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Two-State Poisson Hidden Markov Models for Analysis of Seismicity Activity Rates in West Nusa Tenggara

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Abstract. Awareness of seismicity activity rates could be learned from modeling the earthquake events by utilizing the record of seismicity events data in NTB over time which is associated with count time series data. Poisson Hidden Markov Model (PHMM) has been widely applied in various fields, including earthquake event. Therefore, it would be interesting to implement PHMM on earthquake case in NTB. The data can be analyze using PHMM as we can ignore the over-dispersion and dependency relationship among data. The model is the development of Markov Model that consists of (a) observed state, which can be observed directly and (b) hidden state, which cannot be observed directly because it is hidden. Hidden state in this research is defined as seismicity activity rates classified into a low rate and high rate (2 states). The count time series data of earthquake events will be more informative when it is classified into the seismicity activity levels. This research applied earthquake event data (magnitude $\leq M4.7$ and depth < 60 km) from January 2009 until September 2018, collected from USGS (United States Geological Survey). The parameter estimation method used in this research is the Bayesian method. The objective of this research is to obtain parameters of 2 state Poisson Hidden Markov Model using the Bayesian approach. Model validation measured by MAE (Mean Absolute Error). Based on the result, the average earthquake cases caused by low seismicity activity rate in NTB over time is 1 event whilst the high rate is 18 events. The probability of low seismicity activity rate influenced by the previous rate and the long run behavior (steady state) in NTB is still larger than the high rate. The achieved two-state PHMM is suitable for modeling the earthquake case in NTB with MAE values of 0.4079.

Keywords: Poisson Hidden Markov Models, Earthquake, Bayesian.

1. Introduction

Earthquake is a phenomenon of energy release caused by interference within the earth's crust in the form of wave vibrations that spread to the earth surface [1]. In August 2018, an earthquake hit NTB which caused a negative impact on the surrounding people, as informed by BNPB (National Agency for Disaster Management), among others: many people died and suffered injuries, house damage, public facilities as well as material losses in varied sectors. Therefore, the people in NTB Province need to be aware of the future earthquake. One way that can be done is by modeling the data on the number of earthquakes that have occurred

The number of earthquakes data is related to count time series data, namely time series data which observations come from the sum of several objects or events that have dependency relationship and non-negative integers [2]. The analysis that can be applied to count time series data is Poisson Hidden Markov Model (PHMM).



The PHMM method has been widely used in various fields, including earthquake. Research conducted by several researchers in earthquakes field is PHMM method through the EM (Expectation Maximization) algorithm [3,4], PHMM method and Poisson Process (PP) through the EM algorithm [5], Multivariate PHMM method by EM algorithm [6]. It shows that many PHMM analysis in earthquake cases use the Maximum Likelihood through the EM algorithm. Nonetheless, the parameter estimation method on PHMM used in this study is the Bayesian method.

PHMM is a development of the Markov Model where the next event ($t+1$) only depends on the current event (t), which consists of observed states (a variable that can be observed directly) and hidden states (hidden variables and cannot be observed directly) [7,8]. The observed state in this study is a number of earthquakes data. While the hidden state is a seismicity activity rate, assuming that the rate causes earthquakes which form Markov chain and cannot be observed directly. The achieved number of earthquake data will be more informative when the data is classified into several levels of seismic activity. The number of states used in this study uses two-state, consist of low and high seismicity activity rates.

The data used in this study is the number of earthquakes data in NTB from January 2009 to September 2010 collected from USGS (United States Geological Survey). This study aims to get the parameter estimation in the PHMM analysis using two-states with the Bayesian method approach. The MAE method was used in this study to validate the obtained of two-state PHMM.

An illustration of PHMM concept will be explained in section 2, the research method described in section 3, the results and discussion are presented in section 4, the conclusion of the study and suggestions for further research are shown in section 5.

