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Mathematical description of stress-strain state of trunnion of ball mill taking into account temperature field

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Abstract. The article considers the trunnion of a ball mill in the framework of theory of elasticity, which is subjected to an uneven thermal impact due to the heating of the load. Equations describing the radial displacement of the point inside the trunnion of a ball mill are obtained. The equations describing the movement of trunnion points of the ball mill are derived. The stress-strain state of the trunnion of a ball mill has been determined depending on the temperature of inner and outer surfaces of the trunnion. The deformations and stresses arising on the surface of the trunnion are investigated by numerical methods.

1. Introduction

In the framework of theory of elasticity trunnion of a ball mill is considered, which due to heating of load is subjected to an uneven thermal effect on the volume. As a result, a temperature field arises, which in turn will entail occurrence of thermal deformations and stresses.

The main cause of the stress-strain state of the trunnion of a ball mill is a temperature field, which is uniform along the axis of a ball mill, but changes in the radial direction. As a result we assume that the changes in temperature T in the trunnion of a ball mill take place only in the radial direction [1]:

$$T = T(r), \tag{1}$$

where r varies in the following limits: $R_1 \leq r \leq R_2$ (R_1 is the inner radius of the trunnion of a ball mill, and R_2 is the outer radius of the trunnion of a ball mill).

In case of an axial symmetry a cylindrical coordinate system (r, φ, z) is introduced according to the design scheme presented in Figure 1. The trunnion of a ball mill will be considered as a thick-walled cylindrical tube with a radial temperature change (1).

Neglecting the influence of ends, it can be assumed that the cross-sections that are perpendicular to the OZ axis are in equal conditions and flat.

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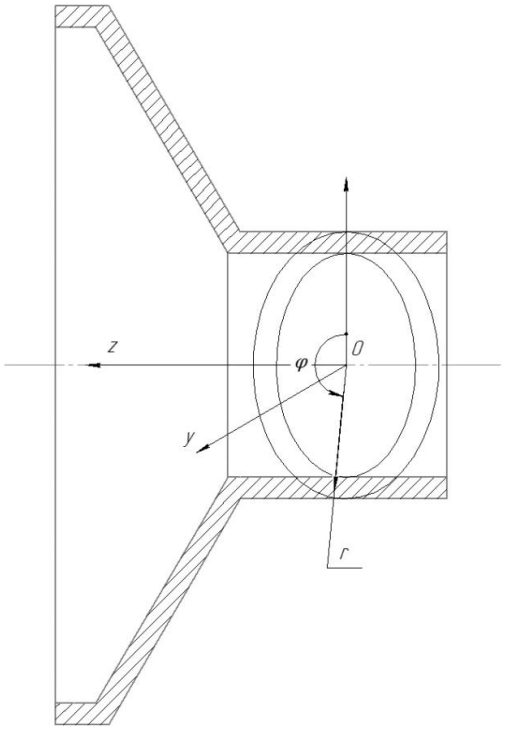


Figure 1. The calculation scheme for choosing a coordinate system.

Based on the assumptions made, it can be concluded that the radial movements of point *U* in the trunnion of a ball mill depend only on one coordinate *r*, the movement in the azimuth direction is absent, and the relative elongation in the direction of the axis *OZ* is constant:

$$\varepsilon_z = const. \tag{2}$$

In turn, the relative elongations in radial direction:

$$\varepsilon_r = \frac{dU}{dr}, \tag{3}$$

in azimuthal direction:

$$\varepsilon_\varphi = \frac{U}{r}. \tag{4}$$

The physical equations describing the stress field in cylindrical coordinates will have the following form [2]:

$$\sigma_z = 2G\varepsilon_z + \lambda\varphi_0 - \eta T(r), \tag{5}$$
$$\sigma_r = 2G\varepsilon_r + \lambda\varphi_0 - \eta T(r), \tag{6}$$
$$\sigma_\varphi = 2G\varepsilon_\varphi + \lambda\varphi_0 - \eta T(r), \tag{7}$$

where $\sigma_z, \sigma_r, \sigma_\varphi$ – respectively, the diagonal components of the stress tender. For brevity, the following notation is entered:

$$\varphi_0 = \varepsilon_z + \varepsilon_r + \varepsilon_\varphi, \tag{8}$$
$$\eta = 2G\alpha\frac{1+\nu}{1-2\nu}, \tag{9}$$
$$\lambda = \frac{2\nu G}{1-2\nu}, \tag{10}$$
$$G = \frac{E}{2(1+\nu)}. \tag{11}$$

where E – elastic modulus; ν – Poisson's ratio; α – the linear thermal expansion coefficient.

