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To cite this article: Popy Febritasari *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **546** 052023

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Generalized Linier Autoregressive Moving Average (GLARMA) Negative Binomial Regression Models with Metropolis Hasting Algorithm

Popy Febritasari^{1*}, Ni Wayan Surya Wardhani¹, Ummu Sa'adah¹

¹Statistic Department, Faculty of Natural Sciences, University of Brawijaya

*Corresponding author: popyfebrita@yahoo.com

Abstract: This paper discusses regression models when the variance in count data is not equal to the mean. It happens in mortality cause of traffic accident data in jurisdiction's territory of Dharmasraya's Police Resort, where the variance is larger than the mean, which is called overdispersion. In this case we used negative binomial regression in time series with generalized linier autoregressive moving average (GLARMA) models. The parameters were estimated using maximum likelihood estimation (MLE) method and metropolis hasting algorithm at 100th burn-in period and 150000 iteration. The prior distribution and the number of iteration in metropolis hasting algorithm had less Mean Square Error (MSE) than MLE method. Prediction for next period using model metropolis hasting algorithm.

Keywords: Overdispersion, Negative Binomial, Metropolis Hasting Algorithm.

1. Introduction

Poisson regression model is basically a regression model that assumes the response variable has Poisson distribution. Researcher extensively uses poisson regression models to solve regression problem of even data or count data. Poisson regression have similar assumption between variance of response variable with mean response variable (equidispersion). Rarely, variance of response variable that larger than the mean resulted in overdispersion. Handling overdispersion poisson model can be done by using Negative Binomial Model. Random sampling X_1, X_2, \dots, X_n of population that have Negative Binomial distribution with parameter p is written as [1]:

$$\begin{aligned} X_i &\sim BN(r, p) \\ f(x_i|p) &= \binom{x_i-1}{r-1} p^r (1-p)^{x_i-r} \\ i &= 1, 2, 3, \dots, n \end{aligned}$$

Prior distribution for $X_i \sim BN(r, p)$ is $p \sim BETA(\alpha, \beta)$. So the prior probability distribution function is:

$$\begin{aligned} \pi(p) &= \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1} (1-p)^{\beta-1} \\ f(x_1, x_2, \dots, x_n|p) &= \prod_{i=1}^n \binom{x_i-1}{r-1} p^r (1-p)^{x_i-r} \end{aligned}$$



$$= \left[\prod_{i=1}^n \binom{x_i - r}{r - 1} \right] p^{nr} (1 - p)^{\sum_{i=1}^n x_i - nr}$$

It is assumed that Negative Binomial Model parameter uses Maximum Likelihood Estimation when the population distribution was known and just based on sample interference. The MLE method estimate the parameter by maximize the likelihood function for joint probability x_1, x_2, \dots, x_n and $\theta_1, \theta_2, \dots, \theta_n$ as a θ parameter function. If random sample is X_1, X_2, \dots, X_n and joint probability function is $f(i, \theta)$ where $i = 1, 2, \dots, n$, then $L(\theta) = \prod_{i=1}^n f(x_i, \theta)$ is the likelihood function. If probability function is $f(x_i, \theta)$ and likelihood function $L(\theta)$, then $w = h(x_1, x_2, \dots, x_n)$ that maximize $L(\theta)$ and $L(w) \geq L(\theta)$ is called Maximum Likelihood Estimation. The probability function for parameter estimation can be written as [2]:

$$\beta^* = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix}$$

$$L(\beta^*) = \prod_{i=1}^n f(Y_i | x_{1i}, x_{2i}, \dots, x_{pi}; \beta, \alpha)$$

$$l(\beta^*) = \log \left\{ \prod_{i=1}^n \left(Y_i | x_{1i}, x_{2i}, \dots, x_{pi}; \beta_0, \beta_1, \dots, \beta_p, \alpha \right) \right\}$$

The MLE is obtained by partial differentiating $l(\beta^*)$ to $\beta_0, \beta_1, \dots, \beta_p$ where $\frac{d l(\beta^*)}{d \beta_i} = 0$. When population distribution is not known, Bayesian method will be applied for parameter estimation. Bayesian method that being used is based on prior distribution population.