2. Literature Review

2.1. Poisson Hidden Markov Model (PHMM)

Each observation using PHMM analysis is produced by one of the m state different Poisson processes called hidden state [3]. PHMM analysis can be used even though over-dispersion (variance is greater than the mean) occurs and there is a dependency relationship between data [9]. These two things are often found when using time series count data. The mean and variance in Poisson distribution can be counted as follows [10]:

$$E(X) = \lambda \quad (2.1)$$

$$Var(X) = E(X^2) - E(X) = \lambda \quad (2.2)$$

There are several characteristics in PHMM [9,11,12];

1. The number of hidden state in the model, symbolized by m .
The m state in PHMM is defined as $\{\lambda_1, \lambda_2, \dots, \lambda_m\}$ where the state at the time interval t expressed by S_t .
2. Observed state i.e. observations in each state can be observed directly and have a non-negative integer value which is symbolized by X , where:
 $X = \{X_1, X_2, X_3, \dots, X_T\}$.
3. Initial state distribution or $\delta(1) = P(S_1 = i) = \delta(i)$, where $\delta_i \in \delta$ and $1 \leq i \leq M$, order $1 \times m$.
4. Transition probability matrix (t.p.m) $\Gamma_{ij}(t) = \{\gamma_{ij}(t)\}$ expressed by Γ , describes the probability for the transition from state λ_i at the time t to state λ_j at the time $t + 1$, where:

$$\gamma_{ij}(t) = P(S_{t+1} = \lambda_j | S_t = \lambda_i) \quad (2.3)$$

where, $j = 1, \dots, m$; any time $t = 1, 2, \dots, T$. The sum of transition probability in each row is 1.

$$\text{Matrix } \Gamma_{m \times m} = \begin{bmatrix} \gamma_{11} & \dots & \gamma_{1m} \\ \gamma_{21} & \gamma_{22} & \dots \\ \vdots & \vdots & \vdots \\ \gamma_{m1} & \dots & \gamma_{mm} \end{bmatrix}$$

The important role of Γ in PHMM is following the MC homogeneity provisions that meet the Chapman Kolmogorov equation:

$$\Gamma(t) = \Gamma(1)^t, \text{ for } t \in T$$

Transition probability matrix Γ in MC have stationary distribution δ if:

$$\delta\Gamma = \delta \text{ and } \delta\mathbf{1}' = \mathbf{1} \quad (2.4)$$

5. *State-dependent probability distribution* or $\phi_i(x)$ expressed with a matrix Φ that consists of element $\phi_i(x)$ order $m \times m$.

$$\phi_i(x) = P(X_t = x | S_t = \lambda_i) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & , \text{ for } x = 1, 2, 3, \dots \\ 0 & , \text{ others} \end{cases} \quad (2.5)$$

6. The marginal probability in PHMM is illustrated by the following equation:

$$P(X_t = x) = \sum_{i=1}^m P(S_t = i)P(X_t = x | S_t = \lambda_i) = \sum_{i=1}^m \delta_i(t)\phi_i(x) \quad (2.6)$$

Therefore, the PHMM model $\{X_t, S_t\}$ has parameters $\theta = \{\lambda, \Gamma, \delta\}$, where:

$$\lambda = [\lambda_i], i = 1, 2, \dots, m; \Gamma = [\gamma_{ij}], i, j = 1, 2, \dots, m; \delta = [\delta_i], i = 1, 2, \dots, m.$$

2.2. Estimating Parameters on PHMM with Bayesian Approach

Estimation of parameters used in this study is using the Bayesian method. The information of the parameter estimation method for PHMM through Bayesian is shown in subsection 2.2.1, subsection 2.2.2 and subsection 2.2.3.

2.2.1. Bayesian Concept

The Bayesian theorem is illustrated as follows [13]:

$$f(\theta|x) = \frac{f(x|\theta)f(\theta)}{f(x)} \propto f(x|\theta)f(\theta) \quad (2.7)$$

where:

$f(\theta|x)$ = posterior distribution, proportional to the product of multiplication among prior distribution and likelihood function.

