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Spatio-Temporal Fay-Herriot Models in Small Area Estimation to Obtain Factors That Affecting Poverty in Polewali Mandar District

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Abstract. Poverty is a classic problem in almost all regions in Indonesia both at the provincial and district levels. Polewali Mandar district is a district with the highest poverty rate in West Sulawesi Province. Handling the problem of poverty requires the availability of data to the smallest level so that the policies taken by the government can be right on target. One method for obtaining poverty estimates reaches the smallest area using the Small Area Estimation (SAE) method. Spatio-Temporal Fay-Herriot models is one of the SAE methods that has considered spatial and time effects. Auxiliary variables are needed in the SAE method to get good estimation results. Availability of auxiliary variables that affect poverty in Polewali Mandar district is very necessary. Therefore the purpose of this study was to obtain auxiliary variables that influence poverty in Polewali Mandar district. Bootstrap parametric procedures are used to obtain factors that influence poverty in Polewali Mandar district, while for spatial weighting matrices use customized contiguity based on the main business fields in the village. The response variable is the percentage of poverty in each village affected by the Susenas sample. The results obtained were the variables that affected poverty in Polewali Mandar district were the percentage of farm worker's families, the percentage of the population receiving Jamkesmas / Jamkesda, ratio of population to number of micro and small industries, average number of household members, ratio of population to number of health facilities and percentage of families living on river banks.

1. Introduction

Poverty is a problem that still occurs in almost all regions in Indonesia. Poverty is a complex problem because it involves many dimensions of life. Poverty is a condition where a person or group of people is unable to fulfill their basic rights to maintain and develop a dignified life. These basic rights include: fulfilling the needs of food, health, education, employment, housing, clean water, land, natural resources and the environment, feeling safe from the treatment or threat of acts of violence and the right to participate in socio-political life [1].

According to the World Bank in [2], poverty is deprivation in welfare. Based on this, it can be seen from several sides. Prosperity is seen conventionally, seen from the monetary side, namely where measuring poverty is comparing income / consumption of individuals with certain limitations, if the individual is below that limit, then it is considered poor. The next view of poverty is not only limited to the size of money, but also includes the amount of nutrients that one of the growth of children is hampered or normal. In addition, in terms of education, for example by using indicators of illiteracy rates. Furthermore, a broader view of poverty is in people who lack basic needs, so that the income and education needed is inadequate. In addition, poor health, insecurity, low self-confidence and a sense of helplessness and not having the right to freedom of freedom.

Polewali Mandar district as one of the "old" districts in West Sulawesi Province, has a condition of facilities and infrastructure that is relatively better than other districts. When viewed from an economic standpoint, Polewali Mandar’s per capita Gross Domestic Product (GDP) is 17,21 million per year, where this figure is higher than the minimum per capita requirement of 4,045 million per year. However, in reality Polewali Mandar district has the highest percentage of poverty in West Sulawesi Province. The percentage of poor people in Polewali Mandar district in March 2016 was 17.06 percent [3]. This value is higher than the percentage of poor people in West Sulawesi Province in the same period which amounted to 11.74 percent. High poverty will result in poor quality of life for people in Polewali Mandar district.

The first step as an effort to tackle the problem of poverty is to know the distribution of the poor to the smallest area, namely the village so that the areas with the most poverty are identified. The concept of poverty according to Statistics Indonesia (BPS) is the inability of the economic side to meet basic food and non-food needs measured from the expenditure side [4]. So the poor are residents who have an average per capita expenditure per month below the poverty line. BPS states that to measure welfare, an approach based on per capita expenditure is used. After that, a minimum standard of welfare indicators was developed to divide the population into poor and not poor. The minimum standard is known as the Poverty Line (GK) which covers basic needs using the Food Energy Intake (FEI) method. The method used to calculate the percentage of poverty by using GK consists of two components, namely the Food Poverty Line (GKM) and the Non-Food Poverty Line (GKNM).

