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Multi-path signal decomposition with white-noise and correlation analysis

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Abstract. Decomposition of composite multi-path signals is a complex problem. The windowing approach in the time domain cannot be used for overlapping signals. While the filtering approach in the frequency domain cannot separate the overlapping signal spectrum. One of the key solutions to this problem is to estimate the time-delay for each signal component. This study discusses techniques for separating multi-path signal components through time-delay estimation by analysing residual signal and its correlation with original signal. The residual signal is the error between the reference and the received signal. Overlapping multi-path signal components are detected in two different approaches. First, when the residual signal is random, the whiteness analysis is applied to detect the signal component. Second, when the whiteness test failed, which means the residual signal has a correlation with the reference signal, the correlation test can then be applied. The simulation results show that this proposed method successfully detected the signal components.

Keywords: time delay estimation, multi-path signal, whiteness test, correlation test

1. Introduction

Multipath or overlapping signal is commonly found in the medical, military, communication, sonar, radar and geophysical fields. It is a composite signal originating from a single signal source. When propagating towards the receiver, the signal does not only pass through one direct path but also passes through another path for several reasons: multiple targets, multilayer structure, and complex property from the medium path of propagation. The time-delay of multipath signal components is very important to know as they accurately separate signal component positions in time when received by the receiver. But this problem is not simple, especially if the multipath signal has been contaminated by noise signals which almost always appear in every single measurement.

Pulse-echo technique is a time-domain technique that is often used to determine the time-delay of individual signals. But this technique is problematic if the received signal consists of several overlapping signals. The overlapping signal phenomenon can occur in the time and frequency domain. Time-overlapping signals occur if time-delay between echoes is shorter than the duration of the source signal. In the frequency domain, difficulties arise when decomposing composite signals because the signal components have identical spectra. This difficulty is often encountered in the separation of audio harmonic signals with the same or similar frequencies. The correlation between frequency and phase of



individual signals is incoherent, causing constructive and destructive interference. As a result, the characteristics of the composite signal become irregular compared to the individual signals. The relative phase of the overlapping individual signals also affects the frequency determination precision and has a strong contribution to the composite signal amplitude. This results in unobserved individual signals.

Various techniques for decomposition of multipath signals have been proposed recently [1–7]. These methods are such as short-time Fourier transform, Wigner-Ville distribution, discrete wavelet transform, discrete cosine transform, chirp transform, and fractional Fourier transform. In this study we propose a different method than those that already exist. The proposed method assumes the multipath signal originates from a known signal source. The received signal is contaminated by measurement noise. The multipath signal is assumed to be a combination of source signals with different amplitude and time-delay. The main idea in this research is to utilize the residual signal (the signal error between the source signal and the multipath signal) to detect the time-delay of the component signal. This residual signal is the main source of information to detect the time-delay position of each component of the multipath signal. Multi-path signal components are detected in two different approaches. When the residual signal is random, the whiteness analysis is applied to detect the signal component. Alternatively, when the whiteness test failed, which means the residual signal has a correlation with the reference signal, the correlation test can then be applied. The correlation coefficient between the shifted source signal and the multipath signal is used to measure the strength of the correlation by which the time-delay of the component signal can be found.

2. Problem Statement

The multipath signal model is illustrated in Figure 1. In the ideal situation, the signal propagates in a direct path. In reality, however, the situation is not simple. The multipath model might be not valid for some environments. Receiver receives not only a direct-path signal but also multi delayed and attenuated copies of the source signal due to reflections from nearby walls and objects.

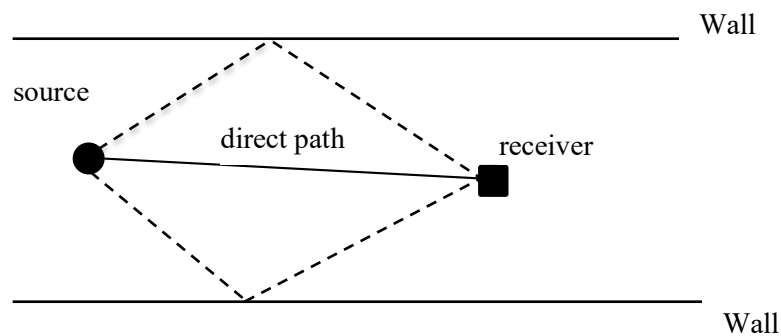


Figure 1. Multipath Signal Model

For an example, given the environment model as shown Figure 1, the source signal is shown in Figure 2a and the receiver will receive the multipath signal with three overlapping components plus noise as shown in Figure 2b. In general, the multipath signal received at the receiver can be represented mathematically as

$$r(t) = \sum_{i=1}^N c_i s(t - \tau_i) + n(t) \quad (1)$$

where $s(t)$ is the source signal, t is the time of propagation, N is the number of components, $n(t)$ is the noise signal, c_i is the amplitude, and τ_i is the time delay of the i -th signal component. The noise

signal is assumed stationary Gaussian which is uncorrelated with the source. The primary problem in this multipath model is to estimate all the time delays in the noisy signal $r(t)$.

