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Geographically Weighted Regression in Cox Survival Analysis for Weibull Distributed Data with Bayesian Approach

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Abstract. Cox survival analysis is a statistical method used in survival data, which examines an event or occurrence of a particular event. In survival analysis, the response variable is survival time, usually called the T failure event. In the development, survival analysis involves spatial effects. One of the spatial effects is point effect, which coordinates of adjacent points will give an influence. Spatial model involves points called Geographically Weighted Regression (GWR). In this research, data distribution used is Weibull distribution, which survival time data is divided into three periods. Parameter estimation used is Bayesian Approach. Bayesian approach is better used in survival analysis that has a lot of censored data. The research purpose is getting survival function, hazard function, and Cox survival model with GWR and Weibull distributed data and determining the prior distribution and posterior distribution in Bayesian approach. The result of this research is reducing the new hazard function from Weibull distribution and changing μ to become the GWR model,

and then obtained model is
$$h(t, \mathbf{x}) = \rho t^{(\rho-1)} \exp \left(\beta_0(u_i, v_i) + \sum_{k=1}^p \beta_k(u_i, v_i) x_{ik} + e_i \right).$$

Parameters in the result are estimated using Bayesian approach.

Keywords: Bayesian, GWR, survival, Weibull.

1. Introduction

Survival analysis is a statistical method used in survival data, which examines an event or time of a particular event. Survival analysis has the advantage of being able to estimate or interpret hazard function or survival function from survival data, compare survival function or hazard function in treatment, and know the effect of independent variables on survival time variables or in this case called Cox regression [1].

In survival analysis, there is censorship of data that causes an individual to not be fully observed until a failure event occurs. So that, in cases with high censored data, the availability of individuals observed until they experience is few events. According to [2], in the censored data the average probability of coverage of the Bayesian approach survival analysis is better than the maximum likelihood. This is because in Bayesian analysis, the number of samples used is not considered and can be used for any distribution.



The Bayesian approach, in addition utilizes sample data obtained from the population also takes into account an initial distribution called priors. Statistical inference with Bayesian approach differs from the classical approach. The classical approach views the parameter θ as a fixed value parameter. While the Bayesian approach views the parameter θ as a random variable that has a distribution, called the prior distribution. Furthermore, the posterior distribution can be determined from the prior distribution so that the Bayesian estimator is obtained which is the mean or mode of the posterior distribution [3].

In the development, survival analysis modeling also includes spatial effects. The advantage of survival analysis with spatial effects is that the occurrence of a survival event is not only influenced by independent variables but also depends on a location (spatial). Spatial effects in general are divided into two, spatial effects of points and spatial effects of area. The point spatial effect uses a geospatial approach that uses geographical location latitude and longitude. Whereas the spatial effect of the area is approached by the location or position of the area where the incident occurs relative to other regions, in this case it is referred to as neighboring.

One model that uses point spatial effects of longitude and latitude is Geographically Weighted Regression (GWR). The GWR model is a development of linear regression models involving geographical factors. The GWR model is a regression method that produces predictors of local model parameters (locally linear regression) for each point or location where the data is collected.

Weibull distribution is one type of distribution that is often used in Survival analysis. The Weibull distribution was introduced by Swedish physicist Waloddi Weibull in 1939. The famous Weibull distribution with flexible distribution. One of the flexibility can be seen from the change in this distribution into other distributions such as exponential distribution depending on changes in the parameters of the scale and shape. Weibull distribution is one of the statistical data models that has a wide range of applications in survival analysis with its main advantage is presenting the accuracy of failure even with very small samples [4].

The purpose of this research was to determine the survival function, hazard function, and survival cox model of the Weibull distribution with the GWR model, and determine the prior distribution and posterior distribution in Bayesian analysis.