2. Small Area Estimation

The method of calculating the percentage of poverty carried out by BPS is not presented for levels below the province or district because of insufficient sample so that if forced it will enlarge standard error so that analysis based on these conditions becomes unreliable. One way to get an estimate of the percentage of poor people is by indirect methods. The indirect method that can be used to overcome this problem is to use the Small Area Estimation (SAE) method [5]. SAE was chosen because it can be used for small area estimates, besides SAE is also able to reduce the standard error [6].

In SAE method there are two estimation levels, namely area levels and unit levels [5]. Area level estimates is the availability of supporting variables, only exists for a certain level of area. Unit level estimates are the auxiliary variables available for each population element *j* in small area *i*. The problem of poverty is not only viewed regionally but also from time to time [7]. One method in SAE that has noticed spatial and time effects is the Fay-Herriot spatio-temporal [8]. Auxiliary variables are needed in the SAE method to get good estimation results [9]. Availability of auxiliary variables that affect poverty in Polewali Mandar district is very necessary. Based on the previous description, the aim of this study is to obtain auxiliary variables that influence poverty in Polewali Mandar district using spatio-temporal Fay-Herriot method.

2.1. Spatio-Temporal Fay-Herriot Models

Spatio-Temporal Fay-Herriot model is one method that uses the Empirical Best Linear Unbiased Predictor (EBLUP) method in estimating parameters. The level of area used in this study. The EBLUP method without spatial and time effects is written as follows:

$$y_d = \mu_d + e_d \qquad d = 1, \dots, D \tag{1}$$

$$\mu_d = \boldsymbol{x'}_d \boldsymbol{\beta} + u_d \tag{2}$$

From equations 1 and 2 are obtained

$$y_d = \boldsymbol{x'}_d \boldsymbol{\beta} + u_d + e_d \tag{3}$$

The EBLUP method by considering the spatial effect so that it becomes the SEBLUP is written as follows:

$$y_d = \mu_d + e_d \quad d = 1, \dots, D \tag{4}$$

$$\mu_d = \boldsymbol{x'}_d\boldsymbol{\beta} + Z(\boldsymbol{I} - \rho\boldsymbol{W})^{-1}\boldsymbol{v} \quad (5)$$

From equations 4 and 5 are obtained

$$y_d = \boldsymbol{x'}_d\boldsymbol{\beta} + Z(\boldsymbol{I} - \rho\boldsymbol{W})^{-1}\boldsymbol{v} + e_d \quad (6)$$

The SEBLUP method is applied if spatial correlation is considered in the model [10]. In addition, if indeed in the spatial autocorrelation test, it can be concluded that there are spatial autocorrelations between the response variables and the area or region observed. The extension of Fay-Herriot's model to the spatio-temporal model is to take into account spatial correlations between neighboring areas and simultaneously combine data from time to T to increase the precision of the use of SAE at the last time T [11], so that for the area d = 1,...,D and time t = 1,...,T, for example $\{\mu_{dt}\}$ become the target characteristic for the area, $\{y_{dt}\}$ is the estimator directly from $\{\mu_{dt}\}$ and $\{x_{dt}\}$ is a column vector that contains the aggregate value of p of the auxiliary variables that are linearly correlated with $\{\mu_{dt}\}$. The expanded Fay-Herriot spatio-temporal model in the first phase is stated as follows:

$$y_{dt} = \mu_{dt} + e_{dt} \quad d = 1, \dots, D, t = 1, \dots, T \quad (7)$$

$$\mu_{dt} = \boldsymbol{x'}_{dt}\boldsymbol{\beta} + Z_{dt} \quad (8)$$

From equations 7 and 8 are obtained

$$y_{dt} = \boldsymbol{x'}_{dt}\boldsymbol{\beta} + Z_{dt}u_{dt} + e_{dt} \quad (9)$$

$$y_{dt} = \boldsymbol{x}'_{dt}\boldsymbol{\beta} + u_{1d} + u_{2dt} + e_{dt} \quad d = 1, \dots, D,\ t = 1, \dots, \mathrm{T} \quad (10)$$

where $\boldsymbol{y}$ is the $n \times 1$ vector of dependent variable, $\boldsymbol{x}$ is $n \times (k+1)$ independen variables matrix, $\boldsymbol{\beta}$ is $(k+1)$ regression parameter vector, u_{1d} is $n \times 1$ error vector effect random area, u_{2dt} is $n \times 1$ error vector effect area-time in equation (10), $\boldsymbol{e}$ is $n \times 1$ error vector in equation (10).

