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Mathematical models of magnetic circuits of sensors of functional diagnostic systems of electric carriers

K K Juraeva and J S Fayzullayev

Tashkent institute of railway engineering, Tashkent, Uzbekistan

E-mail: lade00@bk.ru

Abstract: The article discusses the magnetic circuit of the developed magnetoelastic sensors of mechanical forces used in the systems of functional diagnostics of electric locomotives is investigated, taking into account the distribution of the magnetic resistance of coaxially arranged ring cores, the magnetic capacitance of the ring gap between them, the section of magnetizing and measuring windings. It is shown that the magnetic voltage along the sensor magnetic circuit is nonlinearly distributed and changes its sign at the magnetic neutral point, and the magnetic flux is non-constant and has a minimum value at the magnetic neutral point, and with an increase in the magnetic flux attenuation coefficient in the magnetic conductor magnetic flux along the length of the magnetic circuit increases.

In diagnostic systems of electric locomotives, electromagnetic sensors are widely used to obtain measurement information about electrical (currents, voltages), magnetic (voltages and inductions) and non-electric (displacements, speed, forces, moments, vibration parameters, pressure, temperature, etc.) values [1, 2]. The metrological characteristics of these sensors are mainly determined by the state of the magnetic field in their working gaps.

Numerous studies, for example, [3, 4, 5], are devoted to the study and calculation of the magnetic circuits of electromagnetic sensors and transducers of electrical and non-electrical quantities. At the same time, magnetic systems of newly developed sensors have some peculiarities. In particular, at the Tashkent Institute of Railway Engineers, new designs of magnetoelastic mechanical force sensors have been developed [6, 7]. The peculiarities of the magnetic circuits of these sensors lie in the fact that not only the magnetic resistance of coaxially arranged ring cores and the magnetic capacitance of the ring gap between them, but also the corresponding sections of the magnetizing and measuring windings are distributed in them.

This article investigated the magnetic circuit of the developed force sensors, taking into account the distribution of all the above parameters. The definition of the expressions of magnetic flux and magnetic voltage generated by the distributed sections of the magnetization winding will be performed separately for each section.

Based on the Kirchhoff laws, we make differential equations for magnetic flux and voltage generated by both sections of a distributed winding for an elementary section of a magnetic circuit $d\alpha$ (figure 1 and 2):

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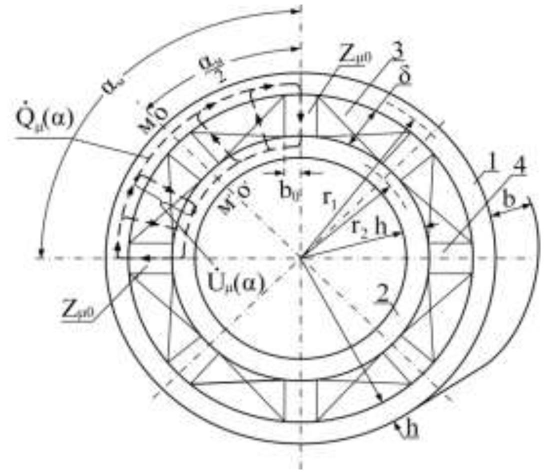

Figure 1. Magnetic circuit of the developed magnetoelastic force sensor.

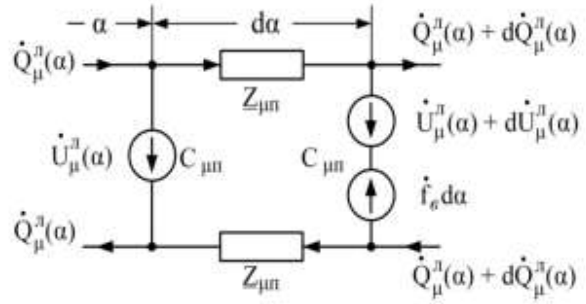

Figure 2. Elementary magnetic circuit with distributed sections of the magnetization winding.

$$\dot{Q}_{\mu}(\alpha)-\dot{U}_{\mu}(\alpha)C_{\mu п}d\alpha-\dot{Q}_{\mu}(\alpha)-d\dot{Q}_{\mu}(\alpha)=0,$$

or

$$\frac{d\dot{Q}_{\mu}(\alpha)}{d\alpha}=-\dot{U}_{\mu}(\alpha)C_{\mu п}, \quad (1)$$

$$-\dot{U}_{\mu}(\alpha)+\underline{Z}_{\mu п}\dot{Q}_{\mu}(\alpha)+\dot{U}_{\mu}(\alpha)+d\dot{U}_{\mu}(\alpha)+\underline{Z}_{\mu п}\dot{Q}_{\mu}(\alpha)=-\dot{f}_{g}d\alpha$$