Substituting (3), (4) into (6) and (7) allows us to obtain the following relations: elasticity modulus;

$$\sigma_r = 2G\frac{dU}{dr} + \lambda\varphi_0 - \eta T(r), \tag{12}$$
$$\sigma_\varphi = 2G\frac{U}{r} + \lambda\varphi_0 - \eta T(r). \tag{13}$$

Let us suppose that, as in the case of solving the problem of a thick-walled ring [1], the following

equilibrium condition must be satisfied:

$$\sigma_r - \sigma_\varphi + r\frac{d\sigma_r}{dr} = 0. \quad (14)$$

Thus, the expressions (12)-(14) are the initial ones for obtaining an equation describing the radial displacement of a point inside the trunnion of a ball mill.

2. Derivation of an equation describing the movement of the trunnion points of a ball mill

To obtain the equation describing the displacement of the point U inside the trunnion of a ball mill under the action of the stress field (5), (12) and (13), let us find the derivative of (12) along the coordinate r [3]:

$$\frac{d\sigma_r}{dr} = 2G\frac{d^2U}{dr^2} + \lambda\frac{d\varphi_0}{dr} - \eta\frac{dT}{dr}. \quad (15)$$

According to (8), taking into account (2), let us calculate:

$$\frac{d\varphi_0}{dr} = \frac{d\varepsilon_r}{dr} + \frac{d\varepsilon_\varphi}{dr}. \quad (16)$$

Substituting (3) and (4) in (16) gives:

$$\frac{dU}{dr} = \frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \frac{U}{r^2}. \quad (17)$$

Based on (12) and (13), we can find:

$$\sigma_r - \sigma_\varphi = 2G\left(\frac{dU}{dr} - \frac{U}{r}\right). \quad (18)$$

Substitution (15), (17) and (18) in equilibrium conditions (14) leads to the following result:

$$2G\left(\frac{dU}{dr} - \frac{U}{r}\right) + 2Gr\frac{d^2U}{dr^2} + \lambda\left(\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \frac{U}{r^2}\right) - \eta\frac{dT}{dr}r. \quad (19)$$

In view of (9) and (10), equation (19) takes the form:

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \frac{U}{r^2} + \frac{\nu}{1-2\nu}\left(\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \frac{U}{r^2}\right) = \alpha\frac{1+\nu}{1-\nu}\frac{dT}{dr}. \quad 20)$$

Equation (20) finally can be rewritten in the following form:

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \frac{U}{r^2} = \alpha\frac{1+\nu}{1-\nu}\frac{dT}{dr}. \quad (21)$$

From a mathematical point of view, relation (21) is the inhomogeneous Euler differential equation. To find the solution (21), we use the variation method of arbitrary constants. For this, according to the method chosen, it is first necessary to find a solution to the homogeneous equation $U_0(r)$:

$$\frac{d^2U_0}{dr^2} + \frac{1}{r}\frac{dU_0}{dr} - \frac{U_0}{r^2} = 0. \quad (22)$$

Equation (22) can be reduced to a homogeneous differential equation of the second order with constant coefficients using the following replacement:

$$\rho = \ln r. \quad (23)$$

Let us calculate:

$$\frac{dU_0}{dr} = \frac{dU}{d\rho}\cdot\frac{d\rho}{dr} = \frac{dU_0}{d\rho}\cdot\frac{1}{r} = e^{-\rho}\cdot\frac{dU_0}{d\rho}. \quad (24)$$

$$\frac{d^2U_0}{dr^2} = \frac{d}{d\rho}\left(e^{-\rho}\cdot\frac{dU_0}{d\rho}\right)\cdot e^{-\rho} = e^{-2\rho}\frac{d^2U_0}{d\rho^2} - e^{-2\rho}\frac{dU_0}{d\rho}. \quad (25)$$