Equation 9 if presented in the form of a matrix is

$$y = \boldsymbol{X\beta} + \boldsymbol{Zu} + \boldsymbol{e} \quad (11)$$

The $\boldsymbol{X}$ in equation 10 is a matrix of the auxiliary variables measuring $D \times p$, $\boldsymbol{\beta}$ is a vector of regression parameters. The deviation of $\boldsymbol{X\beta}$ in this study is the result of Simultaneous Autoregressive (SAR) with the spatial autoregressive coefficient ρ_1 parameter and the $D \times D$ spatial weighting matrix W, namely:

$$u_1 = \{ (\mathbf{I}_D - \rho_1\boldsymbol{W})\ (\mathbf{I}_D - \rho_1\boldsymbol{W})' \}^{-1} \quad (12)$$

where

$$Var(\boldsymbol{u_1}) = E\left[\left(\boldsymbol{u_1} - E(\boldsymbol{u_1})\right)\left(\boldsymbol{u_1} - E(\boldsymbol{u_1})\right)'\right]$$

$$= (\boldsymbol{I}_D - \rho_1\boldsymbol{W})^{-1}E(\boldsymbol{vv'})(\boldsymbol{I}_D - \rho_1\boldsymbol{W'})^{-1} \quad (13)$$

$$Var(\boldsymbol{u_1}) = \sigma_1^2[(\boldsymbol{I}_D - \rho_1\boldsymbol{W})(\boldsymbol{I}_D - \rho_1\boldsymbol{W})']^{-1} = \mathbf{V_u}(\boldsymbol{\theta}) \quad (14)$$

Then to form the likelihood function of $\widehat{\boldsymbol{\theta}}$ it takes the average and variance of $\widehat{\boldsymbol{\theta}}$, namely:

$$E(y) = E[\boldsymbol{X\beta} + \boldsymbol{Zu} + \boldsymbol{e}] \quad (15)$$

$$= \boldsymbol{X\beta} \quad (16)$$

$$Var(\boldsymbol{y}) = E\left[\left(\boldsymbol{y} - E(\boldsymbol{y})\right)\left(y - E(\boldsymbol{y})\right)'\right] \quad (17)$$

$$= \boldsymbol{Z}E(\boldsymbol{uu'})\boldsymbol{Z'} + \boldsymbol{Z}E(\boldsymbol{ue'}) + E(\boldsymbol{eu'})\boldsymbol{Z'} + E(\boldsymbol{ee'}) \quad (18)$$

Because u and e are independent, then

$$Var(y) = \boldsymbol{Z}E(\boldsymbol{uu'})\boldsymbol{Z'} + \boldsymbol{Z}(\boldsymbol{0}) + (\boldsymbol{0})\boldsymbol{Z'} + E(\boldsymbol{ee'}) \quad (19)$$

$$= \boldsymbol{Z}\mathbf{V_u}(\boldsymbol{\theta})\boldsymbol{Z'} + \mathbf{V_e}$$

then $Var(\boldsymbol{y}) = \boldsymbol{Z}\mathbf{V_u}(\boldsymbol{\theta})\boldsymbol{Z'} + \mathbf{V_e} = \boldsymbol{V}(\boldsymbol{\theta})$, So that $\boldsymbol{y}$ is normally distributed with the average $\boldsymbol{X\beta}$ and has a variance of **V (θ)**, with the Probability Density Function (pdf) function as follows:

$$f(y) = \frac{1}{(2\pi)^{\frac{1}{2}}|V(\theta|^{\frac{1}{2}}} exp\left\{-\frac{1}{2}\left(\boldsymbol{y} - \boldsymbol{X\beta}(\boldsymbol{\theta})\right)'\boldsymbol{V^{-1}}(\boldsymbol{\theta})(\boldsymbol{y} - \boldsymbol{X\beta}(\boldsymbol{\theta})\right\}$$