or

$$\frac{d\dot{U}_{\mu}}{d\alpha}=-\left[2\underline{Z}_{\mu п}\dot{Q}_{\mu}(\alpha)+\dot{f}_{g}\right], \quad (2)$$

where $\dot{Q}_{\mu}(\alpha)$ and $\dot{U}_{\mu}(\alpha)$ are respectively the complex values of the magnetic flux in the concentric ferromagnetic cores and the magnetic voltage between them, created by MDS $\dot{F}_{в}$ in both sections of the magnetization winding; $\underline{Z}_{\mu п 1}=\frac{1}{\mu_{б.п.}\mu_0 h_c b_1}$ and $\underline{Z}_{\mu п 2}=\frac{1}{\mu_{б.п.}\mu_0 h_c b_2}$ are the linear values of the complex magnetic resistances of concentric ferromagnetic cores 1 and 2 per unit angle of the magnetic circuit. For convenience of calculation, we take $\underline{Z}_{\mu п 1}=\underline{Z}_{\mu п 2}$ and therefore $\underline{Z}_{\mu п 1}+\underline{Z}_{\mu п 2}=2\underline{Z}_{\mu п}$; $C_{\mu п}=\mu_0\frac{h_c}{\delta}$ is the linear value of the magnetic capacitance (magnetic conductivity by the classical analogy of electric and magnetic circuits) of the annular gap δ between coaxial concentric ferromagnetic cores 1 and 2; $f_g=const$ - the linear value of the magnetizing force of a uniformly distributed section of the winding per unit angle. Geometrical dimensions required for the calculation of the magnetic circuit are shown in figure 1.

The general solution of differential equations (1) and (2) is as follows:

$$\dot{U}_{\mu}(\alpha)=\dot{A}_1 e^{\underline{\gamma}\alpha}+\dot{A}_2 e^{-\underline{\gamma}\alpha}, \quad (3)$$

and the expression for the magnetic flux is found from equation (1) as:

$$\dot{Q}_\mu\ (\alpha) = -\frac{1}{2\underline{Z}_{\mu п}}\frac{d\dot{U}_\mu\ (\alpha)}{d\alpha} - \frac{\dot{f}_g}{2\underline{Z}_{\mu п}} = -\frac{\gamma}{2\underline{Z}_{\mu п}}\dot{A}_1^{л} e^{\underline{\gamma}\alpha} +$$
$$+\frac{\gamma}{2\underline{Z}_{\mu п}}\dot{A}_2\ e^{-\underline{\gamma}\alpha} - \frac{\dot{f}_g}{2\underline{Z}_{\mu п}}, \tag{4}$$

where $\underline{\gamma} = \sqrt{2\underline{Z}_{\mu п} C_{\mu п}}$ is the complex value of the coefficient of propagation of the magnetic flux through the magnetic circuit, $1/deg$; $\underline{A}_1$ and $\underline{A}_2$ are the integration constants.

The partial solutions of equations (3) and (4) will be obtained by determining the integration constants $\underline{A}_1$, and $\underline{A}_2$ for the following boundary conditions:

$$\left.\begin{aligned} \dot{U}_\mu(\alpha)|_{\alpha=0} = \dot{F}_в - \dot{Q}_\mu(\alpha)|_{\alpha=0}\underline{Z}_{\mu 0}, \\ -\dot{U}_\mu(\alpha)|_{\alpha=\alpha_м} = \dot{F}_в - \dot{Q}_\mu(\alpha)|_{\alpha=\alpha_м}\underline{Z}_{\mu 0}. \end{aligned}\right\} \tag{5}$$

Substituting in (3) the values of $\dot{U}_\mu(\alpha)$ and $\dot{Q}_\mu(\alpha)$ according to (5), respectively, at $\alpha = 0, \alpha = \alpha_м$ and solving the system of equations thus obtained, we find the following values $\underline{A}_1$ and $\underline{A}_2$:

$$\underline{A}_1 = -\frac{\dot{F}_в}{2\underline{\Delta}} e^{-\underline{\gamma}\alpha_м} + \dot{F}_b \frac{\underline{\gamma}\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}\underline{\Delta}} e^{-\underline{\gamma}\alpha_м} - \dot{f}_в \frac{\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}\underline{\Delta}} e^{-\underline{\gamma}\alpha_м}$$
$$+\dot{f}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}^2}{8\underline{Z}_{\mu п}^2\underline{\Delta}} e^{-\underline{\gamma}\alpha_м} - \frac{\dot{F}_в}{2\underline{\Delta}} - \dot{F}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}\underline{\Delta}} - \dot{f}_в \frac{\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}\underline{\Delta}} - \dot{f}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}^2}{8\underline{Z}_{\mu п}^2\underline{\Delta}} \tag{6}$$

