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A cascade equivalent *A*-*H*-circuit of a synchronous salient-pole electric machine

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Abstract. Cartesian and cylindrical laminated models are well known in electromagnetic calculations of electric machines. In such models, general solutions of partial differential equations are transformed into four-terminal circuit equations, and this makes possible to synthesize cascade equivalent circuits of the electric machines. In salient-pole machines, solutions of partial differential equations are formed on the base of piecewise continuous Sturm-Liouville eigenfunctions. However, in this case, cascade equivalent circuits cannot be synthesized since it needs many piecewise continuous eigenfunctions in a zone of poles and many smooth functions in a zone of an air gap for ensuring uniqueness of a solution. Meanwhile the author of this paper had offered an approximate method on the base of a single piecewise continuous Sturm-Liouville eigenfunction in a zone of poles and many smooth functions in a zone of the air gap. This method allows transforming an analytic solution of a partial differential equation into four-terminal circuit equations and synthesizing cascade equivalent circuits of salient-pole electric machines. In this paper, a synthesis of an active cell of a cascade equivalent *A*-*H*-circuit of a synchronous salient-pole electric machine is considered. The active cell of the corresponding exciting field is synthesized by means of a solution of Poisson's equation with a single piecewise continuous Sturm-Liouville eigenfunction.

1. Introduction

In the 1950-1970s, many papers appeared [1-4] where electromagnetic calculations of electrical machines on the basis of laminated models were considered. In this model, each layer corresponded to a certain structural zone of the an electrical machine (for example a rotor yoke, a stator yoke, teeth and slots, air gaps). There is integrated electromagnetic field all over the functional volume. In each layer, using simplifying assumptions, it is necessary to find a general solution of the partial differential equation, then, on the grounds of continuity conditions, to work out and solve the system of linear algebraic equations in order to determine unknown integration constants which number is twice as many layers of the model.

However, this area did not have any progress due to difficulty of calculations and unhandiness of results. At the same time, these solutions may be reduced to a system of equations of a standard four-terminal network and, on layer's boundaries, tangential components of electric and magnetic intensities can be regarded as a voltage and a current. On the grounds of continuity of tangential components of electric and magnetic intensities between layers, four-terminal networks must be connected as a cascade connection. As a result, the *E*-*H*-cascade equivalent circuit of the laminated model will be synthesized. Various *E*-*H*-, *B*-*H*- or *A*-*H*- equivalent circuits are well-known [5-10].

In salient-pole machines, solutions of partial differential equations are formed on the base of piecewise continuous Sturm-Liouville eigenfunctions when the zone containing ferromagnetic poles and non-magnetic spaces cannot be transformed into the homogeneous domain [11-13]. However, in this case, cascade equivalent circuits cannot be synthesized since it needs many piecewise continuous eigenfunctions in a zone of poles and many smooth functions in a zone of an air gap for ensure uniqueness of a solution. Moreover, it is a quantity of smooth functions that determines accuracy of a solution in an air gap. Meanwhile the author of this paper had offered an approximate method on the base of a single piecewise continuous Sturm-Liouville eigenfunction in a zone of poles and many smooth functions in a zone of the air gap [14,15]. This method allows transforming an analytic solution of a partial differential equation into four-terminal circuit equations and synthesizing cascade equivalent circuits of salient-pole electric machines.

In this paper, a synthesis of an active cell of a cascade equivalent *A*-*H*-circuit of a synchronous salient-pole electric machine is considered. Let us suppose that poles are fixed on the perfect ferromagnetic rotor yoke where magnetic permeability is infinite. Magnetic permeability of poles is constant. Also, we suppose that a model has a unit length in the line of a machine axis. Because of symmetry, let us consider magnetic field in the range of half a pole pitch.

2. An inductor of a synchronous salient-pole electric machine and a standard active cell of a cascade equivalent *A*-*H*-circuit

Figure 1 shows an analytical model of a synchronous salient-pole electric machine. It should be noted that, in this model, poles and air spaces between them must be confined to coordinate surfaces of the cylindrical coordinate system. Therefore, prismatic poles must be purposively configured as sphenoid. Substituting sphenoid poles for prismatic, it is necessary to keep a pole volume invariable to ensure equivalence.