The likelihood function of y is:

$$L\left(\boldsymbol{\beta}(\boldsymbol{\theta})\right) = L(\boldsymbol{y}) = \prod_{\boldsymbol{1}}^{\boldsymbol{D}} f(\boldsymbol{y})$$

$$L(\boldsymbol{\beta}(\boldsymbol{\theta})) = \frac{1}{(2\pi)^{\frac{1}{2}}|V(\theta|^{\frac{1}{2}}} exp\left\{-\frac{1}{2}(\boldsymbol{y} - \boldsymbol{X\beta}(\boldsymbol{\theta}))'\boldsymbol{V}^{-1}(\boldsymbol{\theta})(\boldsymbol{y} - \boldsymbol{X\beta}(\boldsymbol{\theta})\right\}$$

The estimation of $\boldsymbol{\beta(\theta)}$ parameters using the Generalized Least Square (GLS) method based on [12] is as follows:

$$\boldsymbol{e}'\boldsymbol{V}^{-1}(\boldsymbol{\theta})\,\boldsymbol{e} = (\boldsymbol{y} - \boldsymbol{X\beta}(\boldsymbol{\theta}))'\boldsymbol{V}^{-1}(\boldsymbol{\theta})(\boldsymbol{y} - \boldsymbol{X\beta}(\boldsymbol{\theta})) \tag{20}$$

$$\widehat{\boldsymbol{\beta}}(\boldsymbol{\theta}) = (\boldsymbol{X}'\boldsymbol{V}^{-1}(\boldsymbol{\theta})\,\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{V}^{-1}(\boldsymbol{\theta})\boldsymbol{y} \tag{21}$$

2.2. Spatial Weighting Matrix

Spatial weighting matrix $\boldsymbol{W}$ shows the relationship between the regions in context of contiguity or a function of a distance. In the context of contiguity, the relationship between the region and others can be categorized into several methods, that is linear contiguity, rook contiguity, bishop contiguity, queen contiguity and customized contiguity [13]. The linear, rook, bishop and queen contiguity only consider the proximity of region and its surrounding territories. In the other hand, poverty is a product of various factors, one of which is socio-economic factors is the main employment [14]. Spatial weighting matrix that can be used to consider the main employment is customized contiguity.

2.2.1. Spatial Autocorrelation Test. To identify the spatial dependence is using [15] method, by applying the Moran's I statistic to test the spatial dependencies in the residuals of a regression model. The hypothesis that is used in this test are:

$H_0: I = 0$ (there are no spatial autocorrelation)

$H_1: I \neq 0$ (there are spatial autocorrelation)

With test-statistic

$$Z_{calc} = \frac{I - I_0}{\sqrt{var(I)}}$$

where

$I = \frac{n}{s_0}\frac{\sum_{i=1}^{n}\sum_{k=1}^{n} w_{ij}(X_i-\bar{X})(X_k-\bar{X})}{\sum_{i=1}^{n}(X_i-\bar{X})}$, with $var(I) = \frac{n^2 s_1 - n s_2 - 3 s_0^2}{(n^2-1)s_0^2} - [E(I)]^2$

$E(I) = I_0 = -\frac{\boldsymbol{1}}{\boldsymbol{n-1}}$, $s_0 = \sum_{i=1}^{n}\sum_{k=1}^{n} w_{ik}$, $s_1 = \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}(w_{ik} + w_{ki})$

$s_2 = \sum_{i=1}^{n}(w_{i.} + w_{.i})^2$, $w_{i.} = \sum_{k=1}^{n} w_{ik}$, $w_{.i} = \sum_{k=1}^{n} w_{ki}$

Reject H_0, if $|Z_{calc}| > Z_{\frac{\alpha}{2}}$. The value of $Z_{\frac{\alpha}{2}}$ follows normal standard distribution N (0,1).