Substituting (24) and (25) into (22) allows us obtaining the following equations [4]:

$$\frac{d^2U_0}{d\rho^2} - U_0 = 0. \quad (26)$$

The solution of the differential equation (26) is:

$$U_0 = C_1e^{-\rho} + C_2e^{-\rho}, \quad (27)$$

or taking into account (23) the formula (27) is transformed:

$$U_0 = \frac{C_1}{r} + C_2r. \quad (28)$$

According to the method of variation of arbitrary constants, the desired solution of the inhomogeneous differential equation (21) has the form:

$$U(r) = \frac{C_1(r)}{r} + C_2(r)\cdot r. \quad (29)$$

Differentiation (29) for r allows obtaining the relation:

$$\frac{dU}{dr} = -\frac{C_1(r)}{r^2} + C_2(r) + \frac{dC_1(r)}{dr} \cdot \frac{1}{r} + r \cdot \frac{dC_2(r)}{dr}. \quad (30)$$

According to the method of variation of arbitrary constants, the last two terms in (30) should be equated to zero:

$$\frac{dC_1(r)}{dr} \cdot \frac{1}{r} + r \cdot \frac{dC_2(r)}{dr} = 0. \quad (31)$$

In view of (31), find the derivative of (30):

$$\frac{d^2U}{dr^2} = -\frac{1}{r^2} \cdot \frac{dC_1(r)}{dr} + \frac{2C_1(r)}{r^3} + \frac{dC_2(r)}{dr}. \quad (32)$$

Substituting (30), (31) and (32) into the original equation (21) leads to the following result:

$$-\frac{1}{r^2} \cdot \frac{dC_1(r)}{dr} + \frac{dC_2(r)}{dr} = 0. \quad (33)$$

Based on relations (31) and (33), we find:

$$\frac{dC_1(r)}{dr} = -\frac{\alpha}{2} \cdot \frac{1+\nu}{1-\nu} \cdot r^2 \frac{dT(r)}{dr}, \quad (34)$$

$$2 \cdot \frac{dC_2(r)}{dr} = \alpha \frac{1+\nu}{1-\nu} \cdot \frac{dT(r)}{dr}, \quad (35)$$

The integration of differential relations (34) and (35) leads to the following relations:

$$C_1(r) = -\frac{\alpha}{2} \cdot \frac{1+\nu}{1-\nu} \cdot \int r^2 \frac{dT}{dr} dr + C_{10}, \quad (36)$$

$$C_2(r) = \frac{1}{2}\alpha \cdot \frac{1+\nu}{1-\nu} T(r) + C_{20}, \quad (37)$$

here C_{10} and C_{20} – the arbitrary constants that can be determined using the initial conditions.

We transform the formula (36) by completing the integration in parts:

$$C_1(r) = -\frac{\alpha}{2} \cdot \frac{1+\nu}{1-\nu} \cdot r^2 T(r) + \frac{\alpha}{2} \cdot 2 \cdot \frac{1+\nu}{1-\nu} \int rT(r) dr + C_{10}. \quad (38)$$

Substituting (37) and (38) into (29) allows us obtaining the following expression, which determines the solution of the equation (21):

$$U(r) = \frac{C_{10}}{r} + C_{20} \cdot r + \frac{\alpha}{r} \cdot \frac{1+\nu}{1-\nu} \int rT(r) dr. \quad (39)$$

According to the result of [1], if the constant temperatures T_{R_1} and T_{R_2} are maintained on the inner and outer surfaces of the thick-walled pipe, then for such steady flow temperature distribution over the wall thickness the following formula takes place:

$$T(r) = \frac{T_{R_1} \ln\frac{R_2}{r} + T_{R_2} \ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}. \quad (40)$$

Based on (40), the solution of the equation (21), (39) can be conveniently represented as an integral with a variable upper limit [5]:

$$U(r) = \frac{C_{10}}{r} + C_{20} \cdot r + \frac{\alpha}{r} \cdot \frac{1+\nu}{1-\nu} \int_{R_1}^{r} xT(\alpha) dx. \quad (41)$$

To determine the arbitrary constants C_{10} and C_{20}, it is necessary to use the boundary conditions that are superimposed on the stress values acting on the internal R_1 and external R_2 radii of the mill trunnion:

$$\sigma_r(r = R_1) = \sigma_0, \quad (42)$$

$$\sigma_r(r = R_2) = 0, \quad (43)$$

where σ_0 – the stress value experienced by the internal radius of the mill pin under the load.

