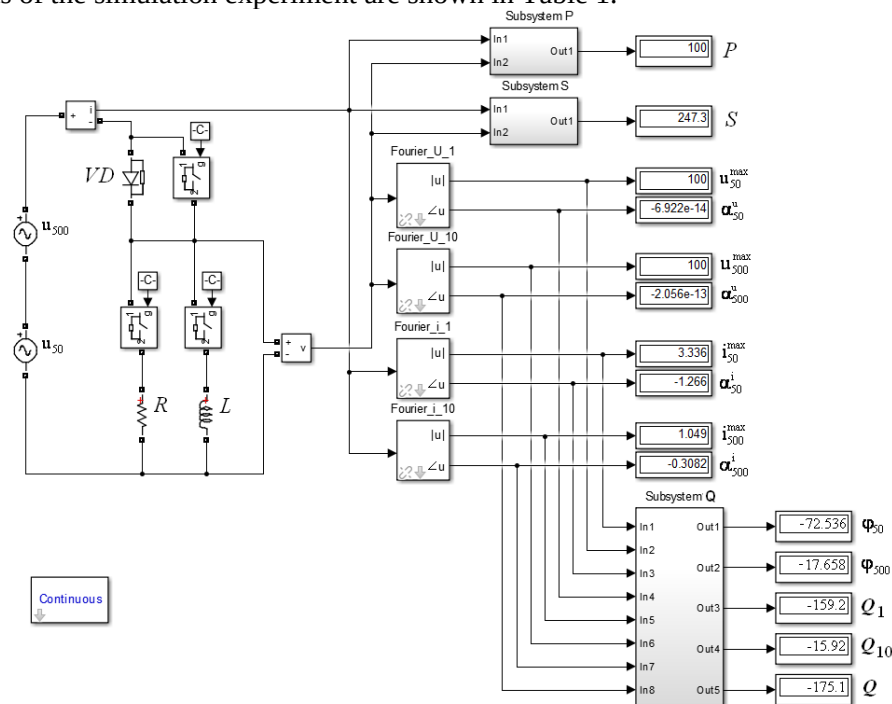


**Figure 2.** The simulation model in Simulink for creating and studying power components

**Table 1.** Parameters of electrical circuits, modes and power.

| $N_0$ | $R$<br>( $\Omega$ ) | $L$<br>(H) | $u_{50}^{\max}$ | $u_{500}^{\max}$ | $i_{50}^{\max}$ | $i_{500}^{\max}$ | $\varphi_{50}$ | $\varphi_{500}$ | $P$ (W) | $Q_1$<br>(VA) | $Q_{10}$<br>(VA) | $D$<br>(VA) | $S$<br>(VA) |
|-------|---------------------|------------|-----------------|------------------|-----------------|------------------|----------------|-----------------|---------|---------------|------------------|-------------|-------------|
| 1     | 100                 | -          | 100             | 0                | 1.000           | 0                | 0              | -               | 50.0    | 0             | 0                | 0           | 50.0        |
| 2     | 100                 | -          | 100             | 100              | 1.000           | 1.000            | 0              | 0               | 100.0   | 0             | 0                | 0           | 100.0       |
| 3     | -                   | 0.1        | 100             | 0                | 3.183           | 0                | -90.0          | -               | 0       | 159.2         | 0                | 0           | 159.2       |
| 4     | -                   | 0.1        | 100             | 100              | 3.183           | 0.318            | -90.0          | -90.0           | 0       | 159.2         | 15.92            | 143.2       | 226.2       |
| 5     | 100                 | 0.1        | 100             | 0                | 3.336           | 0                | -72.5          | -               | 50.0    | 159.2         | 0                | 0           | 166.8       |
| 6     | 100                 | 0.1        | 100             | 100              | 3.336           | 1.049            | -72.5          | -17.6           | 100.0   | 159.2         | 15.92            | 143.2       | 247.3       |
| 7     | 100<br>(+VD)        | -          | 100             | 0                | 0.500           | -                | 0.4            | -               | 25.2    | 0.2           | 0                | 25.0        | 35.7        |