2.3. Normality Test

Spatio-temporal Fay-Herriot models uses normal residual assumptions. Testing the normality assumption using the Kolmogorov-Smirnov test with the following hypothesis:

H_0: $F_n(x) = F_0(x)$

H_1: $F_n(x) \neq F_0(x)$

$F_n(x)$ is an empirical distribution function (based on sample) or cumulative opportunity value (cumulative distribution function) based on sample data.

$F_0(x)$ is a theoretical distribution function (according to hypothesized) or cumulative opportunity value (cumulative distribution function) below Ho.

With test-statistic

$$D = \sup_x |F_n(x) - F_0(x)|$$

Critical area: reject Ho if $D_{calculate} > D_{\alpha,n}$, D_α is a critical value for the Kolmogorov-Smirnov test obtained from the Kolmogorov-Smirnov table. For the selection of variables that are significantly influential in this study used α = 0.10.

3. Experimental Details

The data that is used in this study were secondary data obtained from BPS. The response variable, is the percentage of poverty in each village affected by the Susenas sample, derived from raw data of National Socio-Economic Survey (Susenas) 2011, 2014, 2017. The auxiliary variables are obtained from both of Potensi Desa (Podes) 2011, 2014, 2018 and Kecamatan Dalam Angka Polewali Mandar 2012, 2015, 2018. Unit of observation in this study were 24 villages where the villages in 2011, 2014 and 2017 were selected as Susenas samples respectively, so the total unit of observation is 72. The variables and their scale of measurements are presented in Table 1.

4. Results and Discussion

4.1. Data Exploration

Based on the 2018 Podes data collection, there are three main business fields, the majority of the population from 24 villages is the unit of observation in this study. The three main business fields are agriculture, trade and social services. Figure 1 shows the villages that were the unit of observation along with the main jobs of the village. The main jobs from the villages are the unit of observation.

Table 1. Variables and Their Scale of Measurement

Code	Variable	Scale of Measurement
(1)	(2)	(3)
y	Percentage of poverty	Ratio
x_1	Percentage of farming families	Ratio
x_2	Percentage of farm labor families	Ratio
x_3	Percentage of residents receiving Jamkesmas / Jamkesda	Ratio
x_4	Percentage of residents receiving Surat Keterangan Tidak Mampu	Ratio
x_5	Ratio of population to number of micro and small industries	Ratio
x_6	Average number of household members	Ratio
x_7	Ratio of population to number of educational institutions	Ratio
x_8	Average distance to health facilities	Ratio
x_9	Ratio of population to number of health facilities	Ratio
x_{10}	Ratio of population to number of health workers	Ratio
x_{11}	The percentage of families living on the riverbank	Ratio

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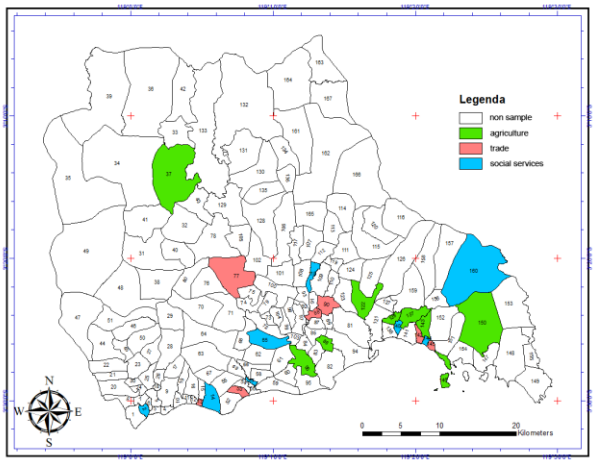

Figure 1. Map of Distribution of Villages that Become Observation Units and Main Employment Fields

4.2. Identification of Spatial Effects

The step to find out the effect of area effectiveness is to test spatial dependence. Testing for spatial dependence uses the Moran's index I statistics. If there were spatial effects, further step was to get what factors influence poverty in Polewali Mandar district by including spatial elements. By using the Moran's I test, Moran's I univariate value was obtained for the percentage variable of the poor population of 0.148 with p-value of 0.01695. The value of Moran's I is significant at the level of $\alpha = 0.10$. When viewed from the p-value smaller than α the decision is to reject H_0, so it can be concluded that there are spatial dependencies for the percentage variable of poor people in Polewali Mandar district, using customized contiguity weighting matrices based on the business field. Based on this, it is reasonable to estimate the percentage value of the poor at the village level by including the spatial approach.