The efforts exerted by the faraway loading *F* in a pipe ball mill 1 cm long, which were defined in [6] by the graphic-analytical method, are presented in figure 2.

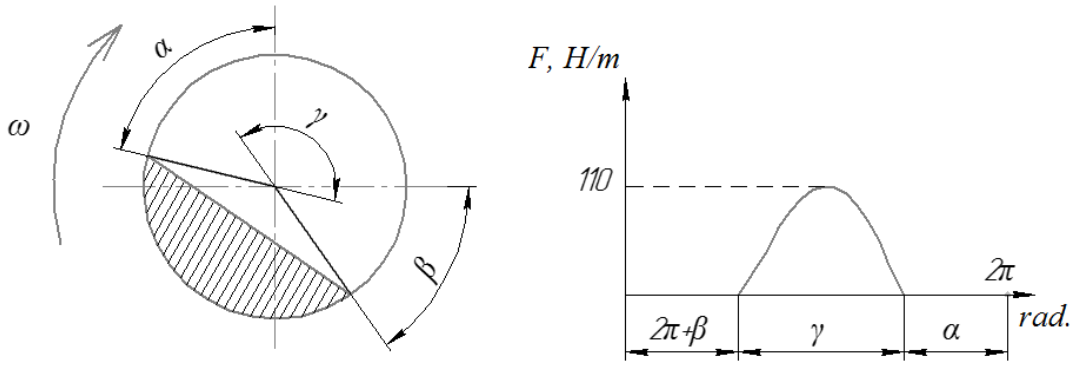


Figure 2. The calculation scheme for estimating the load force exerted on the inner surface of the radius R_1 of the trunnion of a ball mill during cascade operation.

According to the design scheme presented in Figure 2, the volume of the ball-bearing load affects the inner surface of the trunnion of a ball mill when turning through the angle γ, the value of which with the load factor, according to the reference data [6], is related by:

$$\psi = \frac{\gamma}{206^\circ} - 0.374, \tag{44}$$

here the angle γ is given in degrees.

In the transition to a radial measure, the relation (44) can be brought to the form:

$$\psi = 0.278\gamma - 0.374. \tag{45}$$

Based on (45), we find the change in the angular size of the ball-bearing load of the trunnion of a ball mill from the load factor:

$$\gamma = 3.594\psi + 1.344. \tag{46}$$

The obtained expressions (45) and (46) are valid within$1.74 < \gamma < 4.54$.

According to (46), the time Δt, during which the inner surface of the trunnion of the ball mill is subjected to a load, is:

$$\Delta t = \frac{\gamma}{\omega}, \tag{47}$$

where ω – spindle speed.

Taking into account (46), the formula (47) takes the form:

$$\Delta t = \frac{3.594\psi + 1.344}{\omega}. \tag{48}$$

The time of one complete trunnion of a ball mill is equal to:

$$T = \frac{2\pi}{\omega}. \tag{49}$$

Based on (48) and (49) find:

$$\frac{\Delta t}{T} = \frac{3.594\psi + 1.344}{2\pi}. \tag{50}$$

When substituting into (50) the working value of the load factor $\psi = 0.4$, we find:

$$\Delta t = 0.44T. \tag{51}$$

Thus, according to (51), the residence time of the material in the inner part of the ball mill trunnion with the transverse angular size γ under the action of the force of the ball charge is 0.44 from the time of the complete rotation.

The potential energy of deformation of the inner part of the trunnion of a ball mill under the influence of ball-bearing loads accumulates over time $\Delta t/2$ (an increasing part of the graph in Fig. 2). And over time $\Delta t/2$ in the process of unloading, the forces in a reversible form are released, turning into the thermal energy. Therefore, most of the time at 0.56T, the inner part of the trunnion of a ball mill remains unloaded.

