Optimization of Laminated Composites Characteristics via integration of Chamis Equation, Taguchi method and Principal Component Analysis

To cite this article: Syafiq Syarafuddin Salam *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **551** 012110

IOP Conf. Series: Materials Science and Engineering **551** (2019) 012110 doi:10.1088/1757-899X/551/1/012110

Optimization of Laminated Composites Characteristics via integration of Chamis Equation, Taguchi method and Principal Component Analysis

Syafiq Syarafuddin Salam[1], Nik Mizamzul Mehat[1] and Shahrul Kamaruddin[2]

[1]Department of Mechanical Engineering Technology, Faculty of Engineering Technology, Uniciti Alam Campus, Universiti Malaysia Perlis, Sungai Chuchuh, Padang Besar, 02100, Perlis, Malaysia.
[2]Mechanical Engineering Department, Universiti Teknologi PETRONAS, 32610 Seri Iskandar, Perak Darul Ridzuan, Malaysia

E-mail: nikmizamzul@unimap.edu.my

Abstract. To date, composite material has drawn significant attention due to its extended properties in various application. Many factors need to be considered in designing the composites which might leads to complex decision making. Therefore, this study focuses on analyzing and optimizing various factors including combination of material, volume fraction of fiber, ply angle, ply quantity, ply thickness and load applied via integration of the Taguchi method / Principal component analysis along with analytical analysis of Chamis equation. The finite element was adopted in conducting the analysis. Three responses were considered for the laminated composites namely displacement, stress and strain. The findings from the main effects analysis showed that the set of optimum parameters was identified as load applied of 2000N, ply thickness of 0.08 mm, ply quantity of 12 plies, ply angle of 0,90,0 °, material combination of boron/epoxy as well as volume fraction of 65%.

1. Introduction

Laminated composite have been used increasingly in a variety of industrial areas due to their high stiffness and strength to weight ratio, long fatigue life, resistance to electro chemical corrosion, and other superior material properties of composite [1]. A true understanding of their structural behavior is required, such as the deflections, buckling loads and extreme importance for obtaining strong, reliable multi-layered structure failure characteristics [2], [3]. Kelkar et al., [4] for example studied the characteristics of tensile strength, tensile modulus and thermal conductivity of laminated composites. Meanwhile, Fan et al., [5] investigated the fracture toughness, bending strength and work of fracture of the alumina/nickel laminated composites. Many researches were conducted in evaluating the various factors that affecting the properties of laminated composite including combination of material [6], volume fraction of fiber, angle of fiber [7], ply angle, ply quantity and ply thickness [8] that relate with each other. Final design of laminated composite is limited to some extent, which results in complex decision making. This develops dilemma in

IOP Conf. Series: Materials Science and Engineering **551** (2019) 012110 doi:10.1088/1757-899X/551/1/012110

modelling the relation between properties and the characteristics of the laminated composite material. According to Thiago et al., [9] the lamination parameters play an important role as design variables in composite laminates layer optimization. All these influential parameters that affected the quality of the laminated composite need to be systematically controlled. Therefore, there is a needed to have a proper and analytically optimization method. There are many optimization methods that can be implemented for example Taguchi method [10]. However, by adopting the Taguchi method alone will only applicable for single response, whereby in real industry application, the requirement to have a material with multiple properties such as strength, stiffness, lightweight, rigidity which depending on the material functionality. Therefore, in this study, the optimization method of the Taguchi method will be integrated with the Principal Component Analysis (PCA) along with the analytical Chamis equation to investigate the multi response of laminated different composite simultaneously.

2.Methodology

In this study, the finite element simulation was used to conduct the numerical simulation. The robust parameter design of Taguchi method and principal component analysis (PCA) were integrated in conducting the simulation and the overall procedures is elaborated in the following section.
Selection of parameters

Several parameters with four levels of each were selected in this study including load applied, ply thickness, ply quantity, ply angle, combination of materials and volume fraction. Table 1 shows the parameters control and their levels.