The analysis of results shows that a balance of orthogonal power for all modes is achieved and confirms the validity of the equation (7). The total reactive power of the load is determined by the sum of the reactive power of each harmonic according to the equation (6). The distortion power occurs when the form of the load current is not identical to the supply voltage form. It means that combinations of the available harmonics of the voltage ( $h$ ) and the current ( $k$ ) in the product  $u \cdot i$  have zero average values of the power  $N$  during the period due to the general orthogonal properties:

$$N = \frac{1}{T} \int_0^T (U_{hm} \sin(h\omega t + \alpha_h) \cdot I_{km} \sin(k\omega t + \alpha_k)) dt = 0, \quad (10)$$

Therefore, the value  $N$  does not refer to the power  $P$ , and is non-active, i.e. it is not capable to do useful work. When  $h \neq k$  by definition of the equation (6), the power is not the power  $Q$ . Under these conditions the non-active power  $N$  should be considered as the distortion power  $D$ .

In simple conditions of a model experiment, the power  $D$  can be determined analytically. Taking into account that two known harmonics are created artificially, namely, the fundamental and the tenth, we can define:

$$D^2 = S^2 - (P^2 + Q^2)$$

where

$$S^2 = (U \cdot I)^2 = (U_1^2 + U_{10}^2) \cdot (I_1^2 + I_{10}^2),$$

$$P^2 + Q^2 = (P_1 + P_{10})^2 + (Q_1 + Q_{10})^2.$$

Since the powers of each  $i$ -th harmonic are determined according to (4) and (6):

$$P_i = U_i I_i \cos \varphi_i, \quad Q_i = U_i I_i \sin \varphi_i,$$

we get:

$$D^2 = U_1^2 I_1^2 + U_1^2 I_{10}^2 + U_{10}^2 I_1^2 + U_{10}^2 I_{10}^2 - \left( U_1^2 I_1^2 \cos^2 \varphi_1 + U_{10}^2 I_{10}^2 \cos^2 \varphi_{10} + 2U_1 I_1 \cos \varphi_1 \cdot U_{10} I_{10} \cos \varphi_{10} + \right. \\ \left. + U_1^2 I_1^2 \sin^2 \varphi_1 + U_{10}^2 I_{10}^2 \sin^2 \varphi_{10} + 2U_1 I_1 \sin \varphi_1 \cdot U_{10} I_{10} \sin \varphi_{10} \right)$$

After rearrangement, the equation is simplified:

$$D = \sqrt{U_1 I_{10} (U_1 I_{10} - 2U_{10} I_1 \cos \varphi_1 \cdot \cos \varphi_{10}) + U_{10} I_1 (U_{10} I_1 - 2U_1 I_{10} \sin \varphi_1 \cdot \sin \varphi_{10})} \quad (11)$$

Based on theory [7], the reliability of the equation (11) derived analytically, confirms the values of the power  $D$  found by simulation (Table 1).

## 6. Discussion

The research requires the analysis of the most characteristic modes shown in Table 1.

### 6.1. Mode №1

The supply voltage is sinusoidal  $U_{10} = 0$ , the load is clearly active. In this case,  $I_{10} = 0$ ,  $\cos \varphi_1 = 1$ ,  $\sin \varphi_1 = 0$ . Therefore,  $Q_i = U_i I_i \sin \varphi_i = 0$  and  $D = 0$ .

### 6.2. Mode №2

The supply voltage is non-sinusoidal  $U_{10} \neq 0$ , the load is clearly active. In this case,  $\cos \varphi_1 = 1$ ,  $\cos \varphi_{10} = 1$ ,  $\sin \varphi_1 = 0$ ,  $\sin \varphi_{10} = 0$ . From the equation (11),  $D = U_1 I_{10} - U_{10} I_1$ . However, the identity of the ratio between voltage harmonics and current harmonics is true with the clearly active load in AC network:

$$\frac{U_1}{U_{10}} = \frac{I_1}{I_{10}}. \quad (12)$$

Thus, for this mode  $U_1 I_{10} - U_{10} I_1 = 0$ .