On the basis of what was said in the relation (42), the value of σ_0 can be taken equal to zero and therefore:

$$\sigma_r(r = R_1) = 0. \tag{52}$$

3. Determination of the stress-strain state of the trunnion of a ball mill

To determine the constant values ε_z, C_{10} and C_{20}, we must perform the following operations:

- substitute (40) into (41);
- use boundary conditions (43) and (52);
- use the condition of the absence of axial load of a ball mill, which can be written in the following form [7]:

$$2\pi \int_{R_1}^{R_2} \sigma_z r dr = 0. \tag{53}$$

Substituting (40) into (41) gives the following result:

$$U = \frac{C_{10}}{r} + C_{20} \cdot r - \frac{1}{4r(1-\nu)\ln\left({R_2}/{R_1}\right)} \cdot \alpha(1+\nu)(2R_1^2 T_{R_1} \ln\left({R_2}/{R_1}\right) +$$
$$+R_1^2\left(T_{R_1} - T_{R_2}\right) - 2T_{R_1} r^2 \ln\left({R_2}/{r}\right) - r^2\left(T_{R_1} - T_{R_2}\right) - 2T_{R_2} r^2 \ln\left({r}/{R_1}\right). \tag{54}$$

Substituting the resulting equations into (5), (12), (13), taking into account (8) - (10), allows us obtaining a system of two equations for unknown quantities C_{10} and C_{20}. The solution of this system is:

$$C_{10} = \frac{\alpha R_1^2(1+\nu)}{4(1-\nu)}\left[\frac{2\left(R_1^2 T_{R_1} - R_2^2 T_{R_2}\right)}{R_2^2 - R_1^2} - \frac{T_{R_2} - T_{R_1}}{\ln\left({R_2}/{R_1}\right)}\right], \tag{55}$$

$$C_{20} = \frac{\alpha}{2} \cdot \frac{(1-3\nu)}{(1-\nu)}\left[\frac{R_2^2 T_{R_2} - R_1^2 T_{R_1}}{R_2^2 - R_1^2} - \frac{T_{R_1} - T_{R_2}}{2\ln\left({R_2}/{R_1}\right)}\right]. \tag{56}$$

From here, the following relationships are obtained:

$$\sigma_r = \frac{\alpha E}{2(1-\nu)} \cdot \left[\frac{r^2\left(R_2^2 T_{R_2} - R_1^2 T_{R_1}\right) + R_1^2 R_2^2\left(T_{R_1} - T_{R_2}\right)}{r^2\left(R_2^2 - R_1^2\right)} - \frac{T_{R_1}\ln\left({R_2}/{r}\right) + T_{R_2}\ln\left({r}/{R_1}\right)}{\ln\left({R_2}/{R_1}\right)}\right], \tag{57}$$

$$\sigma_\varphi = -\frac{\alpha E}{(1-\nu)(R_2^2 - R_1^2)\ln\left({R_2}/{R_1}\right)} \cdot \left\{\left[\frac{R_1^2 R_2^2}{r^2}\left(T_{R_1} - T_{R_2}\right) + R_1^2 T_{R_1} - R_2^2 T_{R_2}\right]\ln\left({R_2}/{R_1}\right)\right.$$
$$+ \left(R_2^2 - R_1^2\right)\left(T_{R_2}\ln\left({r}/{R_1}\right) + T_{R_1}\ln\left({R_2}/{r}\right)\right) +$$
$$\left. + T_{R_2} - T_{R_1}\right\}, \tag{58}$$

$$\varepsilon_r = \frac{\alpha}{2(1-\nu)}\left[\frac{(1-3\nu)\left(R_2^2 T_{R_2} - R_1^2 T_{R_1}\right)}{R_2^2 - R_1^2} + \frac{2\nu\left(T_{R_2} - T_{R_1}\right) + (1+\nu)\left(T_{R_1}\ln\left({R_2}/{r}\right) + T_{R_2}\ln\left({r}/{R_1}\right)\right)}{\ln\left({R_2}/{R_1}\right)} + \frac{(1+\nu)R_1^2 R_2^2\left(T_{R_1} - T_{R_2}\right)}{r^2\left(R_2^2 - R_1^2\right)}\right], \tag{59}$$

