IOP Conference Series: Materials Science and Engineering

PAPER • OPEN ACCESS

Optimal design based on probability approach and fracture mechanics

To cite this article: A V Pitukhin and I G Skobtsov 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **560** 012085

Optimal design based on probability approach and fracture mechanics

A V Pitukhin and I G Skobtsov

Petrozavodsk State University, 33, Lenin Str., Petrozavodsk, 185910, Russia

E-mail: iskobtsov@mail.ru

Abstract. This paper deals with the statistical fracture mechanical method for the optimal design of forest machine components. The problem of optimal design includes the choice of the objective function and independent design variables and establishment of the system limits. The objective function is determined as the mean total cost that includes the initial cost and the cost of failure according to the failure probability. A quasi-brittle fracture occurs under the influence of the stationary random process of loading. Analytical equations of reliability function estimation are obtained. The algorithm of the random search method includes an interval reduction with regard to zone and functional constraints. The approach to the optimal design problem solution can be applied to obtain optimal geometrical sizes and permissible limit defect values of machine components.

1. Introduction

Forest machines failures are usually classified according to their causes in design defects, manufacture error failure and misuse failure**.** Cracks usually originate in the maximum stress action zone. Therefore, defects arise as a result of imperfection or violation of design rules, a manufacturing process or conditions of exploitation [1,2]. It is necessary to determine geometrical sizes and ultimate defect values of machine components during the design stage.

Optimal design of machine components is an important problem attracting interest of many experts in Russia and abroad [2–4]. The statement of the optimization problem includes definition of the system constrains, selection of the objective function and independent design variables (optimized parameters) [5]. In a number of situations, it is important to achieve maximum life of a machine part (reliability of the system). Therefore, it is better to choose an efficiency index considering both reliability during exploitation and manufacturing cost. To minimize the total expected costs, the single criterion design of the minimum total cost expenses is formulated as follows:

$$C_T(X^*) = \min_{X \in \Omega} C_T(X),$$

when the following conditions are held:

$X_{\min} \le X \le X_{\max}$ – constraint zones,

$F(X) \le 0$ – functional constraints,

where X^* is the optimal value of the design parameters vector X that minimizes the objective function; Ω is the tolerance range of vector *X*, including constraint zones and functional constraints.

The vector of design parameters X is given by:

$$X = \{x_1, \ldots, x_m, l\},$$

where x_j are optimized parameters;

l is the crack-like defect size (length).

We impose side constraints on crack-like defect sizes (lengths), depending on manufacturing processes. Functional constraints may be also imposed on some machine parameters such as mass, stress-strain properties, rigidity, etc.

Mean total expected costs are:

$$C_T(X) = C_1(X) + \sum_{i=1}^{n} Q_i \cdot C_{2i},$$

where C_1 is the initial cost, C_{2i} is the cost of failure and Q_i is the failure probability for the mode of failure i, n is the total number of possible modes of failure.

The aim of this study is to present the fracture mechanics method for the optimal design of forest machine components. Statistical fracture mechanics methods allow the estimate of the reliability function dependence on crack-like defect length.

2. Materials and methods

Evaluation of the reliability function in terms of fracture mechanics. The critical plane-strain fracture toughness K_{1C}, determined experimentally, is assumed as the material resistance index against the quasi-brittle fracturing under single loading [6].

The reliability function should be determined as the probability:

$$R = \Pr(K_{1C} - K_1 \geq 0),$$

where K_1 is the stress intensity factor.

We know [7, 8] that:

$$K_1 = Y(l) \cdot \sigma \cdot \sqrt{\pi \cdot l},$$

where $Y(l)$ is a dimensionless geometry factor, depending on the machine part shape and crack's length (semilength); σ is the maximum applied stress; l is the crack length (semilength).

By assuming that the stress on the machine part is a stationary random process $\sigma(t)$ with mean $\overline{\sigma}$ and spectral density $S_\sigma(\omega)$, the stress intensity factor can be formulated:

$$K_1(t) = Y(l) \cdot \sqrt{\pi \cdot l} \cdot \sigma(t).$$

It is evident that spectral density of the stress intensity factor:

$$S_K(\omega) = Y_I(l) \cdot \sqrt{\pi \cdot l} \cdot S_\sigma(\omega).$$

If $K_1(t)$ – a differentiable stochastic process:

$$\upsilon(t) = \frac{dK_I(t)}{dt}.$$

It is necessary to estimate the probability of the situation when the random process $K_1(t)$ exceeds the bounds of K_{1C} i.e. it will cross the given level (Figure 1).

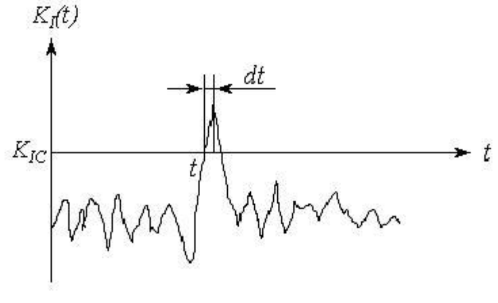

Figure 1. Sample of random process $K_1(t)$.

In general terms the mean number of K_{1C} level crossings (overshoots) during time τ [9]:

$$N_+(\tau) = \int_0^\tau P_{K_{IC}}(t)dt ,$$

where $P_{K_{IC}}(t)$ – temporary probability density. Temporary probability density $P_{K_{IC}}(t)$ presents the mean number of overshoots by the random stationary process $K_1(t)$ per time unit.