Therefore, the power  $D$  does not occur in the AC network with the active load even when the voltage is non-sinusoidal.

### 6.3. Mode №6

The supply voltage is non-sinusoidal, the load is both active and reactive. In this case,  $0 < \cos \varphi_1 < 1$ ,  $0 < \cos \varphi_{10} < 1$ ,  $0 < \sin \varphi_1 < 1$ ,  $0 < \sin \varphi_{10} < 1$ . For this mode after rearranging the equation (11), we get:

$$D = \sqrt{\frac{U_1 I_{10}}{U_{10} I_1} + \frac{U_{10} I_1}{U_1 I_{10}} - 2(\cos \varphi_1 \cos \varphi_{10} + \sin \varphi_1 \sin \varphi_{10})},$$

$$D = \sqrt{\frac{U_1 I_{10}}{U_{10} I_1} + \frac{U_{10} I_1}{U_1 I_{10}} - 2 \cos(\varphi_1 - \varphi_{10})}. \quad (13)$$

Thus, in this mode the power  $D = 0$  under two conditions: the identity (12) and the equation  $\varphi_1 = \varphi_{10}$ . However, the theory says that the resistance of reactive elements is determined by frequency of alternating current flowing through them. Thus, for inductance  $x_L = j2\pi fL$ . Therefore, the value of the harmonic of current flowing through the inductance in the electrical circuit is equal to:

$$I_i = \frac{U_i}{x_L} = \frac{U_i}{j2\pi fL},$$

then:

$$\frac{I_1}{I_{10}} = \frac{U_1}{j2\pi fL} \cdot \frac{j2\pi 10 fL}{U_{10}} = \frac{10U_1}{U_{10}}$$

that contradict equation (12).

For the same reason, the equation (12) will not be proper for the circuit with a capacitance, the reactance  $x_C$  of which also depends on the frequency  $x_C = 1/(j2\pi fC)$ .

Thus, if the reactive load is added in the AC network with the non-sinusoidal voltage both the power  $Q$  and the power  $D$  will occur.

### 6.4. Mode №7

The supply voltage is sinusoidal, the load is active but non-linear. In this case,  $U_1 \neq 0$ ,  $U_i = 0$ , where  $i > 1$ , and only load current is non-sinusoidal in AC network. The resulting harmonics will not be able to do useful work due to the equation (10) in the AC network with the non-sinusoidal voltage.

If switching processes are ignored at the first approximation, the fundamental harmonic of the current will not have the phase shift regarding the fundamental harmonic of the supply voltage, i.e.  $\cos \varphi_1 = 1$  and  $\sin \varphi_1 = 0$ . Under such condition, the value of  $Q = 0$ , and the value  $D$  will be determined from equations (3), (4), (5) and (7) as:

$$D = \sqrt{S^2 - P^2} = \sqrt{\sum_{i=0}^{\infty} U_1^2 I_i^2 - U_1^2 I_1^2} = U_1 \sqrt{\sum_{\substack{i=0, \\ i \neq 1}}^{\infty} I_i^2}. \quad (14)$$

However, if V-I of the load creates the first current harmonic for which  $\sin \varphi_1 \neq 0$ , then the power  $Q$  will occur in the AC network with the sinusoidal voltage. For example, it is typical for controlled rectifiers operation mode which create both the power  $D$  and the power  $Q$  without any reactive elements in their circuits.