$$\varepsilon_\varphi = \frac{\alpha}{2(1-\nu)}\left[\frac{(1-3\nu)\left(R_2^2 T_{R_2} - R_1^2 T_{R_1}\right)}{R_2^2 - R_1^2} - \frac{(1-\nu)\left(T_{R_2} - T_{R_1}\right) - (1+\nu)\left(T_{R_1}\ln\left({R_2}/{r}\right) + T_{R_2}\ln\left({r}/{R_1}\right)\right)}{\ln\left({R_2}/{R_1}\right)} - \frac{(1+\nu)R_1^2 R_2^2\left(T_{R_1} - T_{R_2}\right)}{r^2\left(R_2^2 - R_1^2\right)}\right]. \tag{60}$$

$$\varepsilon_z = \frac{\alpha\left[\left(T_{R_2} - T_{R_1}\right)\left(R_2^2 - R_1^2\right) + 2\left(R_1^2 T_{R_1} - R_2^2 T_{R_2}\right)\ln\left({R_2}/{R_1}\right)\right]}{2\left(R_2^2 - R_1^2\right)\ln\left({R_2}/{R_1}\right)}. \tag{61}$$

The relations obtained determine the change in the stress-strain state of the pin of a ball mill as the function of temperature and the change in the radial distance from the axis of symmetry of the pin of the mill.

To find the deformation of the internal surface of the trunnion of the ball mill, we substitute $r = R_1$:

$$\varepsilon_r(r = R_1) = \frac{\alpha}{1-\nu}\left[\frac{T_{R_1}\left(R_2^2 - R_1^2\right) + \nu T_{R_1}\left(R_2^2 + R_1^2\right) - 2\nu T_{R_2} R_2^2}{R_2^2 - R_1^2} + \frac{\nu\left(T_{R_2} - T_{R_1}\right)}{\ln\left({R_2}/{R_1}\right)}\right], \tag{62}$$

$$\varepsilon_\varphi(r = R_1) = \frac{\alpha}{2(1-\nu)}\left[\frac{(1-3\nu)\left(R_2^2 T_{R_2} - R_1^2 T_{R_1}\right) + (1+\nu)T_{R_1}\left(R_2^2 - R_1^2\right)}{R_2^2 - R_1^2} - \frac{(1-\nu)\left(T_{R_2} - T_{R_1}\right)}{\ln\left({R_2}/{R_1}\right)}\right]. \tag{63}$$

To find the deformations on the outer side of the trunnion of the ball mill in the formula, we must put $r = R_2$:

$$\varepsilon_\varphi(r = R_2) = \frac{\alpha}{2(1-\nu)}\left[\frac{(1-3\nu)\left(R_2^2 T_{R_2} - R_1^2 T_{R_1}\right) - (1+\nu)\left(R_1^2\left(T_{R_1} - T_{R_2}\right) + T_{R_2}\left(R_2^2 - R_1^2\right)\right)}{R_2^2 - R_1^2} - \frac{(1-\nu)\left(T_{R_2} - T_{R_1}\right)}{\ln\left({R_2}/{R_1}\right)}\right]. \tag{64}$$

Let us find the values of the stresses on the inner surface of the trunnion of the ball mill. To perform this operation, we set $r = R_1$:

$$\sigma_z(r = R_1) = \sigma_\varphi(r = R_1) = \frac{\alpha E(T_{R_2} - T_{R_1})}{1-\nu}\left[\frac{R_2^2}{R_2^2 - R_1^2} - \frac{1}{2\ln\left({R_2}/{R_1}\right)}\right]. \quad (65)$$

When substituting $r = R_2$, we obtain stress values for the outer surface of the trunnion of the ball mill:

$$\sigma_z(r = R_2) = \sigma_\varphi(r = R_1) = \frac{\alpha E(T_{R_2} - T_{R_1})}{1-\nu}\left[\frac{R_1^2}{R_2^2 - R_1^2} - \frac{1}{2\ln\left({R_2}/{R_1}\right)}\right]. \quad (66)$$

Thus, the obtained expressions (62) – (67) determine the values of the stress-strain state of the inner and outer surfaces of the trunnion, depending on the temperature inside and outside the trunnion of a ball mill.