$$\underline{A}_2 = \frac{\dot{F}_в}{2\underline{\Delta}} + \dot{f}_в \frac{\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}\underline{\Delta}} - \dot{F}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}\underline{\Delta}} - \dot{f}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}^2}{8\underline{Z}_{\mu п}^2\underline{\Delta}} + \frac{\dot{F}_b}{2\underline{\Delta}} e^{\underline{\gamma}\alpha_м} +$$
$$+\dot{f}_в \frac{\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}\underline{\Delta}} e^{\underline{\gamma}\alpha_м} + \dot{F}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}\underline{\Delta}} e^{\underline{\gamma}\alpha_м} + \dot{f}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}^2}{8\underline{Z}_{\mu п}^2\underline{\Delta}} e^{-\underline{\gamma}\alpha_м}, \tag{7}$$

where $\underline{\Delta} = \left(1 + \frac{\underline{\gamma}^2\underline{Z}_{\mu 0}^2}{4\underline{Z}_{\mu п}^2}\right) sh\left(\underline{\gamma}\alpha_м\right) + \frac{\underline{\gamma}\underline{Z}_{\mu 0}}{\underline{Z}_{\mu п}} ch\left(\underline{\gamma}\alpha_м\right)$.

Substituting the found values of $\underline{A}_1$ and $\underline{A}_2$ into (3) and (4) we finally get:

$$\dot{U}_\mu(\alpha) = \frac{\dot{F}_в}{\underline{\Delta}}\left\{sh\left[\underline{\gamma}(\alpha_м - \alpha)\right] - sh(\underline{\gamma}\alpha)\right\} + \dot{F}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}}{2\underline{Z}_{\mu п}\underline{\Delta}}\left\{ch\left[\underline{\gamma}(\alpha_м - \alpha)\right] -\right.$$
$$\left.-ch(\underline{\gamma}\alpha)\right\} + \dot{f}_в \frac{\underline{Z}_{\mu 0}}{2\underline{Z}_{\mu п}\cdot 2\underline{\Delta}}\left\{sh\left[\underline{\gamma}(\alpha_м - \alpha)\right] - sh(\underline{\gamma}\alpha)\right\} +$$
$$+\dot{f}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}^2}{4\underline{Z}_{\mu п}^2\underline{\Delta}}\left\{ch\left[\underline{\gamma}(\alpha_м - \alpha)\right] - ch(\underline{\gamma}\alpha)\right\}, \tag{8}$$

$$\dot{Q}_\mu(\alpha) = \frac{\underline{\gamma}\dot{F}_в}{2\underline{Z}_{\mu п}\underline{\Delta}}\left\{ch\left[\underline{\gamma}(\alpha_м - \alpha)\right] + ch\left(\underline{\gamma}\alpha\right)\right\} + \dot{F}_в \frac{\underline{\gamma}^2\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}^2\underline{\Delta}}\left\{sh\left[\underline{\gamma}\ (\alpha_м -\right.\right.$$
$$\left.\left.-\alpha)\right] + sh\left(\underline{\gamma}\alpha\right)\right\} + \dot{f}_в \frac{\underline{\gamma}\underline{Z}_{\mu 0}}{4\underline{Z}_{\mu п}^2\underline{\Delta}}\left\{ch\left[\underline{\gamma}(\alpha_м - \alpha)\right] + ch\left(\underline{\gamma}\alpha\right)\right\} +$$
$$+\dot{f}_в \frac{\underline{\gamma}^2\underline{Z}_{\mu 0}^2}{8\underline{Z}_{\mu п}^3\underline{\Delta}}\left\{sh\left[\underline{\gamma}(\alpha_м - \alpha)\right] + sh\left(\underline{\gamma}\alpha\right)\right\} - \frac{\dot{f}_в}{2\underline{Z}_{\mu п}}. \tag{9}$$

It should be noted that if the distribution of the sections of the magnetizing winding is not taken into account, i.e. when $\dot{f}_в = 0$, equations (8) and (9) passes to equations for magnetic circuits with concentrated

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sections of the magnetizing winding [8].

Transforming (8) and (9) using hyperbolic trigonometry, we obtain the following expressions:

$$\dot{U}_{\mu}(\alpha^*) = \left(2\frac{\dot{F}_{\text{в}}}{\underline{\Delta}^*} + \dot{f}_{\text{в}}\frac{\alpha_{\text{м}}\underline{k}_0}{\underline{\Delta}^*}\right)\left[ch\left(\frac{1}{2}\beta\right) + \underline{\beta}\underline{k}_0 sh\left(\frac{1}{2}\beta\right)\right] sh\left[\underline{\beta}\left(\frac{1}{2} - \alpha^*\right)\right], \tag{10}$$