In the case of noise-free signal, the transfer function can be easily found by taking the ratio between the multipath and transmitted signal in the frequency domain:

$$H(f) = \frac{R(f)}{S(f)} = \sum_{i=1}^n c_i e^{-i2\pi f \tau_i} \quad (2)$$

Taking the inverse transformation, we obtain a time domain function:

$$h(t) = \sum_{i=1}^n c_i \delta(t - \tau_i) \quad (3)$$

composed of several impulses $\delta(t)$ with the time delay τ_i . Using $h(t)$, the echo time delays are simply the location of the impulses which can be easily found.

Unfortunately, in practical application the above method is hard to apply given noisy signals.

In the case of noisy signals, the cross correlation may be used to estimate the time delay. However, it only works for the cases of single path signals or multipath but non-overlapping signals.

3. Proposed Method

In this preliminary research the proposed method of estimating the time delays in multipath signals is assumed to work in the discrete time domain. It also assumes the components overlap each other loosely but several samples remain not-overlapping between adjacent components as illustrated in Figure 2b. It detects the time delay of each echo in the time domain with the following steps:

- 1) Normalize the multipath signal with the same scale as the source signal
- 2) Overlay source signal on the normalized multipath signal
- 3) Move forward/shift source signal one sample to the right
- 4) Obtain the error signal (a.k.a. the residual signal) between the shifted source/reference and the normalized multipath signal
- 5) Windowing the residual signal and the normalized multipath signal with the same window size.
- 6) If the windowed residual is white, the time-delay is detected
- 7) If the windowed residual is not white, the correlation test is applied to the windowed reference signal and windowed multipath signal. The time-delay is detected when both signals are highly correlated with high correlation coefficient
- 8) After the time delay is found, the corresponding signal component is removed by replacing the multipath signal with the residual signal
- 9) Go to the 3rd step.

In this paper the 95% confidence interval is used in the whiteness test. The 95% confidence interval corresponds to the range of the residual values that have a 95% probability of being statistically insignificant. The correlation test uses a correlation coefficient R to measure the strength of the association or correlation between the reference signal and the multipath signal after windowed. The window size is carefully chosen not to exceed the distance between adjacent signal components.

4. Simulation Results and Discussion

The source signal in the simulation in this paper is generated by

$$s(t) = a^m(t) e^{-t/u} \cos(2\pi f t + \varphi) \quad (4)$$

where $m = 2.0$, $u = 5 \times 10^{-6}$, $f = 550 \times 10^3$ Hz, and $\varphi = 0$. The source signal (4) is sampled with the sampling frequency ten times the signal frequency f . The received multipath signal is simulated by convolving $s(t)$ with the delayed discrete impulses:

$$h(n) = \sum_{i=1}^3 c_i \delta(n - d_i) \quad (5)$$

where $d_1 = 200$, $d_2 = 300$, $d_3 = 400$ (all in sample unit), $c_1 = 1$, $c_2 = 0.6$, and $c_3 = 0.5$. The first component is normalized already with the same scale as the source signal. The second and the last components are attenuated by respectively 40% and 50%. In order to mimic the real situation, the noise signal is added the multipath signal. The resulting multipath signal has SNR of about 12 dB. Both the source and multipath signals are shown in Figure 2b.