Bayesian method is a method to obtain parameter estimation with sample's information [3]. The Bayesian estimation for θ is defined by[4]:

$$\begin{aligned} \hat{\theta} &= E[\theta | X_1, X_2, \dots, X_n] \\ &= \frac{\int \theta \pi(\theta) f(x_1, x_2, \dots, x_n | \theta) d\theta}{\int \pi(\theta) f(x_1, x_2, \dots, x_n) d\theta} \\ &= \frac{\int \theta f(x_1, x_2, \dots, x_n | \theta) d\theta}{\int f(x_1, x_2, \dots, x_n) d\theta} \end{aligned}$$

Metropolis-Hasting (MH) algorithm is one of the Bayesian method which is used to obtain random sample's sequence from posterior distribution. MH algorithm is used to estimate Negative Binomial *iid*

regression. Sampling for distribution π to estimate the parameter defined by $E_{\pi} f(X)$ where $X \sim \pi$. The

MH algorithm generate random walk density function as well as sample's sequence using accept-reject mechanism. MH algorithm needs large iteration to achieve convergent parameter. Thus, some researchers face obstacle at computation with large iteration.

2. Methodology

Research of negative binomial regression got through some stages. First stage, research data was collected and identified. The data that has been identified will be examined for dispersion assumption and lag of autoregressive moving average. If this fulfil the overdispersion assumption, then negative binomial regression can be used for overdispersion handling. Research use generalized linier

autoregressive moving average's model (GLARMA) for handling lag of autoregressive moving average. Second stage, determination of prior distribution and posterior distribution for GLARMA negative binomial. Third stage, MH algorithm based on prior distribution and its posterior was used to get parameter estimation of GLARMA negative binomial regression and it will be used for predicting response variable of the next period. Fourth stage, response variable of the next period was predicted and mean square error (MSE) of MH algorithm was calculated. Last stage, conclusion was taken according to MH algorithm and MLE.

2.1. Data

The data in this research was secondary data from Traffic Accident's Unit of Dharmasraya's Police Resort West Sumatera that have 63 data [7]. It is divided by 2 part, which were:

- (i) In-sample's data (modeling): Mortality of traffic accident in January 2014-December 2018.
- (ii) Out-sample's data (validation): Mortality of traffic accident in January 2019-March 2019.

The variable that used in this researches were:

y : The numbers of traffic accident's victim who died in jurisdiction's territory of Dharmasraya's Police Resort during January 2014-March 2019

2.2. Flowchart

Figure 1 below shows the research stages:

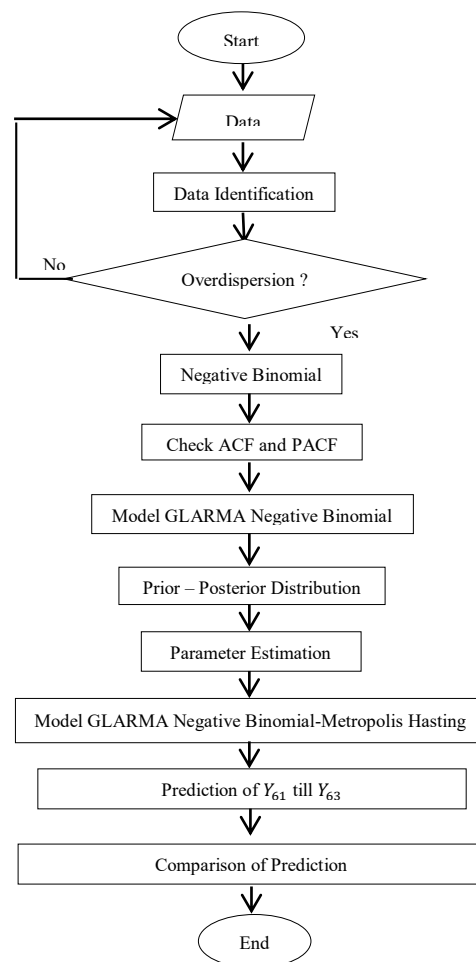


Figure 1. Research flowchart

3. Results and Discussion

3.1. Regression Models

Firstly, data that involved in this study such as data distribution, mean, and variance of response variable were identified. The data had poisson distribution. Variance of response variable that larger than the mean was called overdispersion where variance of Y_t (2.620056) > mean of Y_t (1.91667). Lag of autocorrelation function and lag of partial autocorrelation function have been seen in Figure 2. Autoregressive's orde is subset [1,3] and moving average's orde is [2]. Thus, researchers used generalized linier negative binomial with orde ([1,3], 0, [2]).

