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Development of mathematical models for calculating statics and dynamics of membrane sensitive elements of microelectromechanical control systems

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Abstract. The article is devoted to the development of mathematical models for calculating the statics and dynamics of membrane sensitive elements of microelectromechanical control systems. The nonlinear analysis of membrane sensitive elements and the finite element method are considered. The analysis of various options for membrane elastic sensitive elements of pressure sensors and their theoretical models. The method of numerical simulation, the method of linear analysis of static deformation and the comparison of the results of a theoretical calculation of the deformation profiles with experimental values are used. A method for calculating statics and dynamics, membrane and beam microelectromechanical control systems provides a high degree of adequacy with real stress-strain states in the structures of membrane and beam elastic microelectromechanical control systems, as well as reducing the complexity of engineering calculations.

1. Introduction

The relevance and development prospects of domestic microelectromechanical devices (MMD) are directly related to the development of devices with small mass size, low power consumption, and cost, with an unconditional high level of reliability and stability of the output characteristics. As can be seen from studies [1-3], the development and calculation of models of statics and dynamics of microelectromechanical systems (MEMS) are relevant at the present time. Unfortunately, at present, the domestic MEMS market is mainly represented by import sensors. This indicates not only the absence of the mass consumer of these products, but the absence of a well-developed and proven production technology and the design of MEMS in our country.

Much work is being done in the search for materials used in the production of MEMS, but semiconductor monocrystalline silicon is still the main material. MEMS is based on microelectronics materials and technologies, which allow forming miniature devices on a single chip according to a single group technology. All this allows in a single approach to create sensitive elements and electrical components of MEMS. The stability of the physicochemical properties of monocrystalline silicon when selecting a constructive material when creating MEMS provides a qualitatively new level and high stability of the output characteristics of microdevices. The most popular mechanical devices manufactured by MEMS technology are capacitive and piezoelectric micromechanical accelerometers (MMA).

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2. Materials and methods

Sensitive elements (SE), executed by the technology of volumetric micromechanics before SE, implemented using the surface microprocessing technology, have a number of significant advantages, namely: high metrological and operational characteristics, which is associated with a larger value of inertial mass when using the technology of volume microscopy.

SE piezoelectric MMA compared with capacitive MMA have a more complex manufacturing process, which is associated with the need to create a piezoresistor directly on the surface of elastic suspensions. As mentioned earlier, the output characteristics of MEMS devices are directly related to the technological features of obtaining single crystal MMA [4].

Linear analysis of static deformation, currently widespread in the practice of engineering calculations of thin-walled structures, is suitable only for approximate estimates of their performance and is valid in the immediate vicinity of the initial state, that is, at small values of displacements and deformations of the elastic sensitive element (ESE). This is due both to the nonlinearity of the characteristics of structural materials and to the change in the metric characteristics of the structure itself in the process of deformation. Accounting for these factors is the subject of nonlinear analysis.

The rapidly growing interest in nonlinear analysis of aviation, thin-walled complex structures led to the development of both effective methods for modeling structures and numerical methods for solving systems of nonlinear equations, developing cost-effective algorithms, and so on, that is, creating powerful tools for nonlinear analysis.

Systems where the application of nonlinear analysis becomes extremely important are primarily ESE. The complex conditions of their loading, high levels of stress require a high degree of reliability in assessing the conditions for the safe operation of these critical elements of automated control systems. The desire for weight optimization leads to the use of materials with increased strength, which in turn stimulates the creation of thin-walled structures, in which the changes in their geometry under the action of operational loads cannot be ignored.

An equally important factor is taking into account the influence of non-linearity of deformation of structural materials, since it is only in the interconnection of these factors that sufficiently reliable estimates are obtained.

Thus, it is the thin-walled structures that constitute the class of tasks for which the development of non-linear analysis methods, taking into account the mutual influence of large displacements and non-linearity of structural materials, is of decisive importance [5].

The main consideration in the design calculation is given to the following categories of nonlinearity:

- nonlinearity determined by finite displacements, that is, caused by nonlinearity of the deformation coupling — displacement;
- nonlinearity, determined by the nonlinearity of the connection between strains and stresses, that is, the physical nonlinearity of materials;
- nonlinearity of the mechanical characteristics of the interactions of elements in the structure, that is, structural nonlinearity;
- combination of different categories of nonlinearity.

The main directions for the development of nonlinear analysis are currently being developed in two directions: the improvement of computational models of complex systems in order to ensure their adequacy to real ESE processes and the development of efficient analysis algorithms with an acceptable expenditure of funds and time.