$f(\theta)$ = prior distribution

$f(x|\theta)$ = likelihood function, is probability density function of n variable $X_1, X_2, X_3, \dots, X_n$ expressed in the form of $P(x_1, x_2, \dots, x_n | \theta)$ in formula (2.8).

$$f(x|\theta) = \prod_{i=1}^n f(x_i | \theta) = f(x_1 | \theta)f(x_2 | \theta) \dots f(x_n | \theta) \quad (2.8)$$

2.2.2. Markov Chain Monte Carlo

The Bayesian method aims to obtain estimated parameters from the posterior distribution. However, the posterior distribution is difficult to be used numerically because of its complex calculations. Therefore, in order to overcome the problem, the simulation method needs to be used by MCMC (Markov Chain Monte Carlo). One of the most applied MCMCs is Gibbs Sampler. Each stage in the Gibbs Sampler method is done by generating random values from unidimensional distribution through varied statistical computing which is widely available [13]. The Markov chain is a stochastic process:

$$\theta = \{\theta^{(1)}, \theta^{(2)}, \dots, \theta^{(K)}\} \text{ where } f(\theta^{(k+1)} | \theta^{(k)}, \dots, \theta^{(1)}) = f(\theta^{(k+1)} | \theta^{(k)}).$$

The stages in the MCMC generate a random sample $\theta_k = \theta^{(1)}, \theta^{(2)}, \dots, \theta^{(K')}$ where K' is the number of generated iterations (K) minus the amount of *burn-in* (B). Each function $W(\theta)$ in the interested parameter θ , can obtain the estimated parameter of posterior distribution with the following equation:

$$\hat{E}(W(\theta)|x) = \overline{W(\theta)} = \frac{\sum_{k=1}^{K'} W(\theta^{(k)})}{K'} \quad (2.9)$$

Significance testing of Bayesian estimated parameter was carried out using 95% credible intervals within 2.5% lower limit percentile and 97.5% upper limit percentile. The estimated parameter is said to be significant when the range does not contain a zero value.

One method that can be used to check the convergence of the estimated parameter in Gibbs Sampler is a trace plot (the plot between generated parameter estimation and the number of iterations).

If the generated parameter estimation in the trace plot shows a stable plot, it can be said that the result of the estimated parameters is convergent.

2.2.3. PHMM analysis using Bayesian Approach.

The Gibbs Sampler Steps on PHMM are as follows [14,15,16,17]:

- (1) Determine the likelihood function as equation (2.10). Meanwhile, the prior distribution in estimated parameters $\hat{T}$ and $\hat{\tau}$ is found in equation (2.11) and equation (2.12) below:

$$f(x, s | \hat{\lambda}, \hat{T}) \propto \hat{\delta}_o \left(\prod_{i=1}^m \prod_{j=1}^m (\hat{\gamma}_{ij})^{n_{ij}} \right) \times \left(\prod_{j=1}^m \hat{\lambda}_j^{\sum_t x_{st}} \exp(-\hat{n}_j \hat{\lambda}_j) \right) \quad (2.10)$$

$$\hat{\tau}_j \sim \Gamma(\hat{a}_j, \hat{b}_j) \quad (2.11)$$

$$\hat{T}_r \sim \text{Dirichlet}(\hat{v}_r) \quad (2.12)$$

Estimating parameters for parameter $\hat{\lambda}$ in each state is obtained by equation (2.13):

$$\hat{\lambda}_i = \sum_{j=1}^i \hat{\tau}_j \quad (2.13)$$

- (2) Initialization of estimated parameters $\hat{T}$, $\hat{\tau}$ and $\hat{\lambda}$.

The initialization of the estimated parameter is important so that the iterative process carried out is more effective. Determination of estimated parameter initialization $\hat{T}$, $\hat{\tau}$ and $\hat{\lambda}$ is as follow:

- a. $\hat{\tau}_j$ is achieved based on prior distribution in equation (2.11) with $\hat{a}_j = \hat{b}_j = 0.001$.
- b. $\hat{\lambda}_i$ is obtained from equation (2.13)
- c. $\hat{T}_r$ is done based on prior distribution in equation (2.12) with $\hat{v}_r = 1$.