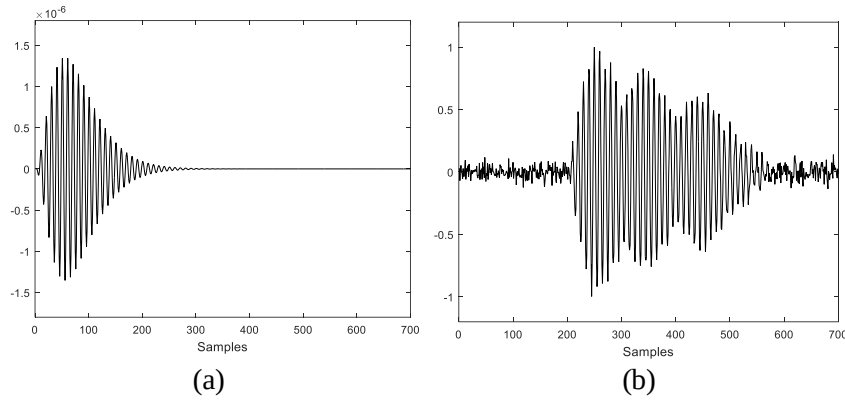


Figure 2. Signal generation: (a) source (b) normalized multipath signal

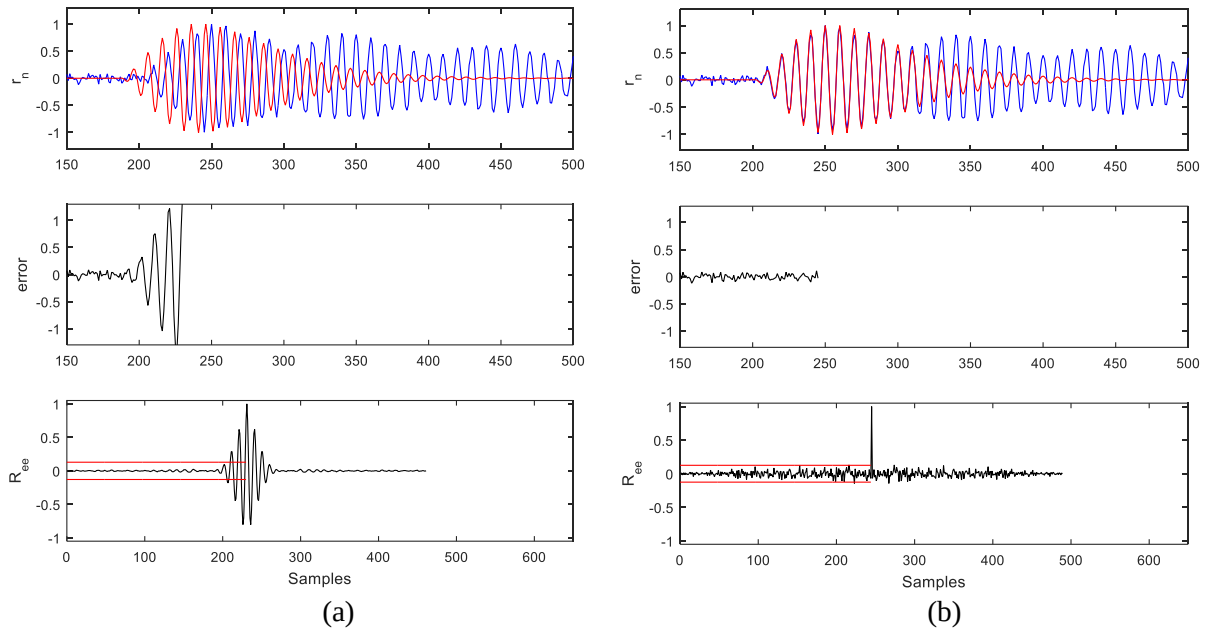


Figure 3. Time delay estimations of multipath signal at: (a) the 186th sample (b) the 200th sample

The time delay estimation of the first component is shown in Figure 3a. The first component of the multipath signal was delayed by 200 sampled, while the reference signal (red color) was shifted by 186 samples, resulting in large signal errors as shown in the second plot in Figure 3a. The signal error in the second plot is windowed with the size of the current sample number plus 45 (i.e., $186+45=231$ samples). The additional sample size of 45 was chosen by assuming that the distance between the time delays of the first and the second component is larger than 45 samples. In other words, the upper limit of the

window lies at the non-overlapping area between the first and the second component. The windowed signal error of 231 samples is to be analyzed with the whiteness test. The whiteness test failed as shown the last plot in Figure 3a where more than 5% of the points of the autocorrelation function were outside the 95% confidence interval (see two red parallel lines in the last plots). The correlation analysis between the reference signal and the multipath shows that both signals were very poorly correlated since it gave a low correlation coefficient $R = -0.744$. These analysis results suggest us to conclude that the sample number of 186 is not the time delay.

After shifted by 200 samples, the reference signal seems to match the exact position of the first component as shown in the top plot in Figure 3b. The resulting signal error shown in the second plot in Figure 3b shows the random signal property. The last plot in Figure 3b shows that the autocorrelation function exhibits the white property of the signal error where only 0.8% samples exceed the 95% confidence interval. The correlation test was also successful where the high correlation coefficient $R = 0.995$ was obtained.