$$\dot{Q}_{\mu}(\alpha^*) = \frac{\underline{\beta}}{2\underline{Z}_{\mu\text{п}}\alpha_{\text{м}}}\left(2\frac{\dot{F}_{\text{в}}}{\underline{\Delta}^*} + \dot{f}_{\text{в}}\frac{\alpha_{\text{м}}\underline{k}_0}{\underline{\Delta}^*}\right)\left[ch\left(\frac{1}{2}\beta\right) + \right.$$
$$\left. + \underline{\beta}\underline{k}_0 sh\left(\frac{1}{2}\beta\right)\right] ch\left[\underline{\beta}\left(\frac{1}{2} - \alpha^*\right)\right] - \frac{\dot{f}_{\text{в}}}{2\underline{Z}_{\mu\text{п}}}, \tag{11}$$

where $\underline{\Delta}^* = \left(1 + \underline{\beta}^2\underline{k}_0^2\right) sh\underline{\beta} + \underline{\beta}\underline{k}_0 ch$; $\underline{\gamma}\alpha_{\text{м}} = \underline{\beta}$; $\underline{k}_0 = \frac{Z_{\mu 0}}{2\underline{Z}_{\mu\text{п}}\alpha_{\text{м}}}$; $\alpha^* = \frac{\alpha}{\alpha_{\text{м}}}$.

Figure 3 shows the curves of the dependence $U_{\mu}^* = f(\alpha^*)$ (a) and $Q_{\mu}^* = f(\alpha^*)$ (b) for different values of β.

Analysis of equations (10), (11) and their curves (figure 3) shows that the magnetic stresses along the magnetic circuit, respectively, with concentrated and distributed magnetizing forces are nonlinearly distributed and change their signs at the magnetic neutral point, and the corresponding magnetic fluxes are non-constant and have the minimum value at the point of the magnetic neutral, and with an increase in the attenuation coefficient of the magnetic flux, the degree of nonlinearity of the distribution.

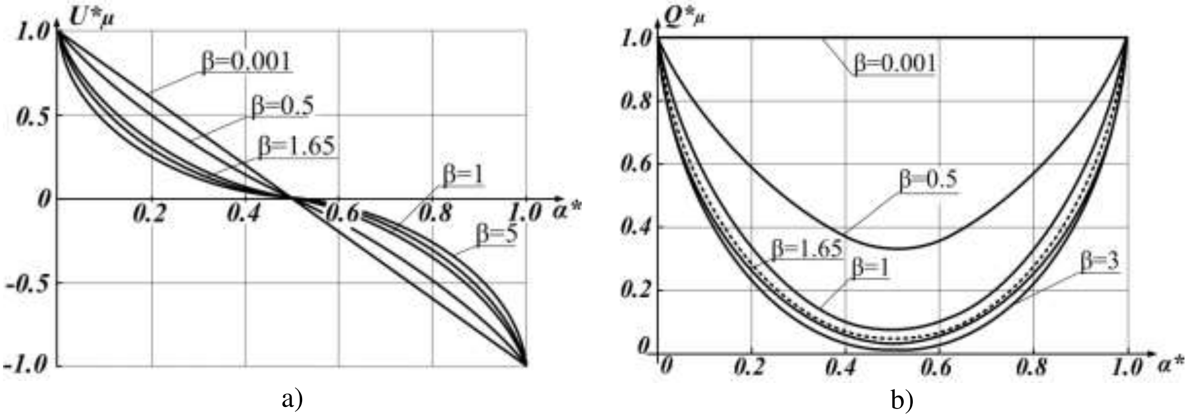

Figure.3. The curves $U_{\mu}^* = f(\alpha^*)$ (a) and $Q_{\mu}^* = f(\alpha^*)$ (b) for different values of magnetic flux attenuation β: solid curves are calculated, and dashed lines are experimental data magnetic voltage and the variability of the magnetic flux along the length of the magnetic circuit increases.

A comparative analysis of the calculated and experimental curves of $U_{\mu}^* = f(\alpha^*)$ and $Q_{\mu}^* = f(\alpha^*)$ magnetic circuits, taking into account the distribution of magnetizing forces, showed that taking into account the distribution of the magnetizing forces of the excitation winding sections significantly (up to 20%) reduces error calculation of magnetic circuits.

Expressions (10) and (11) are mathematical models of magnetic circuits developed by magnetoelastic force sensors with distributed magnetizing forces.

Thus, the study of magnetic circuits developed sensors for diagnostic systems of electric rolling stock of railways developed their mathematical models taking into account the distribution of the magnetic resistance of coaxially arranged ring cores, the magnetic capacitance of the ring gap between them, the section of the magnetizing and measuring windings. It is shown that the magnetic voltage along the sensor magnetic circuit is nonlinearly distributed and changes its sign at the magnetic neutral point, and the magnetic flux is non-constant and has a minimum value at the magnetic neutral point, and with an increase in the magnetic flux attenuation coefficient in the magnetic conductor magnetic flux along the length of the magnetic circuit increases.

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