- (3) Simulation in Hidden State

The hidden state simulation is done through a recursion process wherein the hidden state initialization is $P(s_0 = i | X_0, \theta)$. This recursion process requires forward path simulation from $t = 1$ to $t = T$ and then the backward path from $t = T$ to $t = 1$. The recursion process in the hidden state is as follows:

1. Forward

The prediction step is done by calculating the equation (2.14)

$$a. P(s_t = j | X_{t-1}, \theta) = \sum_{i=1}^m \gamma_{ij} \times P(s_{t-1} = i | X_{t-1}, \theta) \text{ for } j = 1, 2, \dots, m \quad (2.14)$$

Where $P(s_0 = i | X_0, \theta) = \delta(1) = 1/m$ is initial state distribution.

- b. The updating step is done by calculating equations (2.15).

$$P(s_t = j | X_t, \theta) \propto P(s_t = j | X_{t-1}, \theta) \times \phi_i(x) \quad (2.15)$$

Where $\phi_i(x)$ is state dependent probability in formula (2.5).

Repeat steps a and b at $t = 1, 2, \dots, T$.

2. Generating s_T by $P(s_T = j | X_T, \theta)$.

3. Backward

Suppose, the generated result is $s_{t+1} = l$, then the obtained equation (2.16).

$$P(s_t = j | X_T, s_{t+1} = l, \theta) \propto \gamma_{jl} \times P(s_t = j | X_t, \theta) \text{ for } j = 1, 2, \dots, m \quad (2.16)$$

Repeat the step on $t = T-1, T-2, \dots, 2, 1$.

4. Generate s_t from $P(s_t = j | X_T, s_{t+1} = r, \theta)$.

- (4) Updating Estimated Parameters.

The hidden state that was generated earlier is used as a reference in generating estimated parameters both at $\hat{T}_r$ and $\hat{\tau}_j$ in each active state. Matrix Γ can be updated by following posterior distribution:

$$f(\hat{T}_r | x, s) \sim \text{Dirichlet}(\hat{v}_r + \hat{n}_r) \quad (2.17)$$

where:

$\hat{v}_r$ = vector parameters ($v_{r1}, v_{r2}, \dots, v_{rm}$) on transition matrix r -th row.

$\hat{n}_r = \{n_{ij}\}$ = the matrix of the number of transitions from state i to j in the r row.

While vector $\hat{\lambda}_j$ is updated by generating τ_j where $(j = 1, \dots, m)$ by posterior distribution of $\hat{\tau}_j$, namely:

$$f(\hat{\tau}_j | x, s) \sim I(\hat{a}_j + \sum_{t=1}^T x_{jt}, \hat{b}_j + \hat{N}_j) \quad (2.18)$$

where:

$\hat{a}_j$ = form parameters at state j , $\hat{a}_j > 0$; $\hat{b}_j$ = scale parameters at state j , $\hat{b}_j > 0$;

$x_t = \sum_{j=1}^m x_{jt}$ = the sum of contribution given by x_t to be x_{jt} on m ;

$\hat{N}_j = \sum_{t=1}^m I(s_t \geq s)$ = the amount of time when regimes s is active.

After the estimated parameter $\hat{\tau}$ is obtained, the estimated parameter $\hat{\lambda}_i$ will be obtained based on equation (2.13).

- (5) Step (3) and step (4) are repeated as many as K iterations until the results have converged based on the trace plot.
- (6) The results of estimated parameters $\hat{T}_r$ and $\hat{\lambda}_i$ which are the averages of the posterior distribution in equation (2.9) are accomplished.