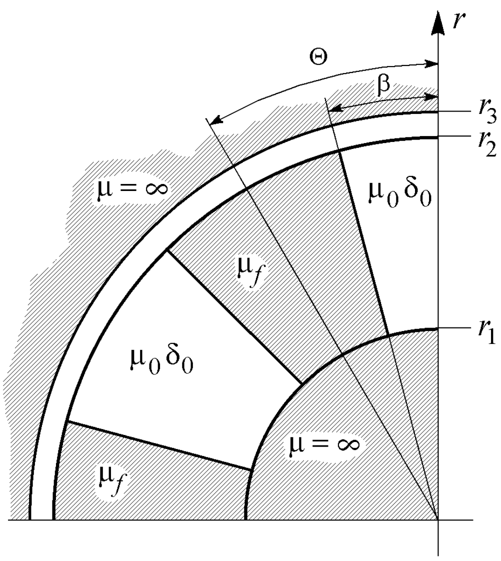

Figure 1. An analytical model of a synchronous salient-pole electric machine.

The exciting field is generated with the current density δ_0 that is located in air spaces between poles. Thus, the exciting current density may be described by a piecewise continuous function in the whole zone, which corresponds to alternate poles and air spaces (in the range of half a pole pitch):

$$\delta_0(\alpha) = \begin{cases} \delta_0, & 0 < \alpha < \beta, \\ 0, & \beta < \alpha < \Theta. \end{cases} \tag{1}$$

Magnetic permeability is also described by a piecewise continuous function in this zone:

$$\mu(\alpha)=\begin{cases}\mu_0, & 0<\alpha<\beta,\\ \mu_f, & \beta<\alpha<\Theta.\end{cases} \tag{2}$$

In cylindrical coordinates, components of a magnetic intensity vector are determined as:

$$H_r(r,\alpha)=\frac{1}{\mu(\alpha)r}\frac{\partial A}{\partial\alpha};\ H_\alpha(r,\alpha)=-\frac{1}{\mu(\alpha)}\frac{\partial A}{\partial r} \tag{3}$$

where A is the z-component of a vector magnetic potential.

In consideration of expressions (3), Maxwell's first equation ($\mathrm{rot}\,\boldsymbol{H}=\delta_0(\alpha)$) turns into Poisson's equation for a vector potential:

$$\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}+\frac{1}{r^2}\frac{\partial^2 A}{\partial\alpha^2}=-\mu(\alpha)\delta_0(\alpha). \tag{4}$$

Let a solution of Poisson's equation (4) have both a single piecewise continuous Sturm-Liouville eigenfunction and a piecewise continuous function that corresponds to exciting field:

$$A(r,\alpha)=-\frac{\mu_0\delta_0 r^2}{4}\begin{cases}1, & 0<\alpha<\beta\\ \dfrac{\sin 2(\Theta-\alpha)}{\sin 2(\Theta-\beta)}, & \beta<\alpha<\Theta\end{cases}+\left[C_1r^n+C_2r^{-n}\right]\begin{cases}\cos n\alpha, & 0<\alpha<\beta\\ K\sin n(\Theta-\alpha), & \beta<\alpha<\Theta\end{cases}, \tag{5}$$

where n is a positive root of a transcendental equation:

$$\frac{\mu_0}{\mu_f}\cos n\beta\cos n(\Theta-\beta)=\sin n\beta\sin n(\Theta-\beta), \tag{6}$$

$$K=\frac{\cos n\beta}{\sin n(\Theta-\beta)}. \tag{7}$$

A tangential component of a magnetic intensity vector is:

$$H_\alpha(r,\alpha)=-\frac{1}{\mu(\alpha)}\frac{\partial A}{\partial r}=\frac{\delta_0 r}{2}\begin{cases}1, & 0<\alpha<\beta\\ \dfrac{\mu_0}{\mu_f}\dfrac{\sin 2(\Theta-\alpha)}{\sin 2(\Theta-\beta)}\end{cases}-\frac{n}{\mu_0 r}\left[C_1r^n-C_2r^{-n}\right]\begin{cases}\cos n\alpha, & 0<\alpha<\beta\\ \dfrac{\mu_0}{\mu_f}K\sin n(\Theta-\alpha)\end{cases}. \tag{8}$$

