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Electromagnetic control of fluid with magnetic particles in the stenosed region

Salah Uddin[1,*], Mahathir Mohamad[1], Mghazali Kamardan[1], Mahmod Abd Hakim Mohamad[2], Suliadi Sufahani[1] and Roslan Rozaini[1]

[1]Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia, Pagoh Educational Hub, 84600 Pagoh, Johor, Malaysia.
[2]Faculty of Mechnical and Manufacturing Engineering, Universiti Tun Hussein Onn Malaysia, Pagoh Educational Hub, 84600 Pagoh, Johor, Malaysia.

Email: salahuddinjalil@gmail.com

Abstract. Hydrodynamic flow of Newtonian blood mixed with magnetic particles (red blood cells) through stenosed region is studied in this paper. The fluid is acted upon by an oscillating pressure gradient in a perpendicular magnetic field. Governing fractional momentum equations are analytically solved for axial velocities. Graphical analysis of velocity profiles against various physiological parameters is developed by coding in Mathematica. Effect of Caputo fractional operator α on axial velocities is also considered. Results show that blood and magnetic particles has increasing effect with maximum height of the stenosis and electric field, whereas, it has a decreasing effect with perpendicular magnetic field. Hence, the flow of blood can be controlled by applying sufficient and strong electric and magnetic field. This is an important fact in medical diagnosis and clinical treatment of various cardiovascular diseases, hypertension and atherosclerosis.

1. Introduction

Vascular fluid dynamics assume a significant share in the advancement of blood vessel illnesses. Local narrowing/stiffening in the lumen of blood vessel section is usually alluded as stenosis. This happens due to deposition of various fatty materials like cholesterol on the endothelium of arterial wall. When a blockade is developed in an artery, one of the most severe consequences is the expanded obstruction and the related fall of the blood stream to the specific vascular bed provided by the artery.

In the comparative study of [1], various simulations were performed for Newtonian, non-Newtonian fluid and Reynolds number. In the stenotic region non-Newtonian blood flow model was found to be laminar and transient flow was observed for Newtonian blood flow model. By using certain assumptions in [2] and [3], the Caputo-Fabrizio fractional order $1<\beta\leq 2$, linear differential equations were transformed to differential equations with integer order and solved for explicit forms. Application towards the mass-spring-damper motion of the obtained result was found in the described literature. In [3], it was observed that with shape parameter n, volumetric flow rate increases. Moreover, in the diverging region, high variation in volumetric flow rate was detected for an inclined artery with magnetic field angle θ, in addition to it, opposite behavior was observed in the converging region.

Under the action of an external magnetic field and oscillating pressure gradient blood flow mixed with magnetic particles in [4] and [5] was reduced. [6] analyzed that for Reynolds number greater than

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1000, flow moves towards turbulence downstream and frequency is connected with the periodic vortex formation. [7] used OpenFOAM programming to compare the influence of his elastic analog blood flow modeling with the analytic solution. [8] model is helpful for physicians in the severe stenosis case and can be extended to more severe case by introducing further rheological and physical parameters. In [9] mathematical simulation was effective in cardiovascular diseases and biomedical engineering.

In the current paper, new definition of Caputo-Fabrizo fractional derivative without kernel (NFD_t) is employed in the governing momentum equations and the resulting equations are obtained by coding in computer software Mathematica.

2. Mathematical Model

We study an in-compressible, laminar and unsteady flow of Newtonian fluid (human blood) mixed with magnetic particles (red blood cells being rich in iron) which is uniformly distributed in an axisymmetric but radially non-symmetric stenosed porous artery. Flow is driven by applying external pressure gradient in the electro-magnetic field.

Blood and magnetic particles are assumed to be flowing in the axial direction (z-axis) under the effect of an exterior electric field, whereas, the magnetic field acts upon perpendicularly to the circular channel of radius R_0 (as shown in the Fig. 1) and δ is the maximum height of the stenosis. Initially blood and magnetic particles are at rest. For $t > 0$ the electrolytic conducting solution is forced by an electromotive force, which depends on blood and magnetic particles flow and strength of electro-magnetic forces as well.