2. Weibull Distribution

The Weibull distribution was discovered by a Swedish physicist, the Weibull Wallodi. A continuous random variable T has Weibull distribution with parameter ρ and scale parameter θ , has a density function [5]

$$f(t) = \mu \rho t^{(\rho-1)} \exp(-\mu t^\rho) \quad (1)$$

The cumulative distribution function is

$$\begin{aligned} F(t) &= \int_0^t f(x) dx \\ &= \int_0^t \mu \rho x^{(\rho-1)} \exp(-\mu x^\rho) dx \\ &= 1 - \exp(-\mu t^\rho) \end{aligned} \quad (2)$$

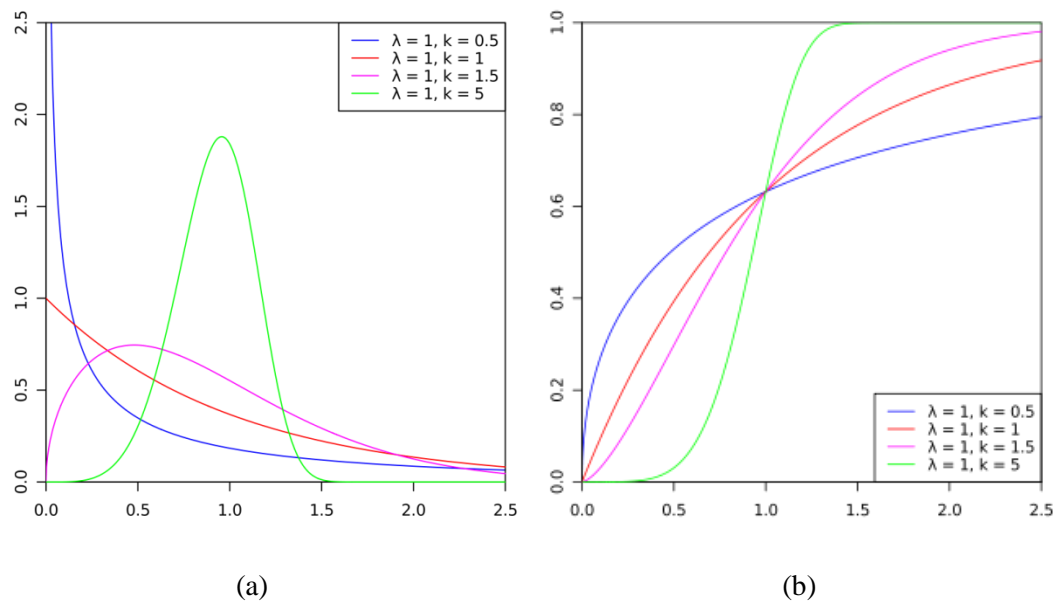


Figure 1. Weibull Distribution Curve. (a) Probability density function; (b) Cumulative distribution function.

Survival analysis is one of the statistical techniques that is useful for carrying out tests on component of durability or reliability and this analysis can be done with Weibull Distribution. The Weibull distribution is an important probability distribution used in characterizing probabilistic behavior from a large number of real-world phenomena. This distribution is useful as a model of failure in analyzing the reliability of various types of systems.

3. Geographically Weighted Regression

GWR is a statistical method used to analyze spatial heterogeneity. Spatial heterogeneity when the same independent variable gives unequal responses at different locations in one study area. The GWR model results in estimating local model parameters for each point or location where the data is observed. In the GWR model, the response variable is estimated by a predictor variable in which each regression coefficient depends on the location where the data is observed [6].

$$y_i = \beta_0(u_i, v_i) + \sum_{k=1}^p \beta_k(u_i, v_i) x_{ik} + e_i \quad (3)$$

when

- y_i : Observation value of response variable
- x_{ik} : Observation value of k predictor variable
- $\beta_0(u_i, v_i)$: intercept
- (u_i, v_i) : the coordinates of the geographical location from the observation
- $\beta_k(u_i, v_i)$: Parameter value of k predictor variable
- e_i : Error.