The modern theory of structural analysis based on the finite element method (FEM) has been developed to such an extent that it can be effectively applied to solving very complex non-linear problems. It is based on the developed theory of elasticity, proven methods for discretization of structures, effective numerical methods for the formation and solution of large systems of nonlinear equations. However, the complexity of nonlinear analysis, which is based on multi-step or iterative algorithms, is incommensurably higher than the complexity of linear analysis [6].

The analysis of nonlinear systems is represented as a continuous process of modeling. Therefore, it is very important that each element of the design model is either located in conditions similar to those of the original system, or each inaccuracy made in the simulation results in an unpredictable accumulation of errors.

Since all solutions of nonlinear analysis are based on various approximations, it is necessary to put up with the inevitable error of each step and its accumulation in the counting process. Therefore, quite often it is necessary to use some heuristic methods for developing a solution strategy and developing specific procedures [7].

The basis of modern achievements of nonlinear analysis is the accumulated experience of rational modeling of complex structures [8]. When calculating complex structures, an important element of the analysis is the comparison of the results obtained using various models, methods and algorithms. It is the complexity of the analysis that becomes the determining factor that gives confidence in the reliability of the result obtained.

The use of modern numerical methods [9-11], in particular, the FEM, to the analysis of small static deformations allows us to naturally reduce the problem of studying elastic elastics of an elastic sensitive element to solving systems of linear algebraic equations of the form:

$$[K]\{U\} = \{Q\}, \tag{1}$$

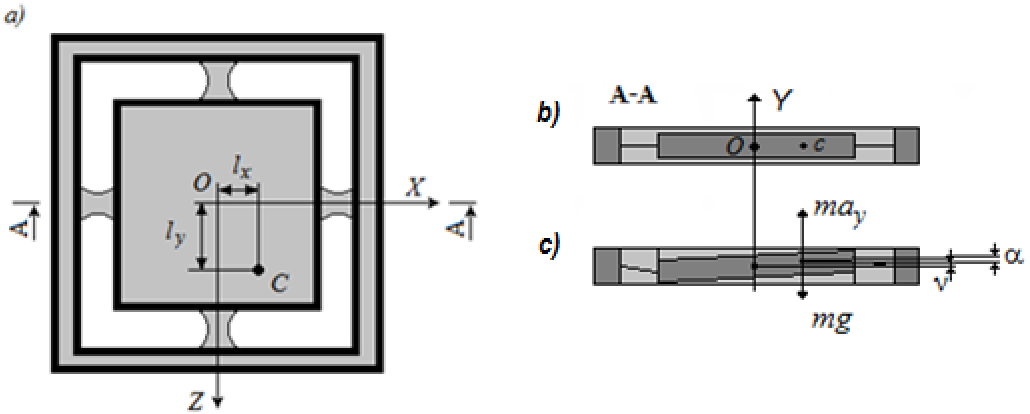
where [K] – stiffness matrix; {U} – displacement vector; {Q} – static load vector.

The problem of nonlinear static analysis of deformable ESE is currently relevant, since the accuracy of the designed sensors and switching devices depends on it.

The assessment of the accuracy of the obtained decisions, which is necessary for making a decision on the continuation or termination of the iterative procedure for finding a solution, can be conducted either directly on the displacement vector or on the residual vector. Good results were obtained when recording the criteria evaluation in the form of:

$$\Delta = \frac{\sum_{r=1}^{m}\left|q_r^{j+1} - q_r^{j}\right|}{\sum_{r=1}^{m}\left|q_r^{j}\right|} \le \varepsilon\,, \tag{2}$$

q_r – r-displacement vector component {U}; m – the number of components of the displacement vector; j – iteration number; ε – small value determined by the required accuracy of the solution. When solving applied problems, the value of ε is recommended to choose in the range ε < (0.001…0.0001).



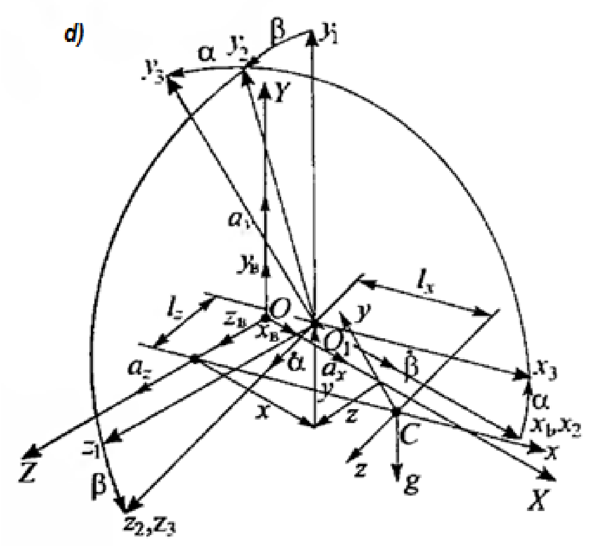


Figure 1. The location of the axes relative to the silicon wafer. a, b – location of the center of mass of the membrane silicon wafer; c – arrangement of single-crystal plates under the effect of acceleration; d – coordinate axes.