Thus, the mode where the power  $D = 0$ , according to results of the analysis of equations (12), and (13) is achieved under the following conditions:

- the equality of phase shifts for all harmonics of currents and voltages:

$$\varphi_i = \varphi_j \quad (15)$$

- the identity of the ratios of values for all harmonics of currents and voltages:

$$\frac{U_i}{U_j} = \frac{I_i}{I_j}. \quad (16)$$

When increasing the number of harmonics of currents and voltages in the model then by analogy with the deduction of the equation (11), the general equation can be derived to calculate the value  $D$ :

$$D = \sqrt{\sum_{i=0}^{\infty} \sum_{\substack{j=1 \\ i \neq j}}^{\infty} U_i I_j (U_i I_j - 2U_j I_i \cos \varphi_i \cdot \cos \varphi_j) + U_j I_i (U_j I_i - 2U_i I_j \sin \varphi_i \cdot \sin \varphi_j)}. \quad (17)$$

There are some versions of deriving similar equations in general terms to find the value  $D$  in the scientific literature. In [13], it is proposed to calculate the value  $D$  according to:

$$D^2 = \sum_{\substack{k,l \\ k \neq l}}^{\infty} U_k^2 I_l^2 - \sum_{\substack{k,l \\ k \neq l}}^{\infty} U_k I_k U_l I_l \cos(\varphi_k - \varphi_l),$$

The most similar equation to (17) is given in [14]:

$$D = \sqrt{\sum_{i=1}^{N-1} \sum_{j=1}^N U_i I_j (U_i I_j - U_j I_i \cos(\varphi_i - \varphi_j)) + U_j I_i (U_j I_i - U_i I_j \cos(\varphi_i - \varphi_j))}.$$

Thus, the components of power  $N$  can be analyzed and calculated. At least for simple AC networks in static mode they can be decomposed into the power  $Q$  by different harmonics and the power  $D$ . Definition of the power  $Q$  and the power  $D$  described in [7] is the most similar to the results obtained analytically and confirmed by simulating.

## 7. Conclusion

In this paper, the analysis of the main modes resulting in the separate occurrence of power  $Q$  and  $D$  in the AC electric network has been performed. This analysis allows concluding that the compensation of the power  $Q$  and the power  $D$  will be more effective if technical device provides a correction of the load current the waveform of which is similar to the waveform of the supply voltage and provides the condition  $P/S \rightarrow 1$ .

The method of direct derivation of analytical equations of the power  $D$  in the AC network was developed, and two simple basic criteria to provide the distortion power compensation have been verified by modeling, namely, the equality of the phase shifts for all current and voltage harmonics and the equality of ratios between the voltages harmonics values and the load currents harmonic values.

## References

- [1] Zevke G V, Ionkin P A, Netushil A V 1963 *Fundamentals of the theory of chains* (Moscow: Gosenergoizdat) p 440
- [2] Akagi H, Kim H 1999 *IEEE Int. Conf. on Power Electr. and Drive Syst.* (Hong Kong) 422-427
- [3] Ango A 1997 *Mathematics for electrical and radio engineers* (Moscow: Nauka) p 780
- [4] Sharon D 1996 *IEEE Trans. on Instrument. and Measur.* **45** 728-733
- [5] Krogeris A F 1993 *AC power* (Riga: Fiz.-energ. in-t Latv. AN) p 149
- [6] 2010 *IEEE Std. Definition for the measurement of Electric Power Quantities under sinusoidal, non-sinusoidal, balanced or unbalanced conditions* (New York)
- [7] Budenau C 1927 *Puissance reactives et fictives* (Buharest)
- [8] Czarnecki L S 1988 *IEEE Trans. on Instrumentation and Measurement* **37(1)** 30-34
- [9] Fryze S 1931 *Przegląd Elektrot* 193-203.
- [10] Maevskiy O A 1978 *Energy performance valve converters* (Moscow: Energia) p 320
- [11] Akagi H, Nabae A 1993 *ETEP* **3(1)** 27-31
- [12] German-Galkin S G 2013 *Virtual laboratories, semiconductors, systems in MatLab Citylink* (St.Petersburg: Lan) p 448
- [13] Bezikovitch A Ya, Shapiro E Z 1980 *Measurement of electrical power in the audio frequency range* (Leningrad: Energia) p 168
- [14] Shklyarskiy Ya E, Bragin A A, Dobush V S 2012 *Oil and Gas Business* **4** 26-32