4. Survival Analysis

Survival analysis is a combination of statistical procedures for data analysis whose variables are "time until an event occurs". In survival analysis, the response variable is usually a survival time. Usually

calling the event T as failure, because the usual events are death, disease incidence or other negative individual experiences. However, in the case of survival, failure is a positive event [1].

Data in survival analysis consists of complete data and incomplete data. Complete data only includes non-censored data (non-sensor), while incomplete data includes censored data and uncensored data. Censored data is data that presents some information whose time of occurrence of an event is not known with certainty and uncensored data is data that presents some information when the occurrence of an event is certainty known.

Survival function is an opportunity or the possibility that an individual can survive more than time t . The T random variable represents survival time and has a chance density function $f(t)$:

$$\begin{aligned} S(t) &= P(T \geq t) \\ &= 1 - P(T \leq t) \\ &= 1 - \int_0^t f(x) dx \\ S(t) &= 1 - F(t) \end{aligned} \quad (4)$$

The hazard function states the chance that an individual experiences an event at an interval $(t, t + \Delta t)$

$$\begin{aligned} h(t) &= \lim_{\Delta t \rightarrow 0} \frac{P(t \leq T < (t + \Delta t) | T \geq t)}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{P(t \leq T < t + \Delta t \cap T \geq t)}{P(T \geq t) \Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{P(t \leq T < t + \Delta t)}{S(t) \Delta t} \\ &= \frac{1}{S(t)} \lim_{\Delta t \rightarrow 0} \frac{P(t \leq T < t + \Delta t)}{\Delta t} \\ &= \frac{F'(t)}{S(t)} \\ &= \frac{f(t)}{S(t)} \end{aligned} \quad (5)$$

One of the objectives of survival analysis is to determine the relationship between survival time and variables that are thought to influence survival time. The Cox survival model equation can be written

$$\begin{aligned} h(t, \mathbf{x}) &= h_0(t) e^{\sum_{j=1}^p \beta_j x_j} \\ &= h_0(t) \exp(\beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p) \\ &= h_0(t) \mu \end{aligned} \quad (6)$$

5. Bayesian Analysis

Bayesian modelling is based on the posterior model that is combining past data as prior information and observation data used as a likelihood function. Inference statistics are only based on sample data

from the population while Bayesian uses sample data information and takes into account the initial distribution or what is called prior distribution [6].

The posterior distribution equation is as follows

$$f(\theta|y) = \frac{f(y|\theta)f(\theta)}{f(y)} \propto f(y|\theta)f(\theta) \quad (7)$$

With $f(\theta)$ is the prior distribution and $f(y|\theta)$ is the Likelihood function.

Based on the Bayes theorem, the initial information used as the prior distribution and sample information expressed by the likelihood function are combined to form a posterior distribution.

There are several prior distribution types known in the Bayesian method:

1. Conjugate prior VS non conjugate prior. Is related prior to the likelihood model pattern from the data.
2. Proper prior VS Improper prior. Is related prior to giving weight / density at each point whether uniformly distributed or not.
3. Informative prior VS Non-Informative Prior. Is related prior to the known or unknown pattern / frequency of distribution of data.
4. Pseudo Prior. Is related prior to the giving of values that are equated with the results of the elaboration of the frequentist opinion [7].

6. Result and Discussion

Based on equation (2) and (3), the survival function is obtained for Weibull distribution data

$$\begin{aligned} S(t) &= 1 - F(t) \\ &= 1 - (1 - \exp(-\mu t^\rho)) \\ S(t) &= \exp(-\mu t^\rho) \end{aligned} \quad (8)$$

Furthermore, based on equations (1), (5), and (7) obtained the hazard function as follows

$$\begin{aligned} h(t) &= \frac{f(t)}{S(t)} \\ &= \frac{\mu \rho t^{(\rho-1)} \exp(-\mu t^\rho)}{\exp(-\mu t^\rho)} \\ &= \mu \rho t^{(\rho-1)} \end{aligned} \quad (9)$$