Let us expand solutions (5) and (8) to Fourier's series:

$$A(r,\alpha)=-\frac{\mu_0\delta_0 r^2}{4}\sum_{k=1,3,5}^{\infty}Q_{0k}\cos kp\alpha+\left[C_1r^n+C_2r^{-n}\right]\sum_{k=1,3,5}^{\infty}Q_k\cos kp\alpha, \tag{9}$$

$$H_\alpha(r,\alpha)=\frac{\delta_0 r}{2}\sum_{k=1,3,5}^{\infty}\Psi_{0k}\cos kp\alpha-\frac{n}{\mu_0 r}\left[C_1r^n-C_2r^{-n}\right]\sum_{k=1,3,5}^{\infty}\Psi_k\cos kp\alpha, \tag{10}$$

where $2p$ is a number of poles and:

$$Q_{0k}=\frac{2}{\Theta}\left[\int_0^\beta\cos kp\alpha\,d\alpha+\frac{1}{\sin 2(\Theta-\beta)}\int_\beta^\Theta\sin 2(\Theta-\alpha)\cos kp\alpha\,d\alpha\right], \tag{11}$$

$$Q_k=\frac{2}{\Theta}\left[\int_0^\beta\cos n\alpha\cos kp\alpha\,d\alpha+K\int_\beta^\Theta\sin n(\Theta-\alpha)\cos kp\alpha\,d\alpha\right], \tag{12}$$

$$\Psi_{0k}=\frac{2}{\Theta}\left[\int_0^\beta\cos kp\alpha\,d\alpha+\frac{\mu_0}{\mu_f\sin 2(\Theta-\beta)}\int_\beta^\Theta\sin 2(\Theta-\alpha)\cos kp\alpha\,d\alpha\right], \tag{13}$$

$$\Psi_k=\frac{2}{\Theta}\left[\int_0^\beta\cos n\alpha\cos kp\alpha\,d\alpha+\frac{\mu_0}{\mu_f}K\int_\beta^\Theta\sin n(\Theta-\alpha)\cos kp\alpha\,d\alpha\right]. \tag{14}$$

When a rotor rotates (and the angular frequency is ω), an observer sees traveling waves of functions (9) and (10), in which the k-th harmonic, in the complex plane, takes the form:

$$\dot{A}_k = -j\frac{\mu_0\delta_0 r^2}{4}Q_{0k} + j\left[C_1 r^n + C_2 r^{-n}\right]Q_k\,, \tag{15}$$

$$\dot{H}_{\alpha k} = j\frac{\delta_0 r}{2}\Psi_{0k} - j\frac{n}{\mu_0 r}\left[C_1 r^n - C_2 r^{-n}\right]\Psi_k\,. \tag{16}$$

Let first harmonics of a vector potential and a magnetic intensity vector be known at boundaries of the piecewise zone named "poles and air spaces". If $r = r_1$ (see Figure 1), then:

$$\dot{A}_1 = -\dot{A}_{10} + j\left[C_1 r_1^n + C_2 r_1^{-n}\right]Q_1\,, \tag{17}$$

$$\dot{H}_1 r_1 = \dot{H}_{10} r_1 - j\frac{n}{\mu_0}\left[C_1 r_1^n - C_2 r_1^{-n}\right]\Psi_1\,, \tag{18}$$

where $\dot{A}_{10} = j\frac{\mu_0\delta_0 r_1^2}{4}Q_{01}$ and $\dot{H}_{10} r_1 = j\frac{\delta_0 r_1^2}{2}\Psi_{01}$.

If $r = r_2$, then:

$$\dot{A}_2 = -\dot{A}_{20} + j\left[C_1 r_2^n + C_2 r_2^{-n}\right]Q_1\,, \tag{19}$$

$$\dot{H}_2 r_2 = \dot{H}_{20} r_2 - j\frac{n}{\mu_0}\left[C_2 r_2^n - C_2 r_2^{-n}\right]\Psi_1\,, \tag{20}$$

where $\dot{A}_{20} = j\frac{\mu_0\delta_0 r_2^2}{4}Q_{01}$ and $\dot{H}_{20} r_2 = j\frac{\delta_0 r_2^2}{2}\Psi_{01}$.