Table 1. Parameters and level studied

Column	Parameter	Level 1	Level 2	Level 3	Level 4
A	Load applied (N)	1000	2000	3000	4000
B	Ply thickness (mm)	0.04	0.08	0.12	0.16
C	Ply quantity	3 plies	6 plies	9 plies	12 plies
E	Ply angle (°)	0, 30, -30	0, 45, -45	0, 60, -60	0, 90, 0
D	Material combination	Kevlar + Epoxy	Boron + Aluminium	E-glass + Polyester	Boron + Epoxy
E	Volume fraction (%)	50	55	60	65

2.1 Selection of Orthogonal Array (OA)

After determining the number of control parameters and their levels, an appropriate OA was established for laying out the design of experiment in the finite element analysis. The selection of an appropriate OA depends on the total degree of freedom (DOF) of the control parameters. In this study, there were six control parameters with four levels of each. As DOF was calculated by subtracting the number of levels with one (DOF = number of levels-1), therefore for six control parameters with four levels each, the total DOF is equal to 18. The total DOF of selected OA must be greater than or equal to the total DOF required for the experiment. Table 2 shows the OA that was used in the simulation.

Analytical calculation by using Chamis equation.

The material properties of Young modulus, Poisson ratio and shear modulus were determined with respected volume fraction as follows:

Young's modulus $$E_1 = E_f V_f + E_m V_m \qquad (1)$$

$$E_2 = E_2 = \frac{E_m}{1-\left\{\sqrt{V_f}\left(1-\frac{E_m}{E_f}\right)\right\}} \tag{2}$$

Poisson ratio

$$\boldsymbol{v_{12} = v_{13} = v_f V_f + v_m V_m} \tag{3}$$

$$v_{23} = \frac{E_2}{2(G_{23}-1)} \tag{4}$$

Shear modulus

$$G_{12} = G_{13} = \frac{G_m}{1-\left\{\sqrt{V_f}\left(1-\frac{G_m}{G_f}\right)\right\}} \tag{5}$$

$$G_{23} = \frac{E_2}{2(1+v_{23})} \tag{6}$$

TABLE 2. OA L_{32} of simulation runs

Trial No	A	B	C	D	E	F
1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	1	1	2	2	3	3
6	2	2	1	1	4	4
7	3	3	4	4	1	1
8	4	4	3	3	2	2
9	1	2	3	4	1	2
10	2	1	4	3	2	1
11	3	4	1	2	3	4
12	4	3	2	1	4	3
13	1	2	4	3	3	4
14	2	1	3	4	4	3
15	3	4	2	1	1	2
16	4	3	1	2	2	1
17	1	4	1	4	2	3
18	2	3	2	3	1	4
19	3	2	3	2	4	1
20	4	1	4	1	3	2
21	1	4	2	3	4	1
22	2	3	1	4	3	2
23	3	2	4	1	2	3
24	4	1	3	2	1	4
25	1	3	3	1	2	4
26	2	4	4	2	1	3
27	3	1	1	3	4	2
28	4	2	2	4	3	1
29	1	3	4	2	4	2
30	2	4	3	1	3	1
31	3	1	2	4	2	4
32	4	2	1	3	1	3

IOP Conf. Series: Materials Science and Engineering **551** (2019) 012110 doi:10.1088/1757-899X/551/1/012110

2.2 Analysis on results by PCA

To overcome the limitation of Taguchi method, the PCA is integrated with Taguchi method. By using PCA, a set of original responses is transformed into a set of uncorrelated components to find the optimal factor or level combination. The application of PCA involves a series of steps that are capable of solving the weakness of the standalone Taguchi method, which requires engineering judgement to handle multiple quality characteristics because the judgement of an engineer increases uncertainty during the decision making. The procedures of implementing PCA in analyzing the results are as follows:

Step 1. Find the normalization data, to avoid discrimination of variables in calculation results

$$X'_i(j) = \frac{X_i(j) - min, X_i(j)}{max, X_i(j) - min, X_i(j)} \tag{7}$$

Step 2. Arrange the data in matrix form. The data taken from step 1 was arranged in matrix form

$$X' = \begin{bmatrix} x'_1 & x'_2 & \dots & \dots & x'_1 \\ x'_2 & x'_2 & \dots & \dots & x'_2 \\ \vdots & \vdots & \dots & \dots & \vdots \\ x'_m & x'_m & \dots & \dots & x'_m \end{bmatrix} \tag{8}$$

Step 3. Calculate the correlation coefficient array of the normalized response.

$$\mathrm{R} = \frac{Cov\ (x'_i(j), x'_i(l))}{\sigma_{i(j)} \times \sigma_{i(l)}} \tag{9}$$

Where, $(\mathrm{j} = 1,2 \dots .\mathrm{n}; \quad \mathrm{l} = 1,2 \dots ..\mathrm{m})$

Step 4. Determination of eigenvalues and eigenvectors. Eigenvalue is the original total variance while eigenvector is the list of coefficient of the original variables.

$$(R - \lambda_k\ I)\ (V_k) = 0 \tag{10}$$

Where, $\Sigma\lambda_k = n; \quad k = 1,2, \dots, n \quad V_k$: Eigenvector, $\mathrm{V} = V_1, V_2 \dots, n$

Step 5. Evaluating the Principle Components (PC) Score. The PC scores can be obtained as linear combination of the original variable and the weighted.

$$\mathrm{Y_k} = \Sigma\, x'_m(i).V_k \tag{11}$$

Step 6. Multiple Quality Characteristic Index (MQCI) to represent all responses of quality characteristic.

$$MQCI = \sum_{i=1}^{n} W_i \times Y_{ij} \tag{12}$$

3. Results and Discussion

3.1 Analytical results by using Chamis equation.

In this study, Young's modulus (E) , Poisson ratio (υ) and shear modulus for the different composites material as in Table 1 are manually calculated by using the Equation 1 to Equation 6 with respected of four selection of volume fraction where are 50%, 55%, 60% and 65%. Young's modulus (E) is used to measure the ability of material to withstand changes in length when under lengthwise tension or compression. Poisson's ratio (υ) is used to determine ratio transverse strain of longitudinal extension strain in the direction of stretching force. Meanwhile, shear modulus (G) is used to define as the ratio of shear stress to the shear strain, where the shear stress is the force which acts is the area on which the force acts to shear strain. The results are tabulated in Table 3.

Table 3. Young's modulus (E) , Poisson ratio (υ) and shear modulus for the different composites material

Composite material	Volume Fraction (%)	E1 (GPa)	E2 (GPa)	E3 (GPa)	υ12	υ13	υ23	G12 (GPa)	G13 (GPa)	G23 (GPa)
Kevlar + Epoxy	50	67.81	14.54	14.54	0.34	0.34	0.33	4.61	4.61	21.80
	55	74.13	16.24	16.24	0.34	0.34	0.33	5.08	5.08	24.36
	60	80.45	18.28	18.28	0.34	0.34	0.33	5.63	5.63	27.42
	65	86.77	20.79	20.79	0.35	0.35	0.33	6.27	6.27	31.19
Boron + Aluminium	50	244.50	168.68	168.68	0.26	0.26	0.33	64.83	64.83	253.02
	55	262.05	181.47	181.47	0.25	0.25	0.33	69.27	69.27	272.21
	60	279.60	195.66	195.66	0.25	0.25	0.33	75.61	75.61	293.48
	65	297.15	211.51	211.51	0.24	0.24	0.33	82.00	82.00	317.27
E-Glass + Polyester	50	37.60	9.87	9.87	0.29	0.29	0.33	9.98	9.98	14.80
	55	41.04	10.98	10.98	0.28	0.28	0.33	4.09	4.09	16.47
	60	44.48	12.32	12.32	0.28	0.28	0.33	4.59	4.59	18.47
	65	47.92	13.94	13.94	0.27	0.27	0.33	5.21	5.21	20.90
Boron + Epoxy	50	212.31	5.32	5.32	0.26	0.26	0.33	5.34	5.34	7.98
	55	233.08	17.33	17.33	0.25	0.25	0.33	6.03	6.03	26.00
	60	253.85	19.75	19.75	0.25	0.25	0.33	6.88	6.88	29.63
	65	274.62	22.80	22.80	0.24	0.24	0.33	7.95	7.95	34.20