2.3. MAE (Mean Absolute Error)

After the estimated parameters are obtained, the estimated data is generated ($\hat{x}_t$). Hereafter, an error value is got from the difference between the actual or training data (x_t) and the estimated data at the time t ($\hat{x}_t$), as equation (2.19).

$$e_t = x_t - \hat{x}_t \quad (2.19)$$

The error value is applied as input to calculate the MAE value that can be used to test the model validity. The following equation for getting the MAE value is [18]:

$$MAE = \frac{1}{n} \sum_{t=1}^n |e_t| \quad (2.20)$$

where: $|e_t|$ = absolute value of error; n = number of expected samples.

3. Research Methodology

3.1. Research Data

The data in this study is the number of earthquakes events in NTB from January 2009 to September 2018 with earthquake magnitude $\geq M4.7$ and shallow earthquake depth < 60 km, collected from the USGS (United States Geological Survey).

3.2. PHMM Analysis with Bayesian Approach

The steps of PHMM analysis using the Bayesian approach are as follows:

- (1) Inputting research data (data training).
- (2) Over-dispersion checking on data training using equations (2.1) and (2.2). If the sample variance value is greater than the sample mean, it is said that the training data is over-dispersed.
- (3) Conducting the analysis of two-state PHMM using Gibbs Sampler simulation methods as below:
 - a) Determine the likelihood function as well as the prior distribution in estimated parameters $\hat{T}$ and $\hat{\tau}$ using equations (2.1), (2.11) and (2.12). While parameter estimating $\hat{\lambda}$ is done based on equation (2.13).
 - b) Perform initialization of parameter estimation $\hat{T}$, $\hat{\tau}$ and $\hat{\lambda}$.
 - c) Generate hidden states using equations (2.14), (2.15) and (2.16).
 - d) Update $\hat{T}_r$, $\hat{\tau}_j$ and $\hat{\lambda}_i$ based on equations (2.17), (2.18) and (2.13).
 - e) Repeat Step (c) and step (d) until "burn in period" and the estimated parameters value are convergence based on the trace plot.
 - f) Obtain the estimated parameters $\hat{T}_r$ and $\hat{\lambda}_i$ from posterior distribution mean in equation (2.9).
- (4) Testing the significance of the parameter estimation results through the Bayesian approach using credible interval on the 2.5% percentile and 97.5% percentile.

(5) Once estimated parameters on the matrix $\hat{T}$ are achieved, then counting stationer state distribution $\hat{\delta}$ based on a formula (2.4).

(6) Generating estimated data based on the results of estimated parameters and calculating the amount of MAE based on equation (2.20).

This study used software *R* (package *R2OpenBUGS* and *coda*) using the coding of Gibbs Sampler in PHMM [16].

4. Result and Discussion

4.1. Over-dispersion Checking Result

The average number of earthquake events in NTB for the period January 2009 - September 2018 with magnitude scale $\geq M4.7$ and depth < 60 km or $E(X)$, is 0.6325, while the variance data or $Var(X)$ is 14.0448. It can be concluded that the data is over-dispersed ($Var(X) > E(X)$). Therefore, PHMM can be continued at the next stage.

4.2. Result of Estimation Parameter of PHMM with Bayesian

PHMM analysis using the Bayesian approach in this study applies 100,000 iterations with 500 burn-in. The convergence of estimated parameters can be seen in the trace plot displayed in Figure 1.