Moran's scatter plot results for the percentage of poor people in Figure 2 show that many observation villages are in quadrants I and III. Quadrant I means villages that have a high percentage of poor people adjacent to villages that have a high percentage with the same majority of businesses. Quadrant III shows villages with a percentage of poor people who are close together and will group together with villages that also have a low percentage of poor people with the majority of the same business field.

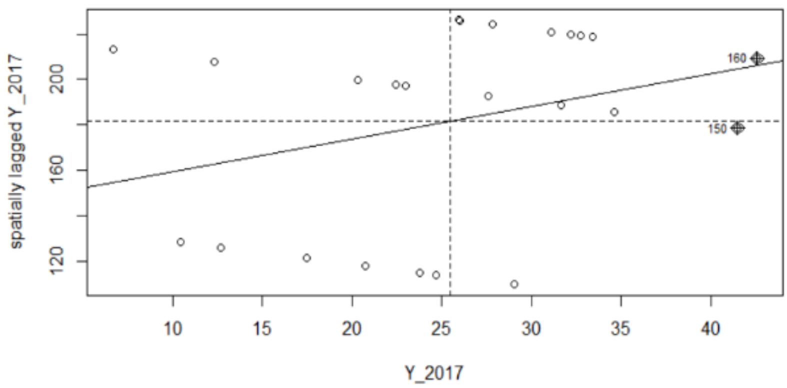

Figure 2. Moran's Scatter Plot on Percentage of Poor Population.

Furthermore, to find out whether there are cases of multicollinearity between the auxiliary variables can be seen from the VIF value, where the VIF value is smaller than 10, it can be concluded that there is no case of multicollinearity.

Table 2. VIF Value of each Auxiliary Variables

Variable	VIF	Information
(1)	(2)	(3)
x_1	2.275	There is no violation of the assumption of multicollinearity
x_2	1.950	
x_3	3.320	
x_4	2.173	
x_5	2.000	
x_6	2.384	
x_7	2.001	
x_8	2.057	
x_9	2.147	
x_{10}	2.970	
x_{11}	1.399	

The results of testing the assumption of multicollinearity on all participating variables indicate that there is no violation of the assumption of multicollinearity, as shown in Table 2. This can be seen from the value of the variance inflation factor (VIF) of all the auxiliary variables whose value is less than 10. The highest VIF value is found in the variable x_3 which is equal to 3.320. Thus the 10 auxiliary variables can be used at the next stage. The VIF values of each of the auxiliary variables can be seen in Table 2.

4.3. Significant Variables Affect Poverty

The method of selecting the auxiliary variables that affect the percentage of the poor is done by issuing auxiliary variables that has the greatest p-value at each stage. The estimation results with 11 variables, as in Table 3, show that the average variable distance to health facilities (x_8) is a variable with the highest p-value, which is equal to 0.710. Then in the second stage the x_8 variable is removed from the model. Then in the third stage the x_7 variable is removed from the model, and so on until finally the sixth stage with the remaining six auxiliary variables which are all significant at the significance level $\alpha = 0.10$.