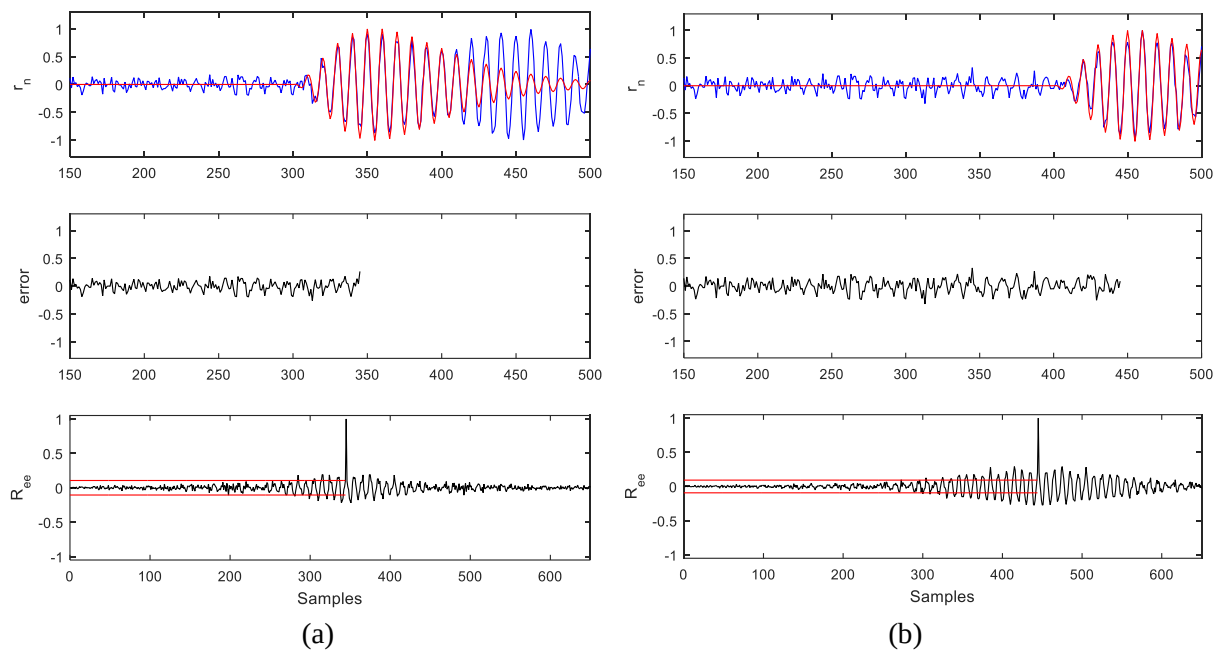


Figure 4. Time delay estimations of multipath signal at: (a) the 300th sample (b) the 400th sample

Figure 4 shows that at the 300th and 400th samples the reference signal seems to match the second and third component of signal. After windowing, there are about 350 samples of the signal errors in Figure 4a and about 450 samples in Figure 4b to be tested. In both cases the windowed signal errors are considered not *white* since there were less than 95% samples of their autocorrelation functions inside the 95% confidence interval as shown in the last plots of Figure 4. In more detail, there are less than 95% samples in Figure 4a and 80% samples in Figure 4b inside the 95% confidence interval. These results mean that the whiteness test failed.

Alternatively, since the reference signal seems to match the signal components at the 300th and 400th samples, it is reasonable to apply the correlation test to the reference signal and the multipath signal at both positions of samples. The correlation test gave the high coefficient correlation of $R = 0.974$ at the 300th sample and $R = 0.975$ at the 400th sample.

Despite the above good test results at the sample number 200, 300, and 400, it is still not sufficient to conclude that those sample positions correspond to the time delay to find. We need to ensure that at some sample positions before and after the sample number 200, 300 and 400 the coefficient correlations must be very low. This should be sufficient criteria.

Table 1 shows the analysis results at some samples around the true time delays in the multipath signal. As can be seen at Table 1 at the sample numbers 200, 300, and 400 (see the bold rows) the correlation coefficients are consistently high. Hence, we conclude that the multipath signal consists of three time delays, i.e., at the sample number 200, 300, and 400. The correlation test is more reliable than the whiteness test.

Table 1. Results of whiteness and correlation tests

Sample number	Outside 95% confidence interval (%)	Correlation coefficient R
186	9.1	-0.744
198	9.5	0.298
199	9.1	0.812
200	0.8	0.995
201	9.8	-0.438
202	9.8	-0.191
298	7.0	0.259
299	8.5	0.777
300	6.1	0.974
301	12.5	-0.399
399	19.9	0.800
400	20.3	0.975

5. Conclusions

The approach for decomposing the multipath signal in discrete time domain with the whiteness test and the correlation test has been presented. The whiteness test was successful to find the time delay of the first component but failed for the remaining ones. Whereas the correlation test was successful to find all the time delays as the correlation coefficients were consistently high at the true sample positions of the time delay of the multipath signal. The correlation test is more reliable than the whiteness test.

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