4. The study of the deformation and strain of the trunnion of a ball mill by numerical methods

The study of the character of the behavior of strains and stresses on the basis of (62)–(66) with a change in the temperature parameters of a pin is carried out by numerical methods. To conduct the research using numerical methods, it is necessary to set the initial numerical values of the constants and the sizes of the worn pin.

The Poisson's ratio, the modulus of elasticity and the linear thermal expansion coefficient have the following values for the pin material - Steel 40:

$$\nu = 0.25, \mathrm{E} = 200\,000\ \mathrm{MPa}, \alpha = 12.4\ \mathrm{grad.}^{-1}$$

Let us consider a mill 3,2x15 m with the following axle dimensions in meters:

$$R_1 = 0.575;\ R_2 = 0.7.$$

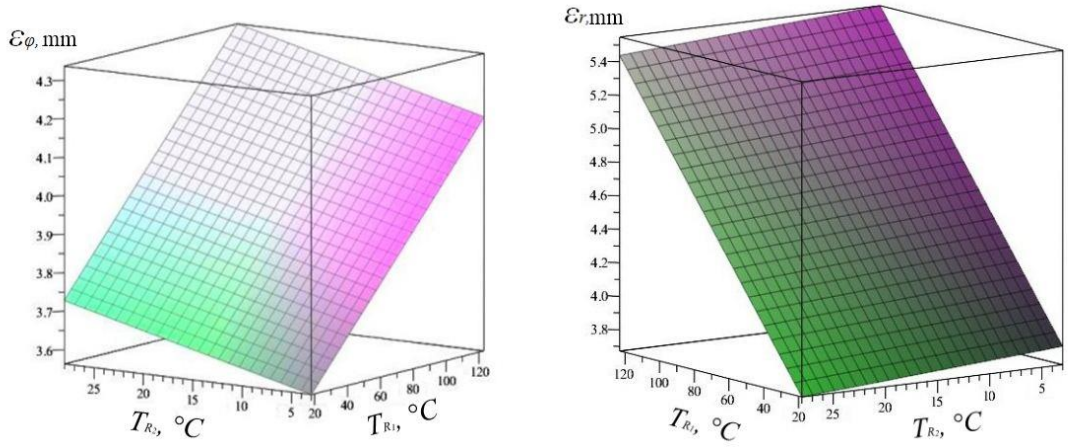
As part of the math package *Maple*, equations (2.79)-(2.86) are solved by the varying parameters within:

$$T_{R_1} = 20 \ldots 120°\mathrm{C},\ T_{R_2} = 5 \ldots 30°\mathrm{C}.$$

The results of the numerical solution of the equations (62)–(66) are presented in the form of graphs in figure 3-5.

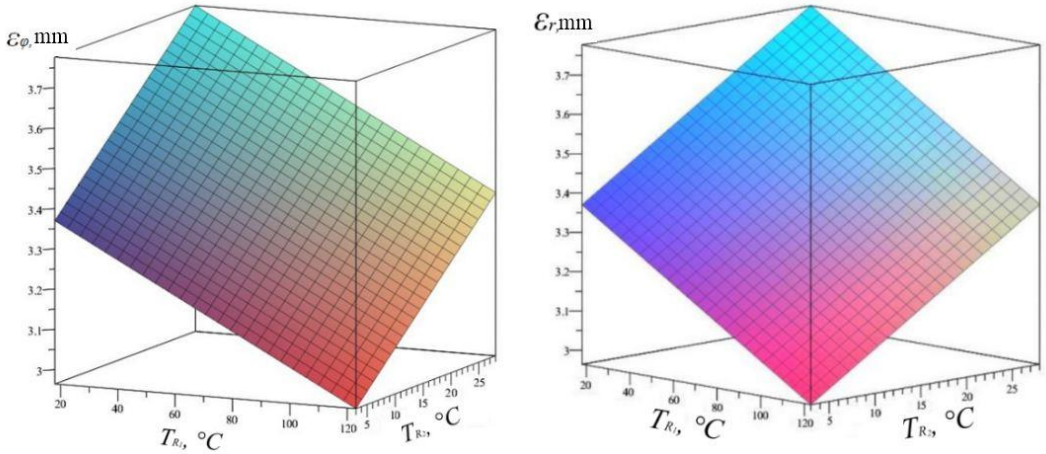
The change in the deformation of the inner surface of the trunnion ($r = R_1$) in the azimuthal and radial directions with changing temperatures of the outer and inner surface is shown in Figure 3. The analysis of these dependences shows that the graphs are monotonically increasing in nature with the changing temperatures: an increase in the temperature of the inner and outer temperature leads to the increase in the deformation of the inner surface of the trunnion in two directions.