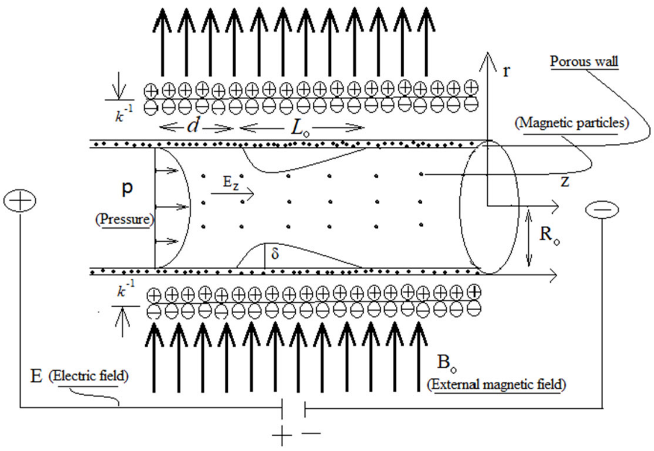

Figure 1. Electromagnetic flow model of blood containing red blood cells

Unsteady flow of blood is due to the influence of applied pressure gradient and is given by

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos(\omega t), \quad A_0 > 0, \tag{1}$$

where A_0 and A_1 are the fluctuating components of the pressure gradient, that is, steady and amplitude pulsatile components.

Axially symmetric but radially non-symmetric geometry of the stenosis is

$$\frac{R_z}{R_0} = \begin{cases} 1 - \xi\{L_0{}^{q-1}(\bar{z} - \bar{d}) - (\bar{z} - \bar{d})^q\}, & \bar{d} < \bar{z} < \bar{d} + L_0 \\ 1, & otherwise \end{cases}, \tag{2}$$

where $\xi = \dfrac{\delta q^{\frac{q}{q-1}}}{R_0 L_0^q (q-1)}, \dfrac{\delta}{R_0} < 1$ and $\bar{z} = \dfrac{\bar{d}+L_0}{q^{\frac{1}{q-1}}}$. R_z and R_0 are the radius of the artery with and without stenosis respectively, L_0 is the length of the stenosis and $\bar{d}$ indicates its location, $q \geq 2$ is the stenosis shape parameter.

Introducing the following dimensionless variables with * notations,

$$A_0{}^* = \frac{\lambda A_0}{\rho u_0}, A_1{}^* = \frac{\lambda A_1}{\rho u_0}, M^* = \frac{u_0 \sigma B_0{}^2 \lambda}{\rho}, \mathrm{Re}^* = \frac{R_0{}^2}{\lambda \upsilon}, R^* = \frac{K N \lambda}{\rho}, G^* = \frac{m}{K\lambda}, K^{2*} = \frac{\varepsilon k^2}{\rho}. \tag{3}$$

Governing non-dimensional fractional momentum equations for the blood and magnetic particles flow under electro-magnetic forces and applied pressure gradient with porous stenotic region in cylindrical coordinates system, (r,θ,z) is given by (after dropping dashes)

$$D_t^{(\alpha)} u = A_0 + A_1 \cos(\omega t) + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + R(v-u) - M u - K^2 \psi(r), \tag{4}$$

where $D_t^{(\alpha)} f(t) = \dfrac{M(\alpha)}{1-\alpha}\displaystyle\int_a^t \dfrac{\dot{f}(\tau)}{\exp\left(\dfrac{\alpha(t-\tau)}{1-\alpha}\right)} d\tau, \alpha \in [0,1]$ and $a \in [-\infty, t), f \in H^1(a,b), b > a$ and $M(0) = M(1) = 1$ (condition of normalization), m is the average mass of magnetic particles, $\lambda = \sqrt{\dfrac{R_0 \rho}{A_0}}$ has a dimension of time t, u_0 is the characteristic velocity, Re is the Reynolds number, R is the particle concentration parameter, G is the particle mass parameter, B_0 is the magnetic field strength, ρ is the fluid density, p is the fluid pressure, N is the number of magnetic particles per unit volume, u is the blood velocity, v is the magnetic particle's velocity, E_z is the electric field axial component, σ is the electrical conductivity, K is the stokes constant, υ is the kinematic viscosity, ρ_e is the net charge density.

According to Newton's second law of motion, magnetic particles movement can be govern by

$$G D_t{}^{(\alpha)} v = u - v. \tag{5}$$

Non-dimensional initial and boundary conditions are

$$\begin{cases} u(r,0) = 0 \text{ and } v(r,0) = 0 \text{ at } r = 1 \\ u(R_0,t) = 0 \text{ and } v(R_0,t) = 0, t > 0 \end{cases}. \tag{6}$$

Using Laplace and finite Hankel transforms on Eq. (3 & 5) together with Eq. (6), final velocity profiles for human blood and magnetic particles (red blood cells) has been obtained. Furthermore, analysis has also been made by coding in computer software Mathematica, where positive root of the Bessel function $J_0\left(r\dfrac{R_z}{R_0}\right)$ has been calculated.