Table 3. Estimation of The Regression Coefficient of The Auxiliary Variables

Estimator of Regression Coefficient	Coefficient value	Standard Error	*t-value*	*p-value*
(1)	(2)	(3)	(4)	(5)
$\hat{\boldsymbol{\beta}}_0$	44.085	20.604	2.140	0.032
$\hat{\boldsymbol{\beta}}_1$	0.047	0.085	0.558	0.577
$\hat{\boldsymbol{\beta}}_2$	0.255	0.190	1.343	0.179
$\hat{\boldsymbol{\beta}}_3$	0.201	0.113	1.776	0.076
$\hat{\boldsymbol{\beta}}_4$	1.104	0.784	1.409	0.159
$\hat{\boldsymbol{\beta}}_5$	-0.010	0.009	-1.151	0.250
$\hat{\boldsymbol{\beta}}_6$	-9.145	4.470	-2.046	0.041
$\hat{\boldsymbol{\beta}}_7$	-0.001	0.003	-0.497	0.620
$\hat{\boldsymbol{\beta}}_8$	0.038	0.103	0.372	0.710
$\hat{\boldsymbol{\beta}}_9$	0.016	0.005	3.139	0.002
$\hat{\boldsymbol{\beta}}_{10}$	-0.002	0.002	-0.825	0.409
$\hat{\boldsymbol{\beta}}_{11}$	0.636	0.378	1.683	0.092

Furthermore, up to the stage of selecting the sixth stage of the auxiliary variables, a significant auxiliary variables was obtained and had the smallest AIC and BIC values of 481.0947 and 506.1380.

A summary of the stages of selecting the auxiliary variables is presented in Table 4 while the results of the estimation of the regression coefficients in the sixth stage are presented in Table 6.

Table 4. Summary of Stages of Auxiliary Variables

Stage	Auxiliary Variables	AIC	BIC
(1)	(2)	(3)	(4)
1	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}$	488.2756	524.7023
2	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_9, x_{10}, x_{11}$	486.2493	520.3993
3	$x_1, x_2, x_3, x_4, x_5, x_6, x_9, x_{10}, x_{11}$	484.3842	516.2575
4	$x_1, x_2, x_3, x_4, x_5, x_6, x_9, x_{11}$	482.6865	512.2832
5	$x_2, x_3, x_4, x_5, x_6, x_9, x_{11}$	481.6132	508.9332
6	$x_2, x_3, x_5, x_6, x_9, x_{11}$	481.0947	506.1380

4.4. Spatio-Temporal Autoregressive Coefficients and Random Effects Variance

There are two autoregressive coefficients and two random effects variance, namely the estimated value of spatial autoregressive coefficient ($\hat{\rho}_1$) and temporal autoregressive coefficient ($\hat{\rho}_2$) and variance on random area effect ($\hat{\sigma}_1^2$) and variance on random time effect ($\hat{\sigma}_2^2$). In the first stage the estimated value of the variance of the random area effects is 63.130 and for the random time effect is 20.326. The estimated value of the area's autoregressive coefficient in the first stage is 0.623, this figure is lower than the time autoregressive coefficient of 0.730. In the sixth stage the estimation of the variance of the effect of the random area is 82.257 and the random time effect is 18.248.

Estimated values for each autoregressive coefficient in both the area and time generated at each stage are all positive. The area and time autoregressive coefficients in the sixth stage were each valued at 0.529 and 0.654, indicating a strong spatial and time relationship between the observation villages and the percentage of poor people using spatial weighting matrices of customized contiguity based on the main businesses in the majority of villages. The estimated value of area and time autoregressive coefficients and the variance of random area time effect are fully presented in Table 5.

Table 5. The Estimation of Spatio-Temporal Autoregressive Coefficients and Random Effects Variance

Stage	Auxiliary Variables	$\hat{\sigma}_1^2$	$\hat{\rho}_1$	$\hat{\sigma}_2^2$	$\hat{\rho}_2$
(1)	(2)	(3)	(4)	(5)	(6)
1	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}$	63.130	0.623	20.326	0.730
2	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_9, x_{10}, x_{11}$	59.995	0.634	20.099	0.749
3	$x_1, x_2, x_3, x_4, x_5, x_6, x_9, x_{10}, x_{11}$	70.003	0.594	19.213	0.689
4	$x_1, x_2, x_3, x_4, x_5, x_6, x_9, x_{11}$	62.852	0.606	19.550	0.707
5	$x_2, x_3, x_4, x_5, x_6, x_9, x_{11}$	48.899	0.618	20.388	0.778
6	$x_2, x_3, x_5, x_6, x_9, x_{11}$	82.257	0.529	18.248	0.654

Based on the procedure for selecting significant variables based on $\alpha = 0.10$, six variables were found that significantly affected poverty in Polewali Mandar district. To ensure that these variables are significant, the residuals are tested first from the value of the actual response variable against the estimated variable value by entering these six variables into the model.