In case of a random stationary process ($P_{K_{IC}}(t) = P_{K_{IC}}$):

$$N_+(\tau) = \tau \cdot P_{K_{IC}} .$$

In case of Gaussian random stationary process:

$$N_+(\tau) = \tau \cdot \frac{1}{2\pi} \cdot \frac{\sigma_\upsilon}{\sigma_K} \cdot \exp\left(-\frac{(K_{IC} - \overline{K}_I)^2}{2\sigma_K^2} \right) = \tau \frac{\omega_e}{2\pi} \exp\left(-\frac{(K_{IC} - \overline{K}_I)^2}{2\sigma_K^2} \right),$$

where $\overline{K}_I$ – the random process mean value:

$$\overline{K}_I = Y_I(l) \cdot \sqrt{\pi \cdot l} \cdot \overline{\sigma} ;$$

σ_K^2 – the random process dispersion:

$$\sigma_K^2 = \int_0^\infty S_K(\omega) d\omega ;$$

σ_υ^2 – the random process rate of dispersion change;

ω_e – the effective frequency of the random process:

$$\omega_e = \left(\frac{\int_0^\infty \omega^2 S_K(\omega) d\omega}{\sigma_K^2} \right)^{1/2} ;$$

in this case:

$$P_{K_{IC}} = \frac{\omega_e}{2\pi} \cdot \exp\left(-\frac{(K_{IC} - \overline{K}_I)^2}{2\sigma_K^2} \right) = \frac{1}{T_e} \cdot \exp\left(-\frac{(K_{IC} - \overline{K}_I)^2}{2\sigma_K^2} \right).$$

If the mean number of overshoots $N_+(\tau)$ is rather low, we can consider overshoot appearances as statistically independent events following the Poisson distribution law [10]. Therefore, the reliability function is estimated under the condition that none overshoots occur during time τ :

$$R(\tau) = \exp(-N_+(\tau)) = \exp\left[-\int_0^\tau P_{K_{IC}}(t) dt \right],$$

In case of Gaussian random stationary process:

$$R(\tau) = \exp\left[-\tau \cdot \frac{\omega_e}{2\pi} \cdot \exp\left(-\frac{(K_{IC} - \overline{K}_I)^2}{2\sigma_K^2} \right) \right].$$

The failure probability:

$$Q(\tau) = 1 - R(\tau) = 1 - \exp\left[-\tau \cdot \frac{\omega_e}{2\pi} \cdot \exp\left(-\frac{(K_{IC} - \overline{K}_I)^2}{2\sigma_K^2} \right) \right].$$

3. Results and discussion

A great many algorithms of optimal problem solution are based on direct search methods. An algorithm of the random search method with an interval reduction can be used to solve the original problem [5]. Statistical modeling of the optimized parameters vector X is possible by using Monte Carlo technique simulation:

$$X_I^1 = \left(x_1^1; x_2^1; \ldots; x_m^1; l^I\right);$$
$$x_j^1 = x_{j\min}^1 + r_j \cdot \left(x_{j\max}^1 - x_{j\min}^1\right),$$

where r_j is a generated random number; $x_{j\max}^1, x_{j\min}^1$ are upper and lower bounds of optimized parameter x_j^1.

If the obtained vector X_I^1 is contained in the acceptable region Ω, including constraint zones and functional constraints, then the objective function value can be defined:

$$C_T^1 = C_T\left(X_I^1\right).$$

The optimization method allows performing computations in accordance with the given number of random tests *N* to obtain values of objective function C_T^2, …, C_T^N .

Therefore, the value X_I^* of the optimized parameters vector that minimizes the objective function $C_T^{I*} = C_T\left(X_I^*\right)$ is obtained. This value is entered for storage and is used as the starting point of the next set of tests; at the same time the boundaries of a new interval are reduced.

Using the same procedure, let us perform *k* test sets and obtain:

$$X_{II}^* \rightarrow C_T^{II*} = C_T\left(X_{II}^*\right); \ldots; \; X_k^* \rightarrow C_T^{k*} = C_T\left(X_k^*\right).$$

The number of test sets depends on the required accuracy of objective function calculations Δ:

$$\left|C_T\left(X_k^*\right) - C_T\left(X_{k-1}^*\right)\right| \le \Delta .$$

4. Conclusion

The proposed approach to the design problem enables us to obtain optimal geometrical sizes and maximum allowable defect values of machine components. The statistical optimization allows us to give recommendations for engineering of forest machine and equipment elements in terms of fracture mechanics.

Acknowledgments

This work was supported by the Strategic Development Program of Petrozavodsk State University (2017-2022).

References

[1] Anderson T L 2004 *Fracture Mechanics: Fundamentals and Applications* (CRC: Press-Book)
[2] Pitukhin A V 1997 *Computers and Structures* **65 (4)** 621-624
[3] Pitukhin A V and Skobtsov I G 2015 *Appl. Mechan. and Mater.* **709** 530-533
[4] Banichuk N V, Ragnedda F and Serra M 2003 *Mechan. Based Des. of Struct. and Machin.* **31(4)** 459–474
[5] Ravindran A, Ragsdell K M and Reklaitis G V 2006 *Eng. Optimiz. Meth. and Applic.* (Hoboken: John Wiley & Sons)
[6] Zhu X and Joyce J A 2012 *Eng. Fract. Mechan.* **85** 1-46
[7] Irwin G R 1948 *Fracturing of Metals* 147-166
[8] Broek D 1982 *Elementary Eng. Fracture Mechan.* (Martinus Nijhoff Publishers: The Hague)
[9] Rise S O 1945 *Bell System Tech. J.* **24** 46-156
[10] Sveshnikov A A 1966 *Appl. Methods of the Theory of Random Funct.* (New York: Pergamon Press)