The general equation of the Cox survival model on the Weibull distribution with GWR is

$$h(t, \mathbf{x}) = \mu \rho t^{(\rho-1)}$$

$h_0(t)$ is a function whose value depends on the value of t so $h_0(t) = \rho t^{(\rho-1)}$. Whereas μ is free of value t .

$$\begin{aligned} h(t, \mathbf{x}) &= h_0(t) \mu = \rho t^{(\rho-1)} \mu \\ &= \rho t^{(\rho-1)} \exp\left(\beta_0(u_i, v_i) + \sum_{k=1}^p \beta_k(u_i, v_i) x_{ik} + e_i\right) \end{aligned} \quad (10)$$

6.1. Prior Distribution

Parameter estimates using the Bayesian method are obtained by determining the prior distribution first. In this study, prior used is conjugate prior and informative prior. The parameter ρ and μ from the Weibull distribution are exponential distribution families. Gamma distribution is an exponential family where the parameters can change like Weibull distribution so that the gamma distribution as prior is used. While the prior distribution for the β parameter using prior informative is the normal distribution. So that it is obtained

$$\rho \sim \text{Gamma}(a, b)$$

$$\beta \sim \text{Normal}(c, d)$$

6.2. Posterior Distribution

From the parameters obtained by equation (8), if δ is the status of the censored data or not, the posterior distribution join is:

$$p(\beta(u_i, v_i), \rho | t, \mathbf{x}, \delta) \propto L(\beta(u_i, v_i), \rho; t, \mathbf{x}, \delta) p(\beta(u_i, v_i)) p(\rho) \quad (11)$$

The Likelihood function can be summarized as follows:

$$\begin{aligned} L(\beta(u_i, v_i), \rho; t, \mathbf{x}, \delta) &= \prod_i \prod_j f_{ij}(t_{ij})^{\delta_{ij}} S_{ij}(t_{ij})^{1-\delta_{ij}} \\ &= \prod_i \prod_j \{h_{ij}(t_{ij}) S_{ij}(t_{ij})\}^{\delta_{ij}} S_{ij}(t_{ij})^{1-\delta_{ij}} \\ &= \prod_i \prod_j h_{ij}(t_{ij})^{\delta_{ij}} S_{ij}(t_{ij}) \\ &= \prod_i \prod_j \left[\rho t^{(\rho-1)} \exp \left(\beta_0(\mathbf{u}, \mathbf{v}) + \sum_{k=1}^p \beta_k(\mathbf{u}, \mathbf{v}) \mathbf{x}_{ijk} + e_i \right) \right]^{\delta_{ij}} \times \\ &\quad \exp \left(- \left(\exp \left(\beta_0(\mathbf{u}, \mathbf{v}) + \sum_{k=1}^p \beta_k(\mathbf{u}, \mathbf{v}) \mathbf{x}_{ijk} + e_i \right) \right) t^\rho \right) \end{aligned} \quad (12)$$

7. Conclusion

In this paper, we can conclude:

1. The survival function of Weibull distribution data is $S(t) = \exp(-\mu t^\rho)$, the hazard function is $h(t) = \mu \rho t^{(\rho-1)}$, and Cox survival model with GWR is $h(t, \mathbf{x}) = \rho t^{(\rho-1)} \exp \left(\beta_0(u_i, v_i) + \sum_{k=1}^p \beta_k(u_i, v_i) x_{ik} + e_i \right)$.
2. The prior distribution in this study uses conjugate prior and informative prior that produce $\rho \sim \text{Gamma}(a, b)$ $\beta \sim \text{Normal}(c, d)$
3. The posterior distribution formed is $p(\beta(u_i, v_i), \rho | t, \mathbf{x}, \delta) \propto L(\beta(u_i, v_i), \rho; t, \mathbf{x}, \delta) p(\beta(u_i, v_i)) p(\rho)$.

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