We assume that the suspension stiffness of the axial MMA (figure 1a) in *x*, *z* coordinates is significantly greater than the stiffness in *y*, α, β coordinates. For this case, the *y* axis is the axis of sensitivity [1].

$$\left.\begin{aligned} & m\ddot{y} + \kappa_{as}\dot{y} + G_y y + m(l_x\ddot{\alpha} + l_z\ddot{\beta}) = F - F_{By}; \\ & J_\beta\ddot{\beta} + \kappa_{d\beta}\dot{\beta} + G_\beta\beta + ml_z(\ddot{y} + l_x\ddot{\alpha}) = -(F + F_{By})l_z - Fz_B + F_{Bz}l_z\beta; \\ & J_\alpha\ddot{\alpha} + \kappa_{d\alpha}\dot{\alpha} + G_\alpha\alpha + ml_x(\ddot{y} + l_z\ddot{\beta}) = (F - F_{By})l_x + Fx_B + F_{Bx}l_x\alpha; \end{aligned}\right\} \tag{3}$$

$$\mathrm{y_c} = \mathrm{y} + \mathrm{l_x}\alpha + \mathrm{l_z}\beta + \mathrm{y_B} \tag{4}$$

where $m(a_y\text{-}g)=F$, $m\ddot{y}_B = F_{By}$, $m\ddot{x}_B = F_{Bx}$, $m\ddot{z}_B = F_{Bz}$; *m* – mass; G_y, G_α, G_β – stiffness, by coordinates y, α, β respectively; $\kappa_{d\alpha}$, $\kappa_{d\beta}$ – damping coefficient in linear and angular coordinates; l_x, l_z – displacement along the x and *z* axes (figure 1a); $J_\beta = J_x + ml_z^2$, $J_\alpha = J_z + ml_x^2$ – moments of inertia of the plate relative to the axes O_1z_1 and O_1x_1, respectively; x_B, y_B, z_B – vibration displacement vectors; α- linear coordinate of the center of the plate angular oscillations.

Figure 2 shows a pressure micro-gauge with a rigid center.

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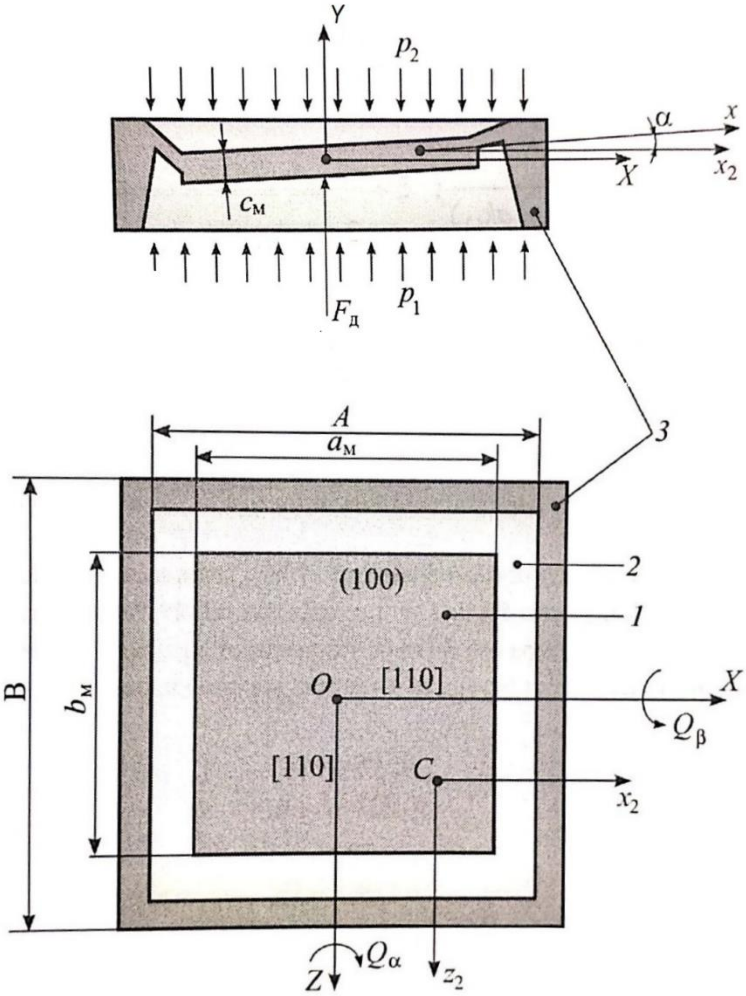