3. Results and Discussion

The main aim of the study is to analyze the blood and iron-rich particles flow patterns due to externally applied electro-magnetic field through stenotic porous artery by means of the new approach of Caputo-Fabrizio fractional derivative (NFD$_t$) without singular kernel. For this purpose few simulations has been made in the local and non-local model against the fixed values given by $A_0 = 5, A_1 = 0.6, G = 0.5, \mathrm{Re} = 2, \omega = \dfrac{\pi}{4}, t = 0.2, r = 0.3.$

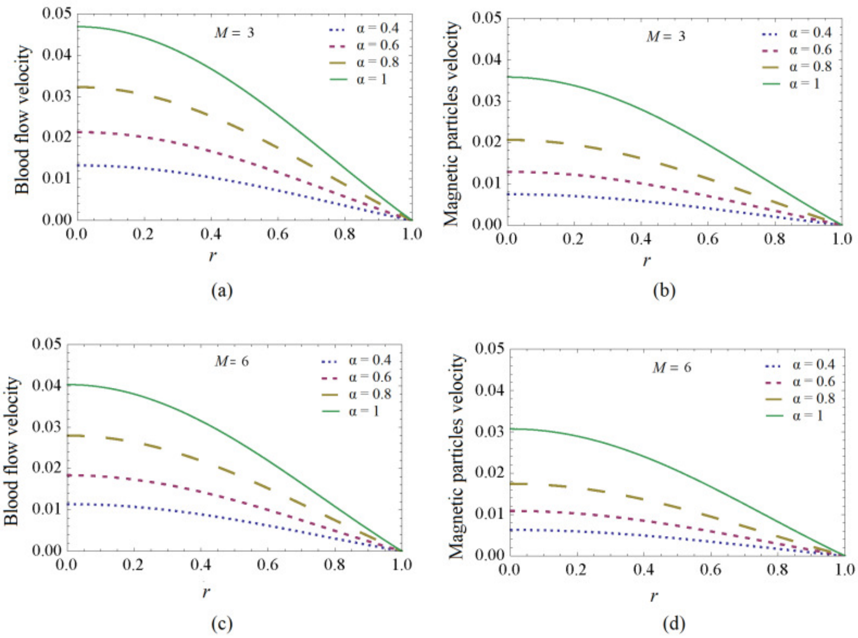

Figure 2. Velocities for different values of magnetic field M against r

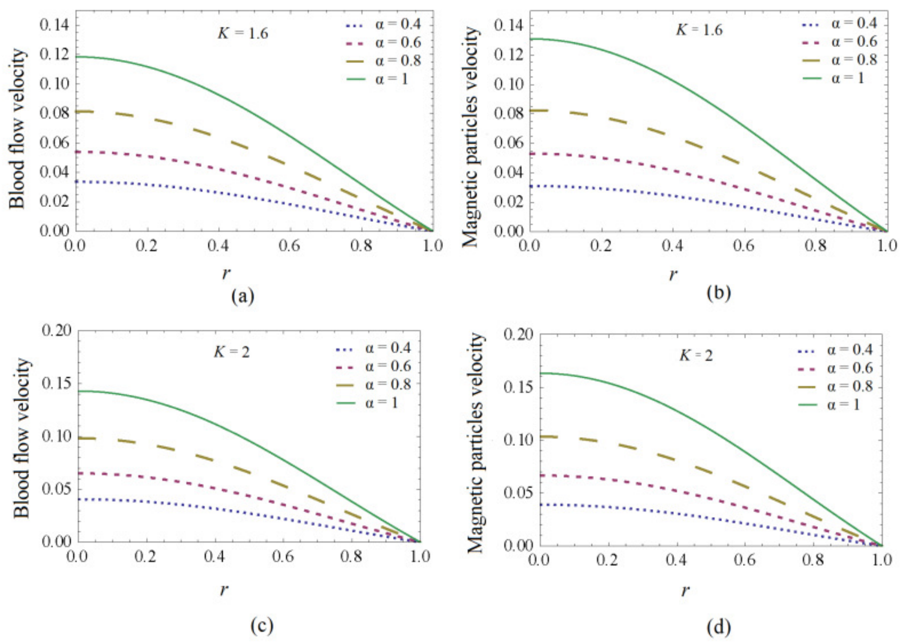

Figure 3. Velocities for different values of electric field K against r

In Fig. 2, axial velocity for blood and magnetic particles is decreased by enhancing the external magnetic field M. However, during the flow, blood moves faster than magnetic particles due to the drag and other retardation forces.

In Fig. 3, the external electric field at different strength is applied against *r*. Both the velocities has increasing effect with *K*. Moreover, particles flow is enhanced due to collision between the charged particles.

4. Conclusions

From the research, we reach at few conclusions which are;

- External electric field has an increasing effect on the velocity profiles, for $\alpha \leq 0.8$, particles velocity is found to be almost equal to blood.
- Magnetized solution decelerates with external magnetic field. Furthermore, magnetic particles move slower than blood.

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