Figure 4 shows the change in the deformation of the outer surface of the axle ($r = R_2$) in the azimuthal and radial directions when the temperatures of the outer and inner surface change. The graphs are linear. As the temperature of the inner surface of the pin increases, the deformation of the outer surface in the azimuthal and radial directions decreases, and as the temperature of the outer surface increases, it increases as well.



a) b)

Figure 3. The change in the deformation of the inner surface of the trunnion with changes in the temperatures of the outer and inner surface: a) in the azimuthal direction, b) in the radial direction

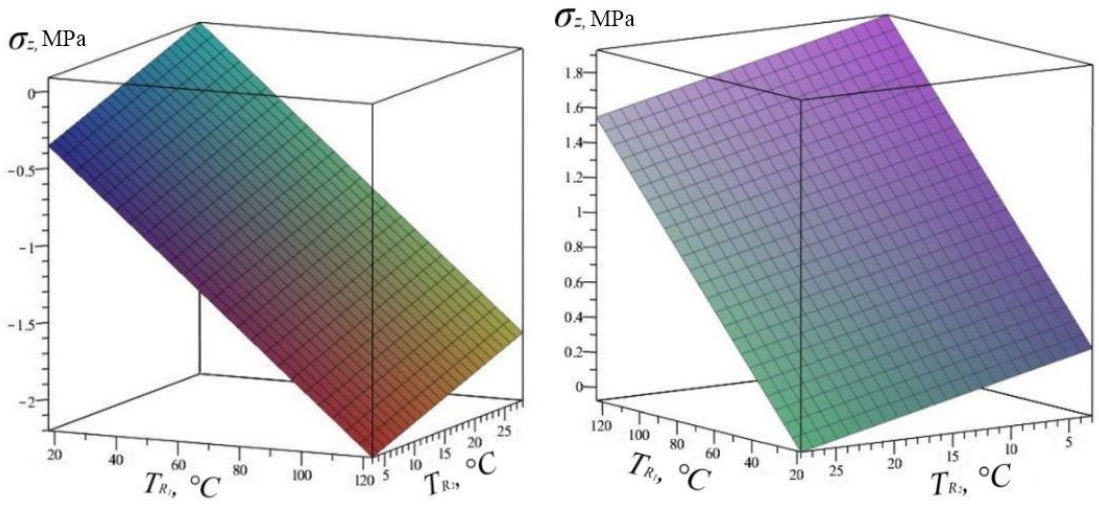


(a) (b)

Figure 4. The change in the deformation of the outer surface of the trunnion with changes in the temperatures of the outer and inner surface: (a) in the azimuthal direction, (b) in the radial direction.

The graphs of the stress variation of the inner and outer surface of the trunnion are shown in figure 5. Negative values on the graph of the stress variation of the inner surface of the trunnion (Figure 5a) indicate the actual direction of the stresses on the surface back to be accepted. This means that increasing the temperature of the inner surface of the trunnion affects the outer surface of the trunnion.

Analysis of the dependence of the stress on the outer surface on the change in temperature of the outer and inner surface shows that as the temperature inside the spigot increases, the stresses on the outer surface increase. In this case, the stresses on the external surface are inversely related to the ambient temperature: as the temperature increases, the stress values decrease slightly.



(a) (b)

Figure 5. The change in the tension of the pin surface with changes in temperature: (a) the inner surface, (b) the outer surface.

5. Conclusions

The trunnion of a ball mill in the framework of the theory of elasticity was considered, which is subjected to an uneven thermal impact due to the heating of the load. The equations describing the radial displacement of a point inside the trunnion of a ball mill were obtained. The equations describing the movement of the trunnion points of the ball mill were derived. The stress-strain state of the trunnion of a ball mill was determined depending on the temperature of the inner and outer surfaces of the trunnion. The deformations and stresses arising on the surface of trunnion were investigated by numerical methods.

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