Buckling loads for sandwich structures having FRP faces and FGM core

To cite this article: A Muc *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **544** 012040

IOP Conf. Series: Materials Science and Engineering **544** (2019) 012040 doi:10.1088/1757-899X/544/1/012040

Buckling loads for sandwich structures having FRP faces and FGM core

A Muc[1], P Kędziora[1] and M Chwał[1]

[1]Institute of Machine Design, Faculty of Mechanical Engineering, Cracow University of Technology, Kraków, Poland
E-mail: olekmuc@mech.pk.edu.pl

Abstract. The aim of the present paper is to present a consistent 2D formulation of sandwiches (CS – classical sandwiches and FGMs – functionally gradient materials) that is accurate enough for a prediction of thermo-mechanical buckling load. Various kinematical hypothesis and variational formulations are studied in order to compare them with 3D finite element modelling and to estimate the effectiveness of 2D approximations. The examples deal with the compressed rectangular plates and shallow cylindrical panels. The effects of boundary conditions on both values of thermal and mechanical buckling loads as well on the variations of bifurcation loads with fibre orientations are discussed in details.

1. Introduction

Classical sandwich (CS) composites constitute a special group of structures used extensively in composite applications. They consist of thin facings made of high strength materials (e.g. FRP) and of a core made of thick and lightweight materials. The motivation for such sandwich structures elements is well-known. In the last decade a new class of sandwich structures has been developed. They are called as functionally gradient materials (FGMs) and distribute the material functions throughout the material body to achieve the maximum heat resistance and mechanical properties. Both types of structures (classical sandwiches and FGM sandwiches) are characterized in the similar manner, i.e. as structures having different properties in the thickness z direction in the form proposed by Qian et al. [1]:

$$p(z)=(p_t-p_b)V(z)+p_b \tag{1}$$

where p denotes a generic material property like modulus, p_t and p_b denote the corresponding properties of the top and bottom faces of the plate, respectively, and V(z) is a function. Here, we assume that moduli E and G, the coefficient of thermal expansion α, and thermal conductivity k vary according to Eqn (1) where the function V(z) is defined in the following way:

$$V(z)=\begin{cases}\sum_{i=1}^{N}\left[1(z-t_i)-1(z-t_{i-1})\right]\text{for CS}\\ \left(\dfrac{z}{t}+\dfrac{1}{2}\right)^n \quad z\in[-t/2,t/2]\ \text{for FGM}\end{cases} \tag{2}$$

Other definitions of the function V(z) can be found in Ref [2] for the grading density of foam core for CS. The first two propositions take into account the use of relations (1) and (2 – for FGM) for degradation in a different direction than thickness. First density grading in the width direction is as follows:

$$\rho(x) = \rho_l - \frac{x(\rho_l - \rho_t)}{w} \tag{3}$$

where ρ_l and ρ_t are the mass densities of closed-cell foam of the leading edge and trailing edge, respectively; w denotes the width of the panel. Second density grading in the length direction is as follows:

$$\rho(x) = \rho_r - \frac{x(\rho_r - \rho_t)}{L} \tag{4}$$

where ρ_r and ρ_t are the mass densities of closed-cell foam of the root edge and tip edge, respectively; L denotes the length of the panel. Next definition of density diagonal grading across the xy plane (in two directions) is as follows:

$$\rho(x, y) = \frac{L - x}{L}\left(\frac{w - y}{w}\rho_u + \frac{y}{w}\rho_l\right) + \frac{x}{L}\left(\frac{w - y}{w}\rho_l + \frac{y}{w}\rho_u\right) \tag{5}$$

where ρ_u and ρ_l are the mass densities of closed-cell foam of upper and lower bounds, respectively. A model for charactering the mechanical properties of FGMs with the regular polygonal cross-section is also developed by introducing the power-law rule in Ref [3].

However, modelling of their mechanical behaviour still leads to many problems, particularly in the description of 1D or 2D structures, such as beams, plates or shells (see e.g. [4,5]). The broader discussion of those problems and solutions can be found in Muc et al. [6-10].

The aim of the present paper is to present a consistent 2D formulation of sandwiches (CS and FGMs) that is accurate enough for a prediction of thermo-mechanical buckling load. Various kinematical hypothesis and variational formulations are studied in order to compare them with 3D finite element modelling and to estimate the effectiveness of 2D approximations – see Muc et al. [6,9]. The examples deals with the compressed rectangular plates and shallow cylindrical panels. The effects of boundary conditions on both values of thermal and mechanical buckling loads as well on the variations of bifurcation loads with fibre orientations are discussed in details.