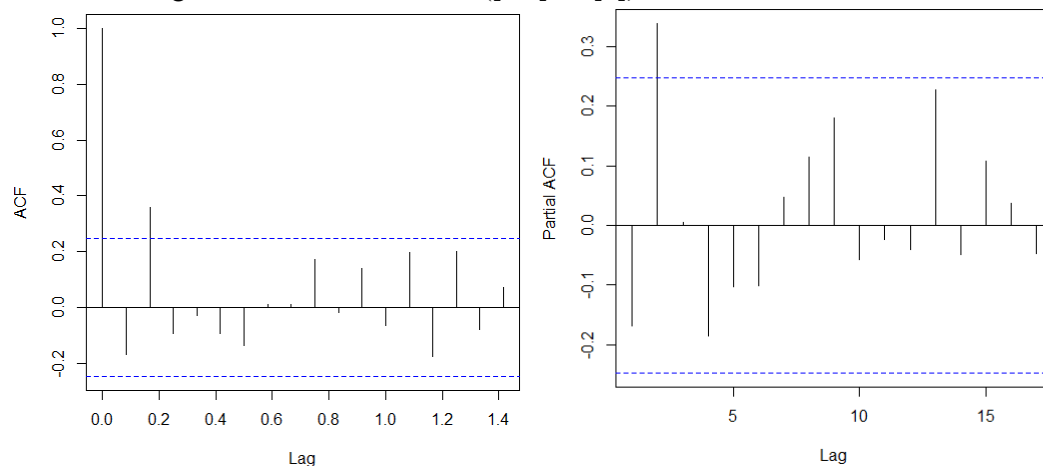


Figure 2. ACF and PACF Plot

Metropolis Hasting algorithm was written as [5]:

1. X_0 at every iteration with $n = 1, 2, \dots, N$
2. Sample $j \sim q_{ij}$, where $Q = \{q_{ij}\}$
3. Generate $U \sim v(0,1)$ where v is uniform distribution at (0,1)
4. Probability

$$\alpha_{ij} = \min \left\{ 1, \frac{\pi_j q_{ji}}{\pi_i q_{ij}} \right\}$$

where

$$X_{n+1} = \begin{cases} j, & \text{if } U \leq \alpha_{ij} \\ \text{new experiment,} & \text{others} \end{cases}$$

Regression model formation used in-sample's data. Metropolis Hasting algorithm was used for parameter estimation in negative binomial models in 100th burn-in period and iteration was 150.000, thus resulted in *mean square error* (MSE) 1.770433. Parameter estimation was provided in Table 1 and it generated the negative binomial regression-MH algorithm model $Y_t = \exp(-0.4312 - 0.21167 Y_{t-1} + 0.04804 Y_{t-3} + 0.44914 \mu_2)$.

Table 1. Results of Metropolis Hasting Algorithm Parameter

	Mean	Standard Deviation
Intercept	-0.43412	1.478
Beta1	-0.21167	0.219
Beta2	0.04804	0.532
Tetha	0.44914	1.521
Sigma	0.29400	-

Better iteration gave more convergent parameter. The convergent level was seen through trace plot with density image as shown in Figure 3. The largest iteration, the more convergent it was, thus resulted to smaller MSE.