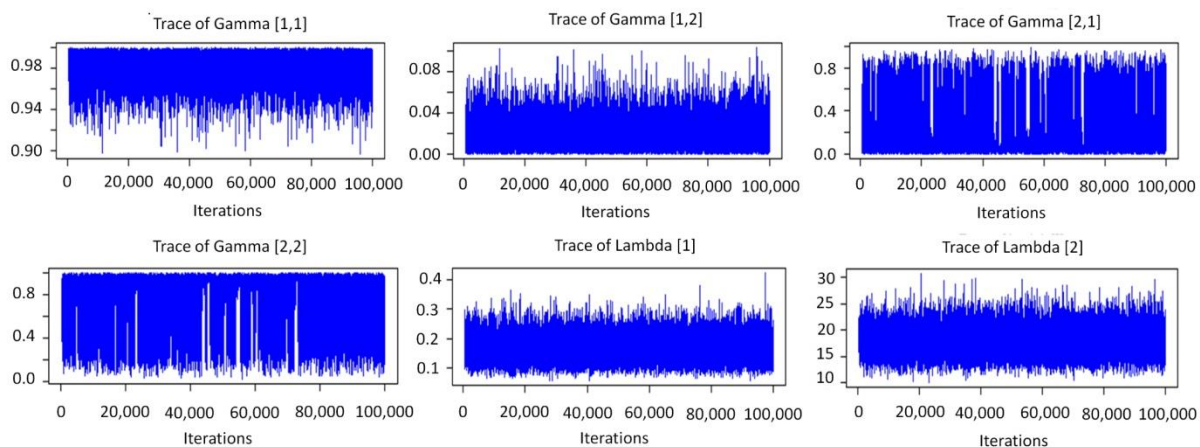


Figure 1. Trace Plots for Estimated Parameters Two-State PHMM with Bayesian

Based on Figure 1, it can be seen that the plot looks stable and does not show a certain pattern. As a result, it can be said that the estimated parameters are convergent. The output of posterior summary results with the Bayesian approach is shown in Table 1. Based on the credible interval value between the 2.5% to 97.5% percentile found in Table 1, the interval does not contain a value of 0. Therefore, it can be said that all estimated parameters in the analysis of two-state PHMM with the Bayesian approach are significant.

Table 1. Posterior Summary Results

Estimated Parameter	Mean	sd*	Credible Interval				
			2.50%	25%	50%	75%	97.50%
$\hat{\lambda}_1$	0.1670	0.0383	0.1002	0.1398	0.1640	0.1912	0.2495
$\hat{\lambda}_2$	18.0467	2.4211	13.6200	16.3500	17.9400	19.6000	23.1200
$\hat{\gamma}_{11}$	0.9836	0.0115	0.9544	0.9779	0.9862	0.9921	0.9980
$\hat{\gamma}_{12}$	0.0164	0.0115	0.002	0.0079	0.0138	0.0221	0.0456
$\hat{\gamma}_{21}$	0.2668	0.1944	0.0159	0.1071	0.2268	0.3903	0.7184
$\hat{\gamma}_{22}$	0.7332	0.1944	0.2816	0.6097	0.7731	0.8929	0.9841

*) sd: standard deviation

The conclusion of estimated parameters of two-state PHMM with the Bayesian approach is in Table 2.

Table 2. Estimated Parameters of Two-State PHMM Parameters with Bayesian

<i>Poisson Rates</i>	$\hat{\lambda} = [0.1670 \quad 18.0467]$
<i>Transition Probability Matrix</i>	$\hat{T} = \begin{bmatrix} 0.9836 & 0.0164 \\ 0.2668 & 0.7332 \end{bmatrix}$
<i>Stationer State Distribution</i>	$\hat{\delta} = [0.9421 \quad 0.0579]$

The estimated parameters of $\hat{\lambda}$ based on Table 2, shows that the average number of earthquake events caused by low seismicity activity rate during the period January 2009 to September 2018 with the magnitude scale $\geq M4.7$ and depth of <60 km is 1 event. While the average number of earthquake events caused by the high rate is approximately 18 events.

Based on the transition probability matrix $\hat{T}$, it can be said that if the level of seismic activity was low in the previous period, the probability of the seismic activity rate being low will still be higher in the next period (0.9836) compared to the high rate (0.0164). Vice versa, if the seismicity activity rate was high in the previous period, the probability of the occurrence of high seismicity activity rate will still be bigger in the next period (0.7332) than the low rate (0.2668).