Table 6. Estimated Regression Coefficient with Selected Auxiliary Variables

Estimator of Regression Coefficient	Coefficient value	Standard Error	*t-value*	*p-value*
(1)	(2)	(3)	(4)	(5)
$\hat{\beta}_0$	48.25426	16.19273	2.979995	0.0029
$\hat{\beta}_2$	0.305989	0.16061	1.905167	0.0568
$\hat{\beta}_3$	0.218099	0.098973	2.203629	0.0276
$\hat{\beta}_5$	-0.01333	0.007433	-1.79388	0.0728
$\hat{\beta}_6$	-9.52484	3.680835	-2.58769	0.0097
$\hat{\beta}_9$	0.018333	0.003977	4.610314	0.0000
$\hat{\beta}_{11}$	0.592059	0.352282	1.680638	0.0928

4.5. Result of Normality Test

The formal test used in testing the normality of this study using the Kolmogorov-Smirnov test. The result is p-value greater than 0.150 while the Kolmogorov-Smirnov (KS) value is 0.059. This value of p-value is greater than the significance level of α = 0.10, which means that the decision failed to reject H_0. It can be concluded that the residuals from the data are normally distributed.

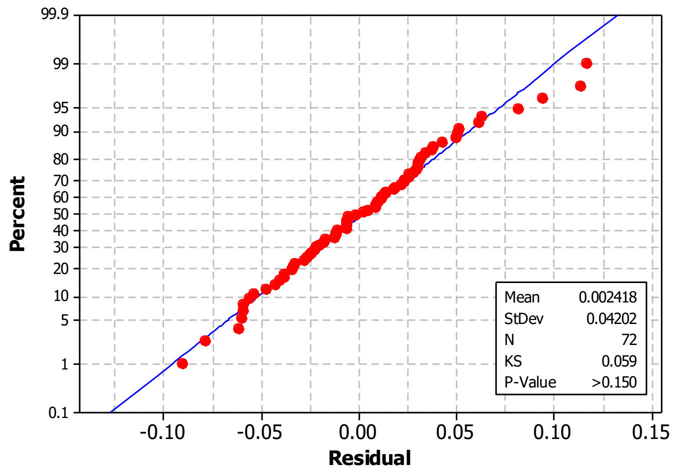

Figure 3. Probability Plot and Residual Normality Test Using the Kolmogorov-Smirnov Test

The results of the selection of significant variables can be modeled as follows:

$$\hat{\theta}_{dt} = 48.2543 + 0.3060x_2 + 0.2181x_3 - 0.0133x_5 - 9.5248x_6 + 0.0183x_9 + 0.5921x_{11}$$

In general, the model suggest that when the others factor held constant, then, if the percentage of farm labor families (x_2) increased by 100%, it will increase the value of the percentage of poverty by 0,3060 in the village *d* at time *t*. Then, if the average number of household members (x_6) in the village *d* at time *t* increased by 100%, it will reduce the value of the percentage of poverty by 9.5248.

5. Conclusion

Poverty is a complex problem that occurs in almost all regions in Indonesia. Polewali Mandar is a district with the highest poverty rate in the province of West Sulawesi. To overcome the problem of poverty must be known to spread the poor at the smallest level. To deal with the problem of poverty in Polewali Mandar district, it is necessary to know the factors that influence poverty. The SAE method can be applied to estimate the percentage of poor people at the smallest level. In SAE, the auxiliary variables that influence poverty are needed to produce valid estimates. From the use of the Spatio-Temporal Fay-

Herriot method there are six factors that influence poverty in Polewali Mandar district namely: the percentage of farm worker's families, the percentage of the population receiving Jamkesmas / Jamkesda, ratio of population to number of micro and small industries, average number of household members, ratio of population to number of health facilities and percentage of families living on river banks.

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