2. Formulation of the problem

In this paper we intend to propose an approach based on the symbolic computations – the use of the Mathematica package. Shear deformation effects are taken into account for different variants of laminate wise and layer wise 2D shell theories. The order of the theory is completely free but it should be fundamental kinematical relations for 3D structures are simplified to 2D equations via assumptions in the displacement field. The structure continuum vector $u_i(x,y,z)$ might be referred to the mid-surface 2D relation by using the following series expansion:

$$u_i^{(k)}\left(x^{(k)} y^{(k)} z^{(k)}\right) = U_i^{(k)}\left(x^{(k)} y^{(k)}\right) + \sum_{r=1}^{P} V_{ri}^{(k)}\left(x^{(k)} y^{(k)}\right)\Phi_r\left(z^{(k)}\right) \tag{6}$$

Using the above relation and the continuity conditions at the top and the bottom of the layers different theories may be formulate, i.e.: the Love-Kirchhoff theory, the first-order transverse shear deformation theory (FSDT) and higher-order transverse shear deformation theories (HSDT).

The formulation is based on the functional of total potential energy in the Lagrange form. An arbitrary form of 2D sandwich structure can be analysed via the required form of the Lame parameters. Then with the aid of the Rayleigh-Ritz formulation buckling load factors can be derived for an arbitrary sandwich structure. The theoretical formulation of this problem is discussed in Ref [11]. Some

IOP Conf. Series: Materials Science and Engineering **544** (2019) 012040 doi:10.1088/1757-899X/544/1/012040

examples are presented in order to demonstrate the performance of such theories in the buckling analysis if sandwich plates, panels and shells.

3. Results and discussions

For compressed rectangular sandwich plates having FRP faces buckling mechanical loads are commonly dependent on fibre orientations.

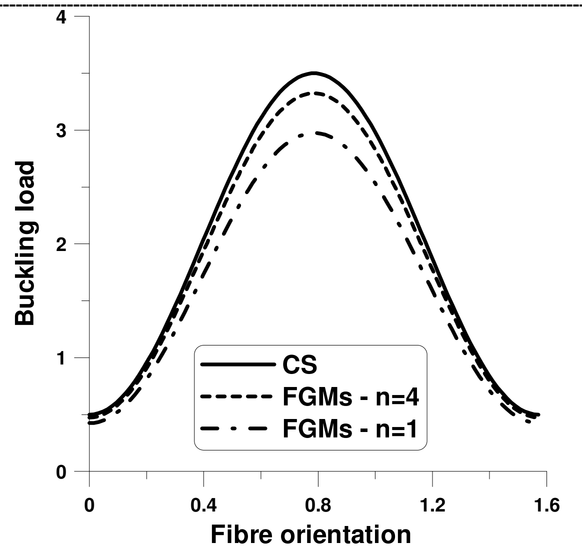

Figure 1. Buckling loads for a compressed square plate – the Love-Kirchhoff theory.

However assuming that the core mechanical properties are derived from relation (1), (2) bifurcation buckling loads are also the function of the parameter n. As it may be seen in Fig.1 the difference between buckling loads for CS and FGM sandwiches decreases as the parameter n grows.

Thermal buckling of the bifurcation type occurs in structures subjected to steadily increasing temperature field. Figure 2 is a plot of buckling loads for combined thermal and mechanical in-plane compressed loads. N'=N/N_0 and N_0 the buckling load value for the laminate subjected to in-plane loads (without thermal loads). As it may be seen the similar situation as previously occurs. With the increasing value of the parameter n the difference between the buckling curves for FGMs and CS disappear.

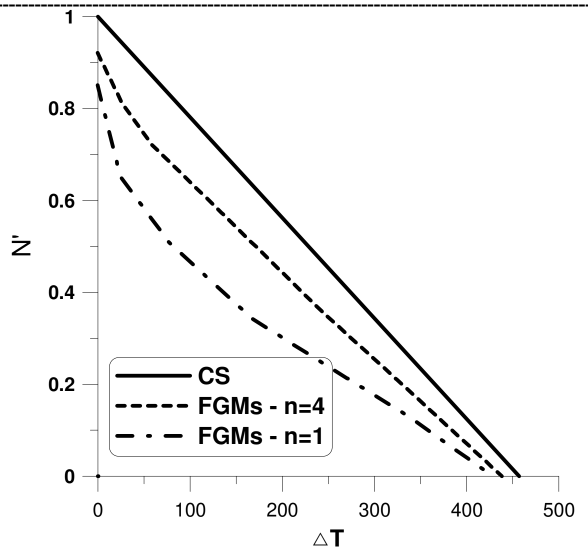

Figure 2. Combined buckling loads.

Figures 3 and 4 demonstrate the influence of the shell theory on buckling mechanical loads. As it may be observed, the growth in the accuracy in 2-D description of plates/shells results in the decrease of buckling loads. It is mainly caused by the core properties. However, as the compressibility of a normal to the plate mid-surface is taken into account (the 2-D approximation called as 11p) the location and distributions of buckling loads have been changed significantly, especially due to large deformations of the core.