Based on the stationary state distribution, the probability of occurrence of earthquakes caused by the low seismicity activity rate in the long run behavior (steady state) is greater (0.9421) than the high rate (0.0579). This means that of 117 earthquake events in NTB during the period January 2009 to September 2018 with magnitude scales $\geq M4.7$ and depths of <60 km, the number of earthquakes caused by low seismicity activity rate in the long run behavior (steady state) was 110 events, while the rest was occurred due to high seismicity activity rate. Even though there were 7 cases of high levels of seismic activity, the government still need to socialize the disaster mitigation information to the public around NTB in order to reduce the impact caused by the earthquake.

4.3. Accuracy Results of the Two-State PHMM

Based on the results of the estimated parameters, a series of estimated data is generated as shown in Figure 2.

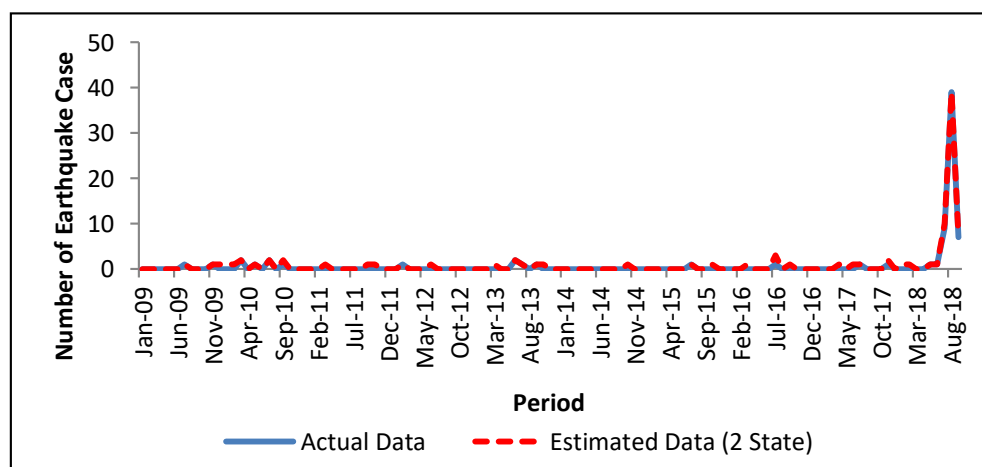


Figure 2. Comparison Plot between Actual Data and Estimated Data of Two-State PHMM.

Based on Figure 2, MAE value is 0.4079. This indicates that the earthquake case in NTB over the period from January 2009 to September 2018 with a magnitude scale $\geq M4.7$ and a depth of <60 km can be modeled using two-state PHMM analysis.

5. Conclusions

The results of estimated parameters of two-state PHMM using the Bayesian approach in the case of earthquakes in NTB during the period January 2009 to September 2018 with a magnitude scale $\geq$ M4.7 and a depth of <60 km are:

$$\hat{\lambda} = [0.1670 \quad 18.0467]; \hat{T} = \begin{bmatrix} 0.9836 & 0.0164 \\ 0.2668 & 0.7332 \end{bmatrix}; \hat{\delta} = [0.9421 \quad 0.0579]$$

Based on the results of estimated parameters, it can be said that the average number of earthquake events caused by the low seismicity activity rate is 1 event while the high rate is approximately 18 events. If the level of seismic activity in the previous period was low, then the occurrence probability of low seismic activity level in the next period (0.9836) will be bigger than the high rate (0.0164). At the same time, if the previous seismicity activity rate was high, the likelihood of high seismicity activity rate in the next period will be bigger (0.7332) than the low rate (0.2668). The probability of low seismicity activity rate in the long run behavior (steady state) in NTB is larger (0.9421) than the high rate (0.0579). Nonetheless, socialization of disaster mitigation towards communities around NTB still needs to be done in order to reduce the impact caused by the earthquake. The MAE in the analysis of two-state PHMM is 0.4079. This value is relatively small, therefore, it indicates that the case of an earthquake in NTB can be modeled using a two-state PHMM analysis.

This study only uses two-state in the PHMM analysis. In the next research, it is suggested to use the number of other states to observe the impact of state changes on the results of PHMM analysis

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