Figure 2. The derivation of the equations of motion of the sensitive element of the pressure microsensor. 1 – hard center; 2 – elastic jumper; 3 – case plate.

The equation of motion of the dynamics of displacement of the center of mass of the microsensor pressure is described by the expression:

$$\left.\begin{aligned}
&m\ddot{y}+\kappa_{dy}\dot{y}+G_y y+m(l_x\ddot{\alpha}+l_z\ddot{\beta})=m(u-g-\ddot{y}_B)+F_d;\\
&J_\beta\ddot{\beta}+\kappa_{d\beta}\dot{\beta}+G_\beta\beta+ml_z(\ddot{y}+l_x\ddot{\alpha})=ml_z(g-u-\ddot{y}_B)+mz_B(g-u)+ml_z\ddot{z}_B\beta;\\
&J_\alpha\ddot{\alpha}+\kappa_{d\alpha}\dot{\beta}+G_\alpha\alpha+ml_x(\ddot{y}+l_z\ddot{\beta})=ml_x(u-g-\ddot{y}_B)+mx_B(u-g)+ml_x\ddot{z}_B\alpha;\\
&y_c=y+l_x\alpha+l_z\beta+y_B.
\end{aligned}\right\} \quad (5)$$

where p_1, p_2 – pressure; F_d – pressure force; g, u – acceleration; $\ddot{y}_B$ – vibration acceleration.

Figure 3 shows a rotary microgyroscope.

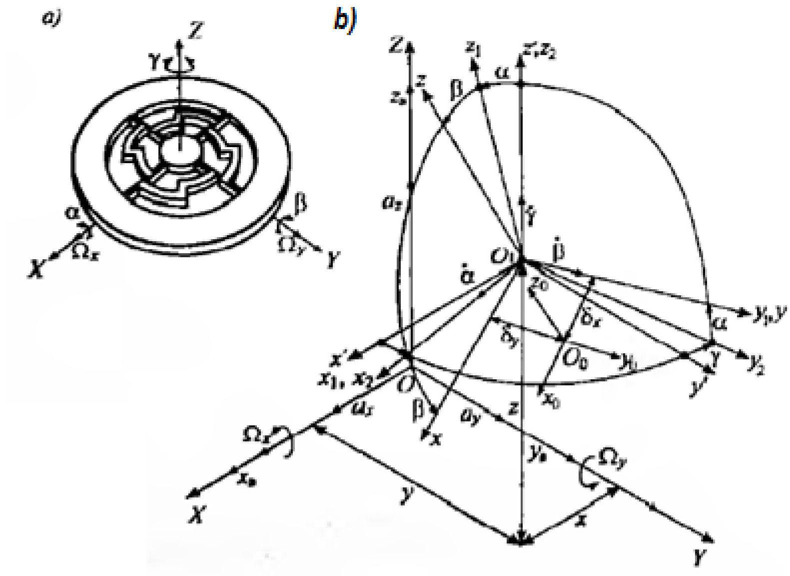

Figure 3. The derivation of the equations of motion of micro gyroscopes: a –kinematic scheme; b – coordinate systems.

The generalized equations of motion of MMG are:

$$\left.\begin{aligned}
&m\ddot{y}+\kappa_{dy}\dot{y}+G_y y+m(l_x\ddot{\alpha}+l_z\ddot{\beta})=m(u-g-\ddot{y}_B)+F_d;\\
&J_\beta\ddot{\beta}+\kappa_{d\beta}\dot{\beta}+G_\beta\beta+ml_z(\ddot{y}+l_x\ddot{\alpha})=ml_z(g-u-\ddot{y}_B)+mz_B(g-u)+ml_z\ddot{z}_B\beta;\\
&J_\alpha\ddot{\alpha}+\kappa_{d\alpha}\dot{\beta}+G_\alpha\alpha+ml_x(\ddot{y}+l_z\ddot{\beta})=ml_x(u-g-\ddot{y}_B)+mx_B(u-g)+ml_x\ddot{z}_B\alpha;\\
&y_c=y+l_x\alpha+l_z\beta+y_B.
\end{aligned}\right\}\quad(6)$$

where b_x, b_y, b_z – damping coefficients for the frame during its movements along generalized coordinates x, y, z; $\ddot{x}_B$, $\ddot{y}_B$ – linear vibration of the base relative to the axis x.