3.2 Results of Finite Element Analysis

The stress, strain and displacement values for each trial conducted by using the selected OA (refer Table 2) obtained from the finite element analysis is normalized to a unit value, ranging from 0 to 1. The normalized values are summed up to represent the stress, strain and displacement simultaneously as shown in Table 4.

Table 4. The result of before and after normalization and MCQI

No Triall	Before normalization			After normalization			MQCI
	Displacement	Stress	Strain	Displacement	Stress	Strain	
1	161.0×10^{-6}	$16.3 \times 10^{+3}$	244.0×10^{-9}	0.448	0.321	0.349	1.406
2	1.5×10^{-6}	$5.5 \times 10^{+3}$	20.6×10^{-9}	0.003	0.085	0.022	-0.639
3	10.0×10^{-6}	$5.7 \times 10^{+3}$	128.0×10^{-9}	0.027	0.091	0.179	-0.224
4	857.0×10^{-9}	$3.0 \times 10^{+3}$	11.0×10^{-9}	0.001	0.031	0.008	-0.799
5	9.9×10^{-6}	$9.3 \times 10^{+3}$	135.0×10^{-9}	0.026	0.168	0.190	-0.022
6	2.8×10^{-6}	$10.0 \times 10^{+3}$	37.8×10^{-9}	0.006	0.183	0.047	-0.351
7	4.8×10^{-6}	$2.7 \times 10^{+3}$	62.3×10^{-9}	0.012	0.026	0.083	-0.618
8	2.0×10^{-6}	$7.6 \times 10^{+3}$	$27.5 \times 10\text{-}^{9}$	0.004	0.131	0.032	-0.509
9	9.4×10^{-6}	$9.5 \times 10^{+3}$	128.0×10^{-9}	0.025	0.172	0.179	-0.039
10	478.0×10^{-9}	$1.6 \times 10^{+3}$	6.5×10^{-9}	0.000	0.001	0.001	-0.885
11	12.5×10^{-6}	$8.3 \times 10^{+3}$	172.0×10^{-9}	0.034	0.147	0.244	0.064
12	26.6×10^{-6}	$18.1 \times 10^{+3}$	71.8×10^{-9}	0.073	0.360	0.097	0.283