Let an equivalent *A*-*H*-circuit correspond to the piecewise zone named "poles and air spaces" (Figure 2). In this equivalent *A*-*H*-circuit, a vector potential is regarded as a voltage, and a tangential component of a magnetic intensity vector multiplied by a radius is regarded as a current. There are both ideal voltage sources ($\dot{A}_{10}$ and $\dot{A}_{20}$) and ideal current sources ($\dot{H}_{10} r_1$ and $\dot{H}_{20} r_2$).

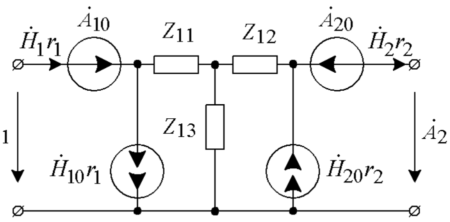

Figure 2. An equivalent *A*-*H*-circuit of the zone named "poles and air spaces".

In the *A*-*H*-circuit, impedances Z_{11}, Z_{12} and Z_{13} do not vary when operation conditions vary. Moreover, various values of C_1 and C_2 (see expressions (17)-(20)) correspond to various operation conditions. Analyzing expressions (17)-(20) in no-load conditions and short-circuit conditions, we can determine impedances:

$$Z_{11} = Z_{12} = \frac{\mu_0 Q_1}{n\Psi_1}\cdot\frac{\left(\frac{r_2}{r_1}\right)^n + \left(\frac{r_1}{r_2}\right)^n - 2}{\left(\frac{r_2}{r_1}\right)^n - \left(\frac{r_1}{r_2}\right)^n}\,, \tag{21}$$

$$Z_{13} = \frac{\mu_0 Q_1}{n\Psi_1}\cdot\frac{2}{\left(\frac{r_2}{r_1}\right)^n - \left(\frac{r_1}{r_2}\right)^n}\,. \tag{22}$$

The transcendental equation (6) has infinitely many roots. At the same time, various values of impedances Z_{11}, Z_{12} and Z_{13} correspond to various roots. If some root results in $Z_{13} >> Z_{11}$, then the

A-H-circuit becomes pointless and it is necessary to use another root in expressions (21) and (22).

3. The cascade equivalent *A-H*-circuit of the synchronous salient-pole electric machine

Figure 3 shows the cascade equivalent *A-H*-circuit of the synchronous salient-pole electric machine.

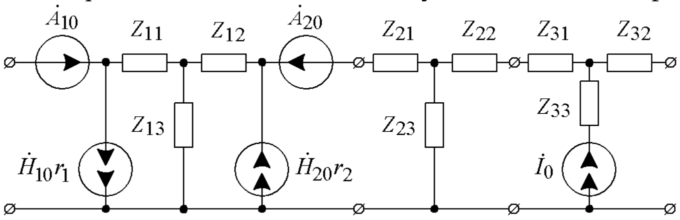

Figure 3. The *A-H*-circuit of the synchronous salient-pole electric machine.

The leftmost active cell (Z_{11}, Z_{12}, Z_{13}) corresponds to rotor poles. The passive cell (Z_{21}, Z_{22}, Z_{23}) corresponds to an air gap. Constants of this cell are given in [10]. For first field harmonic:

$$Z_{21} = Z_{22} = \frac{\mu_0}{p} \frac{\left(\frac{r_3}{r_2}\right)^p + \left(\frac{r_2}{r_3}\right)^p - 2}{\left(\frac{r_3}{r_2}\right)^p - \left(\frac{r_2}{r_3}\right)^p}, \tag{23}$$

$$Z_{23} = \frac{\mu_0}{p} \frac{2}{\left(\frac{r_3}{r_2}\right)^p - \left(\frac{r_2}{r_3}\right)^p}. \tag{24}$$

To take into account an air gap irregularity, it is reasonable to multiply an air gap by $\frac{k_\delta}{\alpha_\delta}$ where k_δ is Carter's factor, and α_δ is a pole overlap factor.