Increasing demands on the reliability and stability of the metrological characteristics of elastic sensitive elements determine the relevance of the issue of ensuring the improvement of the quality of MMD membrane and beam sensitive elements. To solve this problem, there is a need to create new modifications of the known computational and experimental methods for studying the static and dynamic characteristics of this type of elastic sensing element. On the basis of such improved modified methods, it is possible to develop new domestic competitive sensitive elements created by microelectromechanical devices.

3. Conclusion

On the basis of the conducted research the following tasks were solved:

- analysis of various variants of membrane elastic sensitive elements of pressure sensors and their theoretical models;
- checking the adequacy of the developed mathematical models of statics and dynamics of plates and shells that have a hard seal at the edges and experiencing concentrated loads with the results of calculating the basic performance characteristics of sensitive elements of real DMD with distribution parameters was carried out using the finite element method;

- checking the adequacy of the applied numerical simulation when comparing the results of the theoretical calculation of the deformation profiles with experimental values;
- submission of applications for patenting of a new membrane element base of sensors with improved metrological characteristics and enhanced reliability indicators.

The results of the analysis of the statics and dynamics of the elastic sensitive element MMA are based on the system approach and use the numerical finite element method, which is well suited to the study of thin-walled elements. When developing a mathematical model of an elastic sensitive element in a static mode, the methods of probability theory were used.

The results obtained in the work are ensured by the rigor of the tasks set and the mathematical methods used, control of the convergence of approximate solutions and analytical comparisons, where possible, with empirical data and results obtained in other scientific sources, as well as passing examinations in the Russian Agency for Patents and Trademarks and positive decisions on issuing patents for two utility models No. 181216 and No. 181219.

The developed method of calculating statics and dynamics, membrane and beam MEMS control systems provides a high degree of adequacy with real stress-strain states in the structures of the membrane and beam elastic MEMS control systems, as well as reducing the complexity of engineering calculations. The technique is designed for a wide range of specialists in this field.

References

[1] Sun X, Keating A, Parish G 2018 Micromachined microbeams made from porous silicom for dimanic and staticmode sensing sensors and actuators. *Physical* **269** 91-98
[2] Collin J 2015 MEMS IMU carouselling for ground vehicles. *Institute of Electrical and Electronics Engineers* **64 (6)** 2242-2251
[3] Maltsev V N, Nesterenko A A 2016 Investigation of static and dinamic stability of the deformable dynamic spiral magnetic domain. *Phys. of Metals and Metallogr.* **117(3)** 222-229
[4] Lobontiu N 2007 Dynamics of Microelectromechanical Systems Springer US 403
[5] Raspopov V Y 2007 *Micromechanical devices* (Moscow: Mechanical Engineering) 400
[6] Kalinkina M E, Kozlov A S, Labkovskaia R Y, Pirozhnikova O I, Tkalich V L 2018 *14th Int. Sci.-Techn. Conf. on Actual Probl. of Electronic Instrument Eng.* **1(6)** 272-276
[7] Kalinkina M E, Kozlov A S, Labkovskaia R I, Pirozhnikova O I, Tkalich V L, Shmakov N A 2018 Development library of finite elements for computer-aided design system of reed sensors. *J. of Phys.: Conf. Ser.* **1015** 032080
[8] Kozlov A S, Korobeynikov A G, Labkovskaya R I, Pirozhnikova O I 2014 *Cybernetics and programming* **5** 70-77
[9] Bibikov SV, Kalinichenko SV, Obukhov AV 2017 *News of Tula State University. Technical science* **2**(12) 380-391
[10] Maithripala D H S, Jordan M B, Dayawansa W P 2004 Control of an Electrostatic MEMS Using Static and Dynamic Output Feedback. *J. of Dynamic Syst. Measur. and Contr.* **127(3)** 443-450
[11] Manuel P P, Manuel P M, Jose A, Berna G 2009 Chaotic dynamic and control for micro-electro-mechanical systems of massive storage with harmonic base excitation. *Chaos Solitons & Fractals* **39(3)** 1356–1370