13	7.7 X 10^{-6}	5.0 X 10^{+3}	68.8 X 10^{-9}	0.020	0.075	0.092	-0.471
14	624.0 X 10^{-9}	2.1 X 10^{+3}	8.5 X 10^{-9}	0.001	0.012	0.004	-0.852
15	359.0 X 10^{-6}	32.3 X 10^{+3}	687.0 X 10^{-9}	1.000	0.668	1.000	4.631
16	3.2 X 10^{-6}	4.1 X 10^{+3}	16.4 X 10^{-9}	0.008	0.055	0.015	-0.714
17	4.2 X 10^{-6}	5.1 X 10^{+3}	57.5 X 10^{-9}	0.011	0.077	0.076	-0.522
18	3.8 X 10^{-6}	14.4 X 10^{+3}	51.1 X 10^{-9}	0.009	0.279	0.066	-0.079
19	3.8 X 10^{-6}	2.2 X 10^{+3}	52.6 X 10^{-9}	0.009	0.014	0.069	-0.685
20	3.1 X 10^{-6}	8.2 X 10^{+3}	38.8 X 10^{-9}	0.008	0.145	0.048	-0.432
21	1.3 X 10^{-6}	1.5 X 10^{+3}	17.3 X 10^{-9}	0.002	0.000	0.017	-0.848
22	435.0 X 10^{-9}	1.7 X 10^{+3}	5.8 X 10^{-9}	0.000	0.003	0.000	-0.884
23	30.2 X 10^{-6}	16.5 X 10^{+3}	408.0 X 10^{-9}	0.083	0.324	0.591	1.358
24	18.2 X 10^{-6}	47.7 X 10^{+3}	229.0 X 10^{-9}	0.050	1.000	0.328	2.249
25	61.0 X 10^{-6}	37.3 X 10^{+3}	482.0 X 10^{-9}	0.169	0.776	0.699	2.789
26	3.5 X 10^{-6}	14.1 X 10^{+3}	47.7 X 10^{-9}	0.009	0.273	0.062	-0.107
27	3.2 X 10^{-6}	1.6 X 10^{+3}	43.5 X 10^{-9}	0.008	0.002	0.055	-0.745
28	560.0 X 10^{-9}	1.9 X 10^{+3}	8.1 X 10^{-9}	0.000	0.008	0.003	-0.864
29	2.7 X 10^{-6}	2.8 X 10^{+3}	35.6 X 10^{-9}	0.006	0.028	0.044	-0.716
30	895.0 X 10^{-9}	3.6 X 10^{+3}	12.2 X 10^{-9}	0.001	0.046	0.009	-0.762
31	11.3 X 10^{-6}	5.7 X 10^{+3}	152.0 X 10^{-9}	0.030	0.090	0.215	-0.136
32	12.8 X 10^{-6}	16.4 X 10^{+3}	69.1 X 10^{-9}	0.035	0.322	0.093	0.123

3.3 Analysis of the normalized data by PCA

By using PCA, the set of normalized responses is transformed into a set of uncorrelated component, so that the optimal factor or level combination can be found. Table 5 and Table 6 display the eigenvalues and eigenvectors obtained from the PCA, which was conducted by utilizing the statistical software package. The new position coordinate system of the principal component was referred by the principal scores in this analysis. Besides, the linear combination of the original variable and the weighted were obtained by the PC scores. As studied, all the responses composites analysis which include the internal responses (stress, strain, and displacement) were represented by the used of multiple quality characteristic indexes (MQCI). The MQCI was obtained by integrating the total PCs scores via linear combination method correspond to their explanatory power for the total variance in ordered to facilitate the optimization analysis. The results of MQCI are tabulated in Table 4.

Table 5. The eigenvalues and explained percentage of variation for principal component

Component	Eigenvalue	Variability (%)	Cumulative %
1	2.360	78.652	78.652
2	0.504	16.797	95.449
3	0.137	4.551	100.000

Table 6. The eigenvectors for the principal components

Component	Displacement	Stress	Strain
1	0.565	0.541	0.622
2	-0.645	0.761	-0.076
3	-0.515	-0.358	0.779

3.4 Main effects analysis

The main effect analysis is the effect of an independent variable, averaging the levels of any other independent variables. The mean response at each level of the parameters was found out by calculating the average, based on the total MQCI (refer Table 4) for each parameter at different levels. Figure 1 illustrates the main effects analysis for the six parameters studied in this research.