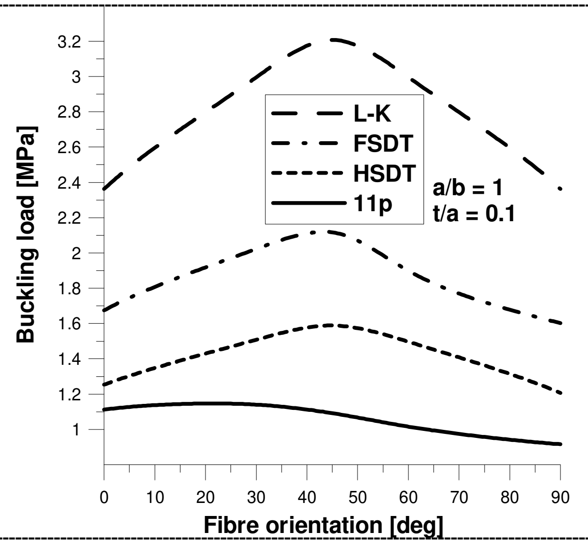

Figure 3. Comparison of buckling loads for different variants of plate theory – square plates (CS).

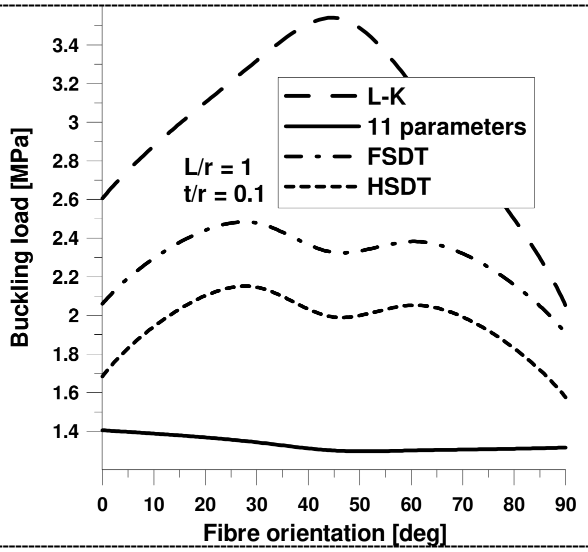

Figure 4. Comparison of buckling loads for different variants of plate theory – cylindrical shells (CS).

The type of boundary conditions affects both values of mechanical buckling loads as well as the variations of bifurcation loads with fibre orientations – see Fig.5. Similar effects have been observed to those for isotropic shells. For clamped edges buckling loads increase comparing with those obtained for simply supported plate/shell structures.

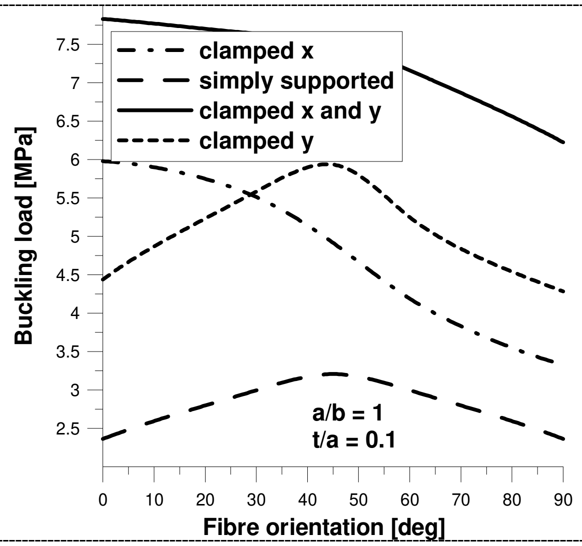

Figure 5. Variations of buckling loads for square plates having different boundary condition (CS).

4. References

[1] Qian L F, Batra R C and Chen L M 2004 *Compos: Part B* **35** 685–97
[2] Peng Jin and Xiaoping Zhong 2016 *Int J Aerospace Eng* **2016** ID 7971435
[3] Xiao-Jian Xu and Jun-Miao Meng 2018 *Compos: Part B* **145** 70–80
[4] Reddy J N and Chin C D 1998 *J. Therm Stresses* **21** 593-626
[5] Shiota I and Miyamoto Y 1997 *Functionally Graded Materials*, ed I Shiota and Y Miyamoto (Elsevier) p 137-41
[6] Muc A, Stawiarski A and Romanowicz P 2018 *Appl. Comp. Mater.* **25** 177-189
[7] Muc A Janczy S and Nogowczyk R. 2007 *16th Int.Conf. on Composite Materials* (Kyoto)
[8] Muc A and Zuchara P 1999 *Archives of Civil Engineering* **45** 289-304
[9] Muc A and Zuchara P 1999 *Comp. Struct.* **48** 145-50
[10] Muc A and Zuchara P 1998 *Mech Compos Mater* **34** 203-11
[11] Muc A 2003 *Mechanics of Fibrous Composites* (Kraków: Księgarnia Akademicka) (in Polish)