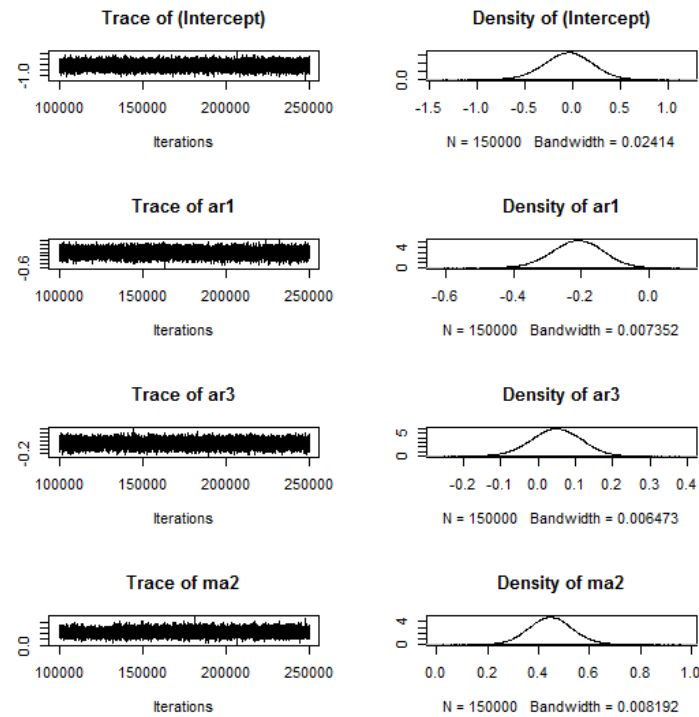


Figure 3. Trace plot and density plot of Metropolis Hasting Algorithm

3.2. Prediction of Response Variable

Regression model obtained from GLARMA negative binomial-metropolis hasting algorithm was used for predicting the numbers of traffic accident's victim who died in jurisdiction's territory of Dharmasraya's Police Resort during January 2019-Maret 2019. The prediction result had MSE 0.66667. In Figure 4, the prediction result traffic accident's victim who died in jurisdiction's territory of Dharmasraya's Police Resort during January 2019-Maret 2019 ($Y_{61} - Y_{63}$) was shown. GLARMA negative binomial-MH algorithm regression and GLARMA negative binomial-MLE models can be used for predicting. For the next periode (Y_{61} till Y_{63}), both of models was simulated for predicting Y_{61} till Y_{63} .

Table 2. Comparison of Prediction Y_{61} till Y_{63} (January 2019 – March 2019)

Time	Actual	MLE	MH
January 2019	0	2	1
February 2019	0	1	1
March 2019	1	2	1
MSE	-	2.00	0.66667

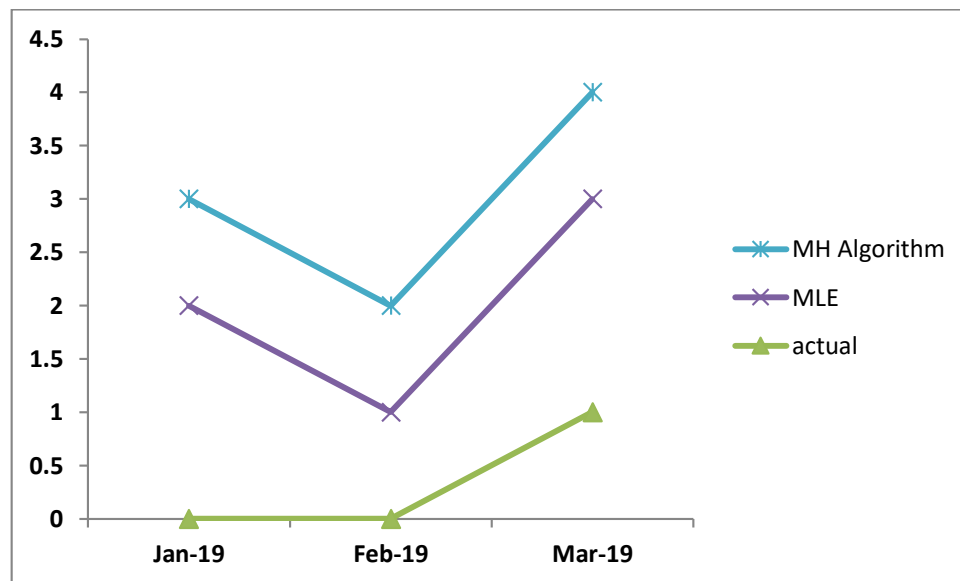


Figure 4. Plot of data actual and prediction using MLE and Metropolis Hasting Algorithm

4. Conclusion

Parameter estimation of GLARMA negative binomial models using metropolis hasting algorithm generated better results. GLARMA negative binomial-metropolis hasting algorithm regression models defined by $Y_t = \exp(-0.4312 - 0.21167 Y_{t-1} + 0.04804 Y_{t-3} + 0.44914 \mu_2)$ with MSE 1.770433. It is concluded that Metropolis Hasting algorithm can minimize the mean square error for optimizing the estimation of GLARMA Negative Binomial parameters. But for the next research, we can formulate models using other estimation such as Gibbs sampler, Kalman filter with state model, etc.

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