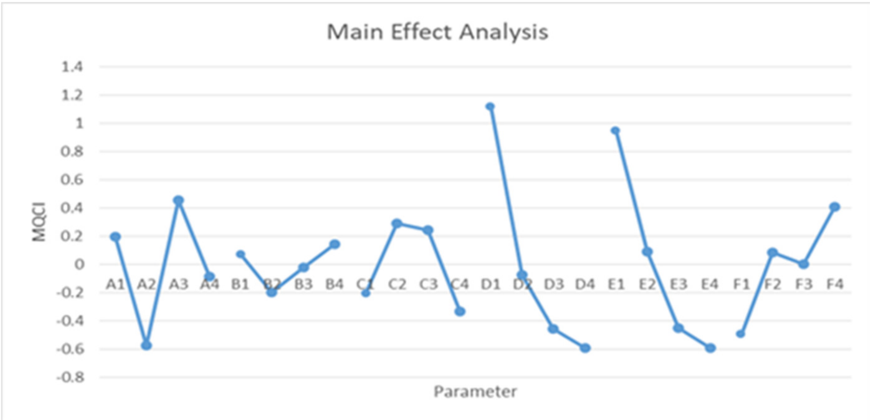

Figure 1. Main effects plot for MQCI

Figure 1 shows that the displacement, stress and strain which are represented by the used of MQCI are affected significantly by variations in the parameters. The lesser magnitude of MQCI will result in better parameter characteristics. The increment of load applied (A) from 1000 N to 2000N decreases the MQCI, yet then rapidly increased at 3000 N of load applied 3. By contrast, the ply thickness (B), an opposite trend is observed, shows the value of average MQCI slightly decreases at Level 1 but greatly increases from Level 2 to Level 4. Besides, the average of MQCI for ply quantities (C) is decreased gradually from Level 2 until Level 4. For ply angle (D) and material combination (E), similar trends can be observed for the decrement of the MQCI value as the levels of each increase. On the contrary, an opposite trend occurs on volume fraction (F) where the increasing of the fraction values apparently increases the value of MQCI in general.

From the result of main effect analysis, the best combination of parameters and levels could easily obtain by selecting the level of each parameter with the lowest average of MQCI. Referring to Figure 1, A2, B2, C4, D4, E4 and F4, show the lowest values of average MQCI for parameters A, B, C, D, E and F respectively. As a result, the optimal parameter setting which statistically results in stress and strain components as well as spatial displacement, are predicted to be A2, B2, C4, D4, E4 and F4. The set of optimum parameters is identified as load applied of 2000N, ply thickness of 0.08 mm, ply quantity of 12 plies, ply angle of 0,90,0 °, material combination of boron/epoxy as well as volume fraction of 65%.

4.Conclusion

Based on the findings of the research, it can be concluded that, by using the integration of the Taguchi method and PCA, the significant mathematical model function can be obtained in order to predict the optimal parameters in designing the different composites with variation of parameters. The set of optimum parameters is identified as load applied of 2000N, ply thickness of 0.08 mm, ply quantity of 12 plies, ply angle of 0,90,0 °, material combination of boron/epoxy as well as volume fraction of 65%.

Acknowledgments

The author would like to acknowledge support from the short term grant under a grant number of 9001-00523, Universiti Malaysia Perlis (Unimap) and Universiti Teknologi Petronas.

References

[1] Anish A.K *et al* 2018 *Structures*, 95-110.
[2] Rousseau M *et al* 2005 *Biomaterials* **26(31)** 6254-6262.
[3] Nikbakt S *et al* 2018 *Composite Structures* **195** 158-185.
[4] Kelkar A. *et al* 2016 *Materials Chemistry and Physics* **176** 136-142.
[5] Fan H *et al* 2017 *Ceramics International* **43 (13)** 10632-10636.
[6] Fiore V *et al* 2018 *Polymer Testing* **69** 310-319.
[7] Luis N *et al* 2017 *Composites Part B: Engineering* **128** 53-66.
[8] Chen X *et al* 2018 *Composite Structures* **192** 336-346
[9] Dutra T.A *et al* 2015 *Composite Structures* **133** 166-177.
[10] Mehat N.M *et al Journal of Physics: Conference Series* , **908 (1)** 012009
[11] Pattanaik A *et al* 2016 *Engineering Science and Technology*, *an International Journal* 710-716.