The third cell is a stator tooth-slot zone simulating armature reaction field. Constants of this cell are given in [10] too:

$$Z_{31} = \frac{\mu_0 b_2^2}{b_2^2 - b_1^2} \ln\frac{b_2}{b_1} - \frac{\mu_0}{2};\ Z_{33} = -\frac{\mu_0}{4}\frac{b_1^2 + b_2^2}{b_2^2 - b_1^2} + \frac{\mu_0 b_1^2 b_2^2}{\left(b_2^2 - b_1^2\right)^2} \ln\frac{b_2}{b_1}, \tag{25}$$

$$\dot{I}_0 = \dot{\delta} h \frac{b_1 + b_2}{4\pi} z_2, \tag{26}$$

where b_1 is a slot width on the air gap; b_2 is a slot width on the stator yoke; h is a tooth height; z_2 is a number of stator teeth; $\dot{\delta}$ is a current density in stator slots.

Both first (Z_{11}, Z_{12}, Z_{13}) and third (Z_{31}, Z_{32}, Z_{33}) cells have open terminals since yokes of a rotor and a stator have infinite permeability. Here a tangential component of a magnetic intensity vector is equal zero.

4. Test results

To verify the cascade equivalent circuit, tests were carried out: calculations by means of both the cascade equivalent circuit and numerical simulations (*FEMM* 4.2).

Calculation data are given. A number of poles is 8; a rotor diameter is 632 mm; a pole height is 83.4 mm; a pole relative permeability is 500; $\Theta = 2\beta$; an air gap is 4 mm; a number of stator teeth is 96; a tooth height is 53 mm; a slot width (on an air gap) is 11.11 mm. A slot width (on a stator yoke) is 14.67 mm; Carter's factor is 1.22; a pole overlap factor is 0.73; an exciting current density is 0.51 A/mm^2; a current density in stator slots (the first harmonic amplitude) is 2 A/mm^2; the armature reaction is quadrature-axis.

Figure 4 shows a normal component of a magnetic induction vector on the stator's surface (first field harmonic). A solid line corresponds to the cascade equivalent circuit and points correspond to the numerical simulation.

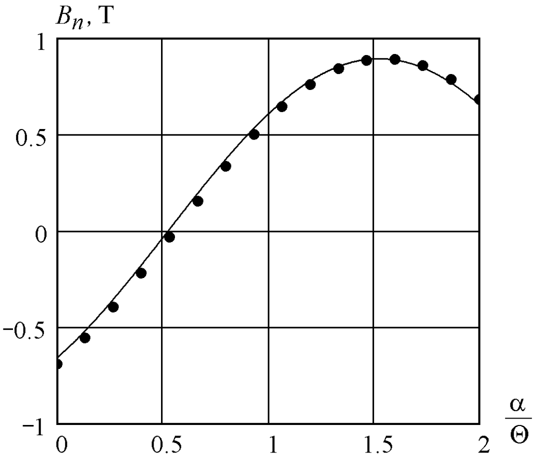

Figure 4. A normal component of a magnetic induction vector on the stator's surface (first field harmonic): a solid line corresponds to the cascade equivalent circuit; points correspond to the numerical simulation.

Calculations of the cascade equivalent *A*-*H*-circuit and the numerical simulation give identical results. That indicates correctness of modeling.

5. Conclusion

The author offers to use a cascade equivalent *A*-*H*-circuit of a synchronous salient-pole electric machine for modeling the electromagnetic field. This cascade equivalent *A*-*H*-circuit is synthesized by fundamentals of the electromagnetic theory and the circuit theory.

In the *A*-*H*-circuit, prismatic poles must be configured as sphenoid. Substituting sphenoid poles for prismatic, it is necessary keep a pole volume invariable.

In the transcendental equation, it is necessary to choose a root correctly to ensure correct values of impedances of the cascade equivalent circuit.

To take into account an air gap irregularity, it is reasonable to increase an air gap.

As follows from calculations, cascade equivalent circuit and numerical simulations give identical results that indicate correctness of modeling.

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