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Accounting of externalities in the development of environmental engineering methods

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Abstract. This paper is on the problem of effective business management related to the segment of long-term participants of various business activities, comprehensively interacting with the social and business environment. The study considers the periodicity of most economic processes, which has a complex characteristic. The optimization criterion was a functional, which included an assessment of the internalization of the impact of the results of such activity. As a result, a mathematical model was obtained, which serves as the basis for expert systems aimed at accounting of externalities of a wide range of nationwide, institutional, business and social activity.

1. Introduction

The modern economy on a nationwide scale reflects not only the systemic interaction of enterprises linked by common interests and obligations. A deeper approach shows that the most difficult to formalize [1] are the effects of externalities that accompany most types of activity in a developed modern society. Such consequences of economic activity must be considered when planning for a sufficiently long time horizon.

Note that both positive and negative externalities refer not only to the impact on the environment or to the conditions of economic activity of other participants in the economic pool [2]. The social impact of the damage is much larger, and it needs to be assessed in order to include it in the system of economic indicators of a particular entity [3].

The most important feature of any kind of activity is the market uncertainty [4], which can be considered using the mathematical tools technique of the theory of stochastic processes. In addition, the seasonality factor [5] should be considered. This makes it possible to reflect the uneven demand for goods and services, for company and transport work load, and for the environment. Thus, it is necessary to involve mathematical modeling with the use of periodic functions.

When producing a mathematical model, the balance [6] between the additional load on the enterprise, i.e. on the internalization, and profitability indicators of its activities should be found. The same applies to the non-profit segment of state structures, when social damage and aggregate social and economic efficiency are assessed.

When solving the problem, the authors used methods based on the use of the theory of calculus of variations, differential equations, mathematical games and search for optimal control.

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2. The main formalisms

When describing processes within the framework of the implementation of a mathematical model that allows applying the methods of finding optimal solutions, we introduce several formalisms. Let us adopt the following notations:

$\Lambda(t)$ is the dependence of the load on resources; t - time; $\lambda_i(t)$ (where $i = 1,2,...$) is the periodic component of the decomposition $\Lambda(t)$; T is a period that can generally be taken equal to a calendar year. Then we can write the expansion of the assumed function in a Fourier series [7] in the following form:

$$\Lambda(t) = \Lambda_0 + \sum_{i=1}^{+\infty} \Lambda_i Cos(2\pi \frac{i}{T} t + \psi_k) = \sum_{i=1}^{+\infty} \lambda_i(t),$$

which gives the dependence of the mathematical expectation [8] in separable functions.

Next, we derive differential equations by analogy with the Erlang system. To do this, we introduce: $\nu_i(t)$ - periodic functions reflecting the capabilities of shared resources; $E_k(t)$ - system states; probabilities corresponding to them: $p_i(t) = \varphi_i(t) + \alpha_i(t)$, where functions $\varphi_i(t)$ are periodic with $T/2\pi i$ period; $\lim\limits_{t\to\infty} \alpha_i(t) = 0$ for all possible i, and $0 \le p_i(t) \le 1$. The sense of the functions $\varphi_i(t)$ consists in stationary probabilities of the values $p_i(t)$. To solve the problem, we make the sequence:

$$p'_{0i}(t) = -\lambda_i(t) p_{0i}(t) + \nu_i(t) p_{1i}(t),\ p'_{1i}(t) = -\nu_i(t) p_{1i}(t) + \lambda_i(t) p_{0i}(t)$$

and add the condition of the complete group: $\sum_i p_i(t) = 1$. Here, after conversion to the form:

$p'_{0i}(t) = -\left[\lambda_i(t) + \nu_i(t)\right] p_{0i}(t) + \nu_i(t)$, and having defined the initial conditions:

$p_{0i}(0) = \alpha_i \le 1$, $p_{1i}(0) = 1 - \alpha_i \le 1$ for any i in the range of legal values,

we obtain a system of differential equations [9], integrated in quadratures.

3. Formula derivation

Let us perform the necessary transformations and substitutions. Then, the corresponding solution has the following form:

$$p_{0i}(t) = \exp\left\{-\int_0^t \left[\lambda_i(x) + \nu_i(x)\right] dx\right\} \left[\alpha_i + \int_0^t \nu_i(x) \exp\left\{-\int_0^t \left[\lambda_i(x) + \nu_i(x)\right] dx\right\} dx\right] \quad (1)$$

It should be noted that the ability to analytically express the dependence $p_i(t)$ fundamentally changes the algorithm [10] itself, which underlies expert decision-making systems. Since the functions $\lambda_i(t)$ and $\nu_i(t)$ are periodic, we introduce the $\varphi_i(t)$ substitution of the form: $\varphi_i(t) = \int_0^t \left[\lambda_i(x) + \nu_i(x)\right] dx$, which will make it possible to integrate the system. Furthermore, for the analysis of periodic processes, we also denote k satisfying the inequality: $kT \le T < (k+1)T$.

This allows us to obtain the resulting expression [11] for calculating the function $\varphi_i(t)$: $\varphi_i(t) = ka_i + \int_0^\tau \left[\lambda_i(z) + \nu_i(z)\right] dz$. Note that the following notation is accepted:

$a_i = \int_0^T \left[\lambda_i(x) + \nu_i(x)\right] dx$, $\tau = t - kT$. Similarly, to derive the final solution [12], which reflects the dynamics of a periodic process, it is necessary to express the functionals, which enter into the equation. Thus, the substitution of the periodic integral expansion

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$$\int_0^t v_i(x)\exp[\varphi_i(x)]dx = \sum_{l=1}^{k} \int_{(l-1)T}^{lT} v_i(x)\exp[\varphi_i(x)]dx + \int_{kT}^{t} v_i(x)\exp[\varphi_i(x)]dx$$

gives the desired result for $p_{0i}(t)$ - found probabilities in the following form, which is convenient for implementation in practical applications:

$$p_{0i}(t) = \exp[-\varphi_i(\tau)]\left[\frac{b_i}{\exp(a_i)-1} + \int_0^\tau v_i(x)\exp[-\varphi_i(x)]dx\right] + \exp[-ka_i - \varphi_i(\tau)]\left[\alpha_i - \frac{b_i}{\exp(a)-1}\right]$$

at the same time, the coefficient b_i means the value: $b_i = \int_0^T v_i(x)\exp[\varphi_i(x)]dx$.

A multidimensional set of the analytically expressed dependencies $p_{0i}(t)$ serves as the basis for the creation of expert systems, where processes are of a periodic nature of a complex form.

The result of the obtained analytical solution is the formula of convergence of the elementary components (1) of a nonlinear periodic process. The accuracy of modeling is determined only by the degree of expansion [13] of any depth.

4. Calculation

Let us calculate the internalization and form a mathematical model. Consider the operating of N enterprises that have externalities as a negative external effect. Reduction of the effect of externalities can be achieved by additional costs. Then $\overline{P} = (p_1, p_2, ..., p_N)$ is the optimal equilibrium solution search vector. The search is performed inside the N-dimensional situation cube [14]. For illustration purposes, consider Table 1 for $N = 3$.

Table 1. Calculation of costs of enterprises and the model of externalities.

Situation	Enterprise 1 (Costs)	Enterprise 2 (Costs)	Enterprise 3 (Costs)	Model: Vector
0,0,0	ω	ω	ω	$(1-p_1)(1-p_2)(1-p_3)$
0,0,1	ω	ω	0	$(1-p_1)(1-p_2)p_3$
0,1,0	ω	0	ω	$(1-p_1)p_2(1-p_3)$
0,1,1	$\psi^* + \omega$	ψ^*	ψ^*	$(1-p_1)p_2p_3$
1,0,0	0	ω	ω	$p_1(1-p_2)(1-p_3)$
1,0,1	ψ	$\psi + \omega$	ψ	$p_1(1-p_2)p_3$
1,1,0	ψ	ψ	$\psi + \omega$	$p_1p_2(1-p_3)$
1,1,1	ψ^*	ψ^*	ψ^*	$p_1p_2p_3$

In table 1, the amount of extra costs ω that enterprises incur when choosing a strategy related to reducing damage from externalities is given for calculation of losses. We also considered the difference in the size of enterprises [15] as the difference of the actual damage ψ^* and ψ.

Let us introduce the notation: $\overline{p}_i = 1 - p_i$, $\forall i$, and multiply the following vector:

$$(\overline{p}_1\overline{p}_2\overline{p}_3, \overline{p}_1\overline{p}_2p_3, \overline{p}_1p_2\overline{p}_3, \overline{p}_1p_2p_3, p_1\overline{p}_2\overline{p}_3, p_1\overline{p}_2p_3, p_1p_2\overline{p}_3, p_1p_2p_3)$$

and the corresponding vectors of Table 1 together term by term.

Applying the rule, for each enterprise, we obtain two inequalities corresponding to the condition of market balance. For the first one, we write down the correspondence to the lower bound:

$$-\omega(1-p_1)(1-p_2)(1-p_3) - \omega(1-p_1)(1-p_2)p_3 - \omega(1-p_1)p_2(1-p_3) -$$

$$(\psi^* + \omega)(1-p_1)p_2p_3 - \psi p_1(1-p_2)p_3 - \psi p_1p_2(1-p_3) - \psi^* p_1p_2p_3 \geq$$
$$\geq -\omega(1-p_2)(1-p_3) - \omega p_2(1-p_3) - \omega(1-p_2)p_3 - (\psi^* + \omega)p_2p_3.$$

We also obtain the inequality reflecting the upper bound:

$$-\omega(1-p_1)(1-p_2)(1-p_3) - \omega(1-p_1)(1-p_2)p_3 - \omega(1-p_1)p_2(1-p_3) -$$
$$(\psi^* + \omega)(1-p_1)p_2p_3 - \psi p_1(1-p_2)p_3 - \psi p_1p_2(1-p_3) - \psi^* p_1p_2p_3 \geq$$
$$\geq -\psi p_2(1-p_3) - \psi(1-p_2)p_3 - \psi^* p_2p_3.$$

After the conversion, we obtain their final form:

$$\left.\begin{array}{r} \omega p_1 - \psi p_1p_2 - \psi p_1p_3 + 2\psi p_1p_2p_3 \geq 0 \\ -\omega(1-p_1) + \psi(p_2+p_3)(1-p_1) - 2\psi p_2p_3(1-p_1) \geq 0 \end{array}\right\} \quad (2)$$

Similarly, we rewrite the equations for the other participants of the activity accompanied by externalities. They can be combined into the system:

$$\left.\begin{array}{r} \omega p_2 - \psi^* p_2p_3 - \psi p_1p_2 + 2\psi p_1p_2p_3 \geq 0 \\ \omega(1-p_2) - \psi^*(1-p_2)p_3 - \psi p_1(1-p_2) + 2\psi p_1(1-p_2)p_3 \geq 0 \\ \omega p_3 - \psi^* p_2p_3 - \psi p_1p_3 + 2\psi p_1p_2p_3 \geq 0 \\ \omega(1-p_3) - \psi^*(1-p_3)p_2 - \psi p_1(1-p_3) + 2\psi p_1(1-p_3)p_2 \geq 0 \end{array}\right\} \quad (3)$$

The feasible control set is inside the multidimensional situation cube (figure 1).

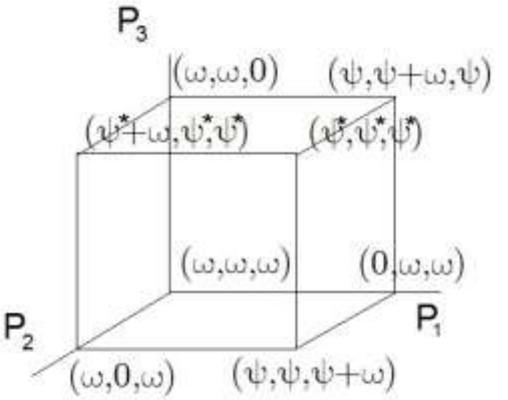

Figure 1. The feasible solution set.

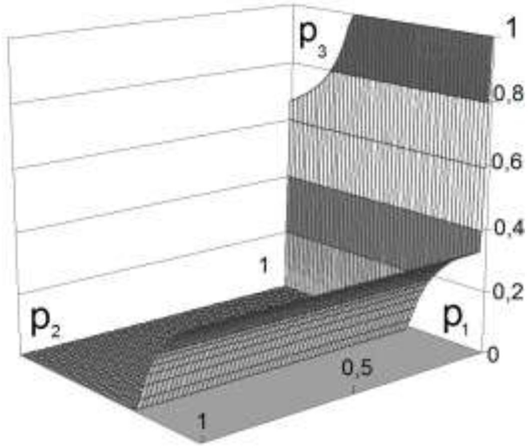

Figure 2. Solution boundary zone.

Figure 1 shows it for the case when $N = 3$. Economic indicators are marked at the vertices.

Solving the jointly obtained equations, we obtain regions $\{p_1, p_2, p_3\}$ in multidimensional space. These regions contain a set of Nash equilibrium situations. The form of inequalities lets us conclude that the solutions are at the intersection of hyperbolic surfaces and portions of planes. Figure 2 shows the modeling results for the first enterprise. The equations are the hyperbolic surfaces [16] with several intersection points, which give the desired solution. In addition to the trivial case $(\overline{p}_1\,\overline{p}_2\,\overline{p}_3) = 0$, we obtain two more solution vectors that give stable [17] states.

The second and third vectors of value of equilibrium for strategies have the following components:

$$\left\{ \frac{\omega - \psi\dfrac{\psi + \sqrt{\psi^2 - 2\omega\psi}}{2\psi} + 2\psi\left[\dfrac{\psi + \sqrt{\psi^2 - 2\omega\psi}}{2\psi}\right]^2}{\psi\left[\left(\psi + \sqrt{\psi^2 - 2\omega\psi}\right)/\,2\psi\right]^2}, \frac{\psi + \sqrt{\psi^2 - 2\omega\psi}}{2\psi}, \frac{\psi + \sqrt{\psi^2 - 2\omega\psi}}{2\psi} \right\}$$

$$\left\{ \frac{\omega - \psi \dfrac{\psi - \sqrt{\psi^2 - 2\omega\psi}}{2\psi} + 2\psi \left[\dfrac{\psi - \sqrt{\psi^2 - 2\omega\psi}}{2\psi} \right]^2}{\psi \left[\left(\psi - \sqrt{\psi^2 - 2\omega\psi} \right) / 2\psi \right]^2}, \frac{\psi - \sqrt{\psi^2 - 2\omega\psi}}{2\psi}, \frac{\psi - \sqrt{\psi^2 - 2\omega\psi}}{2\psi} \right\}$$

5. Application of results

Each of the three solutions obtained reflects different situations. The trivial solution corresponds to the dispositive method of legal regulation. It reflects the equilibrium [18] in the absence of participants' costs for the transformation of external effects into internal ones. Solution 2 corresponds to the presence of restrictive measures in the form of regulatory norms, which are, however, not as radical as solution 3 is. The applied administrative regulators are economic in their nature [19], they prescribe certain behavior to economic entities, and this affects the state of the economic system. The stated principles are easily scaled for any number of participants, as well as for arbitrarily complex dependencies that reflect economic indicators.

6. Conclusions

The stated principles, summarized in a mathematical model, can be used as the basis for making a number of managerial decisions. Currently, when determining the feasibility of emerging externality internalizing, management structures or administration bodies use heuristic methods [20] not based on a verified, scientifically based calculation. The amounts of expenses that must be borne by various members of the business community are assigned in the same way. At the same time, there is no dynamic analysis and search for optimal solutions. The reason is the complexity of considering a variety of factors, and the lack of correct theoretical models [21], which take into account economic indicators. The results obtained in this paper, first of all the mathematical model, the developed solution, can serve as the basis for expert system. Automatic processing of statistics and economic accounts allow to integrate this system into the concept of a digital city. Linking results to economic indicators is